A solvable two-body Dirac equation in one space dimension

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A solvable Hamiltonian for two Dirac particles interacting by instantaneous linear potentials in (1 + 1) dimensions is discussed. The system presents no Klein paradox even if the coupling is rather strong, so particles remain bound. The four independent components of the wave function describing the system resemble the nonrelativistic oscillator eigenfunctions. Although the Hamiltonian is not fully covariant, the effective frequency of the oscillator obeys a typical relativistic Doppler law. In contrast to the nonrelativistic treatment, eigenstates are intrinsically coupled with the overall translational motion of the system.

Une hamiltonienne soluble pour deux particules de Dirac interagissant par des potentiels linéaires instantanés en (1 + 1) dimensions, est discutée. Le système ne présente pas de paradoxe de Klein même si le couplage est plutôt fort, de sorte que les particules demeurent liées. Les quatre composantes indépendantes de la fonction d'onde décrivant le système ressemblent aux fonctions propres de l'oscillateur non relativiste. Bien que l'hamiltonienne soit pas pleinement covariante, la fréquence effective de l'oscillateur obéit à une loi Doppler relativiste typique. Contrairement au traitement non relativiste, les états propres sont couplés intrinsèquement avec le mouvement de translation de l'ensemble du système.

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1. Introduction

There exist a number of potentials depending linearly on spatial coordinates for which the one-body Dirac equation is exactly solvable (1). Such potentials appear in the study of the motion of particles in uniform electric and magnetic fields and more recently in the description of the dynamics of quarks. Since linear electrostatic potentials present no binding of particles, Ito *et al.* (2) and Cook (3) introduced a new type of linear interaction to explain baryon spectra. The resulting relativistic equation can be solved exactly. Recently, Moshinsky and Szczepaniak (4) gave this problem the name Dirac oscillator. These authors have argued that the one-body Dirac equation representing a relativistic oscillator should be linear in both coordinates and momenta, as the Schrödinger equation for the standard oscillator is quadratic in both coordinates and momenta. They found the eigenstates and eigenvalues in a fairly straightforward fashion, since the equations satisfied by the components of the Dirac wave function turn out to be the Schrödinger equation for the standard oscillator plus a strong spin-orbit coupling (this term is absent in (1 + 1) dimensions). Hence, the Dirac oscillator equation presents the binding of particles and could be an alternative way to describe the confinement of quarks (5). The electromagnetic potential associated with this interaction has been found by Benitez *et al.* (6).

Ravndal (7) has considered a different way to obtain oscillator-type solutions for relativistic wave functions. This author has introduced an equal admixture of vector plus scalar quadratic potentials in the one-body Dirac equation, which then reduces to a Schrödinger-like harmonic oscillator equation. Nevertheless the basic wave equation of Ravndal's model is less symmetric than that of the Dirac oscillator, in the sense that it is linear in momenta but quadratic in coordinates.

The two above mentioned relativistic oscillator models (4, 7) use the one-body Dirac equation as a starting point. For two interacting Dirac particles, however, the applicability of these models is only justified whenever one of the particles is much heavier than the other. In general situations, when the masses of particles are comparable or just equal, recoil effects become important, especially in a relativistic regime. Hence a relativistic two-body equation must be used. The corresponding two-body problem was first discussed by Cook (3), in both (1 + 1) and (3 + 1) dimensions. He assumed that the dynamics of the system was described by the Bethe–Salpeter equation. Moshinsky *et al.* (8) generalized the Dirac-oscillator Hamiltonian in (3 + 1) dimensions to a two-body system with equal masses and whose center of mass is at rest. The aim of this work is to introduce a solvable model for two Dirac particles with arbitrary masses in (1 + 1) dimensions, which resembles the harmonic oscillator coupling in the nonrelativistic limit. The basic equation we use is the two-body Dirac equation as defined by Glöckle *et al.* (9). The two particles are considered to be at equal times, so the equation is not manifestly covariant. In fact, for interactions depending upon the spatial separation between particles, covariance is only guaranteed if the interaction is δ-shaped, because only a mathematical point has a relativistically invariant shape. In spite of this simplification, we start with that equation to construct a two-body Dirac-oscillator model. The degree of noncovariance of the composite system will also be examined.

We found it most appropriate to deal with linear interactions rather than quadratic interactions. We have not been able to reduce the two-body Dirac equation for an equal mixture of vector and scalar quadratic potentials to a Schrödinger-like oscillator equation, unless the coupling is weak. Fortunately, the situation is more favourable in the case of potentials depending linearly on the distance between the particles. The two-body Dirac equation we consider approaches the results found by Moshinsky and Szczepaniak (4), if one of the constituent particles has infinite mass. We will focus mainly, however, on the case of two particles with equal masses. A detailed analysis of the binding energy will allow us to discuss some interesting features of the composite system, which are not found in the single-particle treatment.

2. General equations

We follow the notation of Glöckle *et al.* (9) in studying the Dirac equation in one space dimension for two interacting particles of masses m_1 and m_2. The basic equation describing the steady states of the system reads as follows:

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$$[2.1] \quad H\Psi = E\Psi$$

Here the Hamiltonian has the form

$$[2.2] \quad H = \sum_{k=1}^{2} H_k + V_{\mathrm{int}}$$

H_k being the one-particle Dirac Hamiltonian defined as

$$[2.3] \quad H_k = \alpha_k p_k + \beta_k m_k, \qquad k = 1, 2$$

and the interaction potential V_{int}, which will be specified below, is assumed to be dependent only upon the relative coordinate $x_1 - x_2$. Space coordinates and momenta satisfy the usual relations

$$[x_k, p_l] = i\delta_{kl}$$

The 2×2 Dirac matrices α and β anticommute and are traceless Hermitians with square unity. We choose the representation

$$[2.4] \quad \alpha = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

For an interaction depending on $x_1 - x_2$, it is appropriate to make a canonical transformation to total and relative coordinates and momenta. Thus we introduce

$$X = \mu_1 x_1 + \mu_2 x_2$$

$$P = p_1 + p_2$$

$$x = x_1 - x_2$$

and

$$p = \mu_2 p_1 - \mu_1 p_2$$

where

$$\mu_k = \frac{m_k}{(m_1 + m_2)}$$

To obtain the specific form of the interaction potential V_{int} we replace p_1 and p_2 by $p_1 - im_1\gamma_1\omega\beta_1(x_1 - x_2)$ and $p_2 - im_2\gamma_2\omega\beta_2(x_2 - x_1)$, respectively, in the free Hamiltonian $H_1 + H_2$. Therefore, the interaction potential is found to be

$$V_{\mathrm{int}}(x) = -i\omega(\gamma_1 m_1 \alpha_1 \beta_1 - \gamma_2 m_2 \alpha_2 \beta_2)x$$

which is linear in the relative coordinate x. ω denotes the oscillator frequency and the γ's are dimensionless quantities depending on the particle masses, such that γ_1 and γ_2 approach 1 and 0, respectively, as m_2 becomes very large (i.e., one particle much heavier than the other). This choice ensures the existence of a one-body Dirac oscillator equation, as defined by Moshinsky and Szczepaniak (4), for the lighter particle in that limiting case. Hence we can choose $\gamma_1 = \mu_2$ and $\gamma_2 = \mu_1$.

Taking into account the above prescriptions, the form of the two-body Hamiltonian [2.2] becomes

$$[2.5] \quad H = (\mu_1\alpha_1 + \mu_2\alpha_2)P + (\alpha_1 - \alpha_2)p + \beta_1 m_1 + \beta_2 m_2 - i\mu\omega(\alpha_1\beta_1 - \alpha_2\beta_2)x$$

where $\mu = m_1 m_2/(m_1 + m_2)$ is the reduced mass of the system. Note that the Hamiltonian obtained is Hermitian and independent of the total coordinate X. Since $[P, H] = 0$, the momentum P of the center of mass (cm) is a constant of motion. In addition, the Hamiltonian is invariant under the interchange of particles, $H(1, 2) = H(2, 1)$.

In solving [2.1] we expand Ψ in terms of the eigenvectors of the operator $\beta_1\beta_2$.

$$[2.6] \quad \Psi(x) = \psi_{++}(x)\chi_{++} + \psi_{+-}(x)\chi_{+-} + \psi_{-+}(x)\chi_{-+} + \psi_{--}(x)\chi_{--}$$

where

$$\beta_1\beta_2\chi_{++} = +\chi_{++}$$
$$\beta_1\beta_2\chi_{+-} = -\chi_{+-}$$
$$\beta_1\beta_2\chi_{-+} = -\chi_{-+}$$
$$\beta_1\beta_2\chi_{--} = +\chi_{--}$$

The system may be described by four independent components. According to Glöckle *et al.* (9), it is convenient to use combinations of $\psi_{\pm\pm}$

$$[2.7] \quad \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{pmatrix} \exp(iPX) = \frac{1}{\sqrt{2}} \begin{pmatrix} \psi_{++} + \psi_{--} \\ \psi_{++} - \psi_{--} \\ \psi_{-+} + \psi_{+-} \\ \psi_{-+} - \psi_{+-} \end{pmatrix}$$

P being the eigenvalue of the momentum of the cm. We find the following four coupled equations

$$[2.8a] \quad P\varphi_3 + (m_1 + m_2)\varphi_2 - E\varphi_1 + 2i\mu\omega x\varphi_4 = 0$$

$$[2.8b] \quad [(\mu_1 - \mu_2)P + 2p]\varphi_4 + (m_1 + m_2)\varphi_1 - E\varphi_2 = 0$$

$$[2.8c] \quad P\varphi_1 - (m_1 - m_2)\varphi_4 - E\varphi_3 = 0$$

$$[2.8d] \quad -[(\mu_1 - \mu_2)P + 2p]\varphi_2 + (m_1 - m_2)\varphi_3 + E\varphi_4 + 2i\mu\omega x\varphi_1 = 0$$

This set of equations describes the motion of two particles of arbitrary masses m_1 and m_2 under the action of the interaction given above. At this point, we do not solve it explicitly. However, it is an easy matter, although laborious, to check that φ_4 satisfies a Schrödinger-like oscillator equation, with an effective frequency depending on the particle masses, the energy, and the momentum P. Hence, the resulting states are certainly bound, no matter how small ω is. The composite system is free from the Klein paradox; unlike the case of particles interacting through Lorentz-vector linear potentials (the time component of a four vector) the *leakage* of particles to infinity is avoided. To our understanding, this result is one of the main virtues of the linear interaction we have introduced.

3. Reduction to one-body Dirac oscillator

To get insight into the problem, we first consider the limiting case in which one of the masses, say m_2, becomes very large. Denoting $E_1 = E - m_2$ the total energy of the light particle and then taking the limit $m_2 \to \infty$, [2.8] reduce in the cm frame ($P = 0$) to

$$[3.1a]\quad \varphi_1 = \varphi_2$$

$$[3.1b]\quad \varphi_3 = \varphi_4$$

$$[3.1c]\quad (p_1 + im_1\omega\, x_1)\varphi_4 + (m_1 - E_1)\varphi_2 = 0$$

$$[3.1d]\quad (p_1 - im_1\omega\, x_1)\varphi_2 - (m_1 + E_1)\varphi_4 = 0$$

where we have set $x_2 = 0$. Thus from the definition of the φ's,

$$\psi_{++} = \sqrt{2}\varphi_2$$

$$\psi_{-+} = \sqrt{2}\varphi_4$$

$$\psi_{+-} = \psi_{--} = 0$$

Using the last two equations, one can write in a more closed notation

$$[3.2]\quad [\alpha_1(p_1 - im_1\omega x_1\beta_1) + \beta_1 m_1 - E_1]\begin{pmatrix}\psi_{++}\\ \psi_{-+}\end{pmatrix} = 0$$

This is nothing but the Dirac oscillator equation for the light particle as proposed by Moshinsky and Szczepaniak (4). Note that it may be directly obtained from the one-body free Dirac equation replacing p_1 by $p_1 - im_1\omega\, x_1\beta_1$. The whole system is described by just two independent components, as usual for a single Dirac particle in (1 + 1) dimensions.

Equation [3.2] is easily decoupled to give

$$[3.3a]\quad \psi_{-+}(x_1) = \frac{1}{E_1 + m_1}(p_1 - im_1\omega x_1)\,\psi_{++}(x_1)$$

and

$$[3.3b]\quad (p_1^2 + m_1^2\omega^2 x_1^2)\,\psi_{++}(x_1) = (E_1^2 - m_1^2 + m_1\omega)\,\psi_{++}(x_1)$$

The upper component ψ_{++} satisfies the standard harmonic oscillator equation, whose solutions are readily found. Using the normalization condition

$$\int dx_1\,(|\psi_{++}(x_1)|^2 + |\psi_{-+}(x_1)|^2) = 1$$

we get

$$[3.4a]\quad \psi_{++}(x_1) = N_n H_n(x_1\sqrt{m_1\omega})\exp\left(-\frac{1}{2}m_1\omega\, x_1^2\right)$$

and with the aid of [3.3a]

$$[3.4b]\quad \psi_{-+}(x_1) = -2inN_n\frac{(m_1\omega)^{1/2}}{E_1 + m_1} \times \mathrm{H}_{n-1}(x_1\sqrt{m_1\omega})\exp\left(-\frac{1}{2}m_1\,\omega\, x_1^2\right)$$

where n is a nonnegative integer. The normalization constant N_n is given by

$$[3.5]\quad N_n^{-2} = 2^n n!\left(\frac{\pi}{m_1\omega}\right)^{1/2}\left[1 + \frac{2nm_1\omega}{(E_1 + m_1)^2}\right]$$

The energy of the particle is quantized according to

$$[3.6]\quad \begin{aligned} E_1 &= +m_1, & n &= 0\\ E_1 &= \pm(m_1^2 + 2m_1\omega\, n)^{1/2}, & n &= 1, 2\ldots \end{aligned}$$

Two values (positive and negative) of the energy are possible for each value of ω and $n \geqslant 1$, so this potential can bind particles as well as antiparticles. The fact that bound states actually appear in pairs (except for the ground state) also occurs for the Dirac equation with Lorentz scalar interactions (10). Note that ψ_{-+} (ψ_{++}) vanishes in the nonrelativistic limit $E \gtrsim m$ ($E \lesssim -m$) for weak coupling $m_1 \gg \omega$, and ψ_{++} (ψ_{-+}) agrees with the usual Schrödinger eigenfunction.

The energy of the light particle depends on the mass m_1; however, this situation is quite different from the results found for the Dirac equation with linear scalar interactions. In the former case E_1 reaches the value m_1 as the interaction is adiabatically switched off, while in the latter case the energy of the particle vanishes (11).

The lowest lying energy levels of the particle ($E_1 \geqslant m_1$) can be approximately given by $E_1 - m_1 = \omega\, n$ in the weak-coupling limit, with $n = 0, 1, \ldots$ The ground-state energy is shifted downward by $\omega/2$ in relation to the standard oscillator, but the spacing between levels is the same. At high energy, however, the energy levels rise as the square root of the frequency, and the spacing of levels becomes dependent on the quantum number n.

4. Two particles with equal masses

In this section we discuss the case of two interacting particles with equal masses $m_1 = m_2 \equiv m$. Hence [2] leads to the following set of equations

$$[4.1a]\quad \varphi_1 = \frac{4\,m}{E^2L^2(P) - 4\,m^2}\left(p + i\frac{E\,\omega\,x}{4}\right)\varphi_4$$

$$[4.1b]\quad \varphi_3 = \left(\frac{P}{E}\right)\varphi_1$$

$$[4.1c]\quad \varphi_2 = \frac{2EL^2(P)}{E^2L^2(P) - 4\,m^2}\left(p + i\frac{m^2\,\omega}{E\,L^2(P)}x\right)\varphi_4$$

and

$$[4.1d]\quad [p^2 + m^2\omega^2(P)x^2]\varphi_4 = \frac{1}{L^2(P)} \times \left(\frac{E^2L^2(P)}{4} - m^2 - m^2\frac{\omega}{E}\right)\varphi_4$$

where $\omega(P) \equiv \omega/2L(P)$ is the effective frequency and $L(P) \equiv (1 - P^2/E^2)^{1/2}$ is the Lorentz-contraction factor. We obtain a standard oscillator equation for the component φ_4. The effective frequency $\omega(P)$ of the oscillator depends on the energy and momentum of the cm and it transforms as $\omega(P) = \omega(0)/L(P)$, where $\omega(0) = \omega/2$. Consequently, a change in the effective frequency does take place according to the relativistic Doppler law, so $\omega(P)$ is in fact covariant despite the noncovariance of the instantaneous two-body Dirac equation.

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From [4.1d] one can easily find φ_4, and the other three components can be directly calculated from φ_4 using [4.1a]–[4.1c]. In the cm frame, the components of the eigenfunction are written as

$$[4.2a] \quad \psi_{++}(x) = N_n H_{n+1}\, [x\sqrt{m\omega(0)}]\, e^{-m\omega(0)x^2/2}$$

$$[4.2b] \quad \psi_{-+}(x) = -iN_n \frac{E-2m}{2\sqrt{m\omega(0)}} H_n\, [x\sqrt{m\omega(0)}]\, e^{-m\omega(0)x^2/2}$$

$$[4.2c] \quad \psi_{+-}(x) = +iN_n \frac{E-2m}{2\sqrt{m\omega(0)}} H_n\, [x\sqrt{m\omega(0)}]\, e^{-m\omega(0)x^2/2}$$

$$[4.2d] \quad \psi_{--}(x) = 2nN_n \left(\frac{E-2m}{E+2m}\right) H_{n-1} \times [x\sqrt{m\omega(0)}]\, e^{-m\omega(0)x^2/2}$$

and using the normalization condition

$$\int \mathrm{d}x\, [|\psi_{++}(x)|^2 + |\psi_{-+}(x)|^2 + |\psi_{+-}(x)|^2 + |\psi_{--}(x)|^2] = 1$$

the normalization constant is found to be

$$[4.3] \quad N_n^{-2} = 2^{n+1} n! \left(\frac{\pi}{m\omega(0)}\right)^{1/2} \times \left[n + 1 + \frac{(E-2m)^2}{4m\omega(0)} + n\left(\frac{E-2m}{E+2m}\right)^2\right]$$

The energy levels are given through the expression

$$[4.4] \quad E_n^2(P)\, L^2(P) = 4m^2\left[1 + \frac{\omega}{E_n(P)}\right] + 4m\omega\left(n + \frac{1}{2}\right) L(P)$$

n being a nonnegative integer. since the right-hand side of [4.4] is not Lorentz invariant, we come to the conclusion that the cm motion affects the internal structure of the system. Energy levels change when the system is boosted, but not according to any Lorentz transform. Nevertheless, if momentum P is much smaller than the rest mass of the constituent particles, we get

$$[4.5] \quad E_n(P)\, L(P) \cong E_n(0)\left[1 + \mathrm{O}\left(\frac{\omega}{m}\right)\right]$$

and the system approaches covariance (i.e., $E(P)$ transform according to the Lorentz law) in the weak-coupling limit. In the relativistic regime for strong coupling $E \gg 2m \gg \omega$ the covariance of the system is broken. This is because the particles

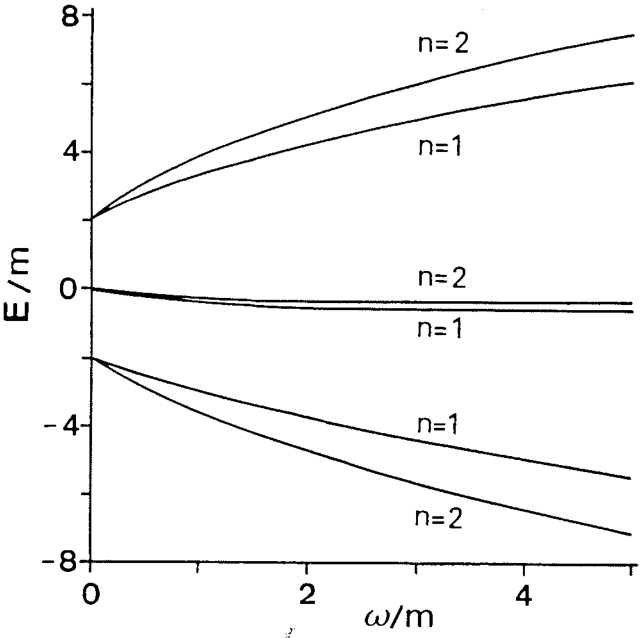

FIG. 1. Energy levels ($n \geq 1$) as a function of the oscillator frequency for particle–particle (upper lines), particle–antiparticle (middle lines), and antiparticle–antiparticle (lower lines) systems.

were initially considered to be at equal times, so a nonzero-ranged potential makes it impossible to construct a Lorentz-boost operator (9).

In the cm frame ($P = 0$), [4.4] reduces to a polynomial of degree 3 in the variable E_n. Therefore, for a fixed n there exist three real solutions. For $n = 0$, however, only two of them are physically acceptable solutions. The remaining one, which is found to be $E = -2m$, irrespective of ω, leads to a vanishing wave function. For $n \geq 1$ the three solutions are valid (Fig. 1). Following Coutinho *et al.* (12), we classify the bound states of the composite system as normal and abnormal. The energy of normal bound states approaches $\pm 2m$ in the vanishing interaction limit, whereas for abnormal solutions the energy reaches zero in that limiting situation. In view of the Dirac hole theory, one should consider normal solutions to represent bound states of particle–particle and antiparticle–antiparticle systems, while abnormal solutions would describe particle–antiparticle pairs. Accordingly, the energy of the composite system when the interaction is adiabatically switched off approaches the sum of the constituent masses (+m+m, −m−m, and +m−m).

The energy of normal states in the cm frame can be approximately calculated as

$$[4.6] \quad E_n^2(0) \cong 4m^2\left[1 + \frac{\omega}{m}(n+1)\right]$$

so the energy levels also appear in pairs, within that approximation (see Fig. 1). For weak coupling, the energy of the lowest lying normal state is

$$E_n(0) - 2m \cong \omega\,(n+1)$$

and the spacing between energy levels is

$$E_{n+1}(0) - E_n(0) \cong \omega$$

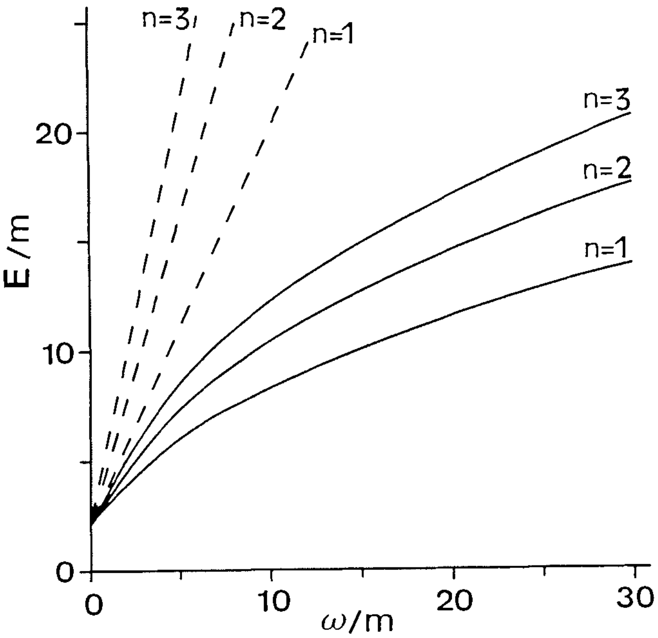

FIG. 2. Depression of relativistic energy levels (solid lines) for particle–particle systems compared with the nonrelativistic prediction (broken lines).

as occurs in the nonrelativistic oscillator. On the contrary, if ω is much larger than the rest mass, $E_n(0)$ rises as the square root of ω. Hence, the effect of relativity is a depression of the oscillator levels from the nonrelativistic prediction, as shown in Fig. 2. The result that the levels are no longer evenly spaced is also found in more sophisticated relativistic oscillator models, based on the Bethe–Salpeter equation (13).

To obtain the energy of the abnormal solutions as $\omega \rightarrow 0^+$, we consider the limit $E_n \rightarrow 0^-$ in [4.4] in the cm frame. In so doing, we find

[4.7] $$E_n(0) \cong -\omega + \frac{\omega^2}{m}\left(n + \frac{1}{2}\right), \qquad m \ll \omega$$

The ground and excited states of the particle–antiparticle system have nearly the same energy $-\omega$, since the splitting of the levels due to quantum number n is of higher order in ω. The spacing of the energy levels is then given by

$$E_{n+1}(0) - E_n(0) \cong \frac{\omega^2}{m}$$

being smaller than the spacing between normal state levels.

In the opposite case, for which the coupling strength is much larger than the rest mass of the constituent particles, abnormal energy levels are given by

[4.8] $$E_n(0) = -\frac{2m}{2n + 1}, \qquad m \gg \omega$$

Therefore, the binding energy of the particle–antiparticle system for strong coupling becomes independent of the coupling constant and it is of the order of the rest mass of the particles, as depicted in Fig. 1. For large n values, the spacing between levels is given approximately by

$$E_{n+1}(0) - E_n(0) \cong \frac{m}{n^2}$$

Having disposed of the energy levels for normal and abnormal states, we briefly examine the behaviour of the eigenfunction components given by [4.2], in the nonrelativistic limit for weak coupling. In that limiting case, the large components of the eigenfunction for particle–particle ($E \cong 2m$) and antiparticle–antiparticle ($E \cong -2m$) systems are ψ_{++} and ψ_{--}, respectively, as can be easily seen by inspection of [4.2] and [4.3]. On the contrary, abnormal solutions ($E \cong 0$) predominantly consist of ψ_{+-} and ψ_{-+}, while ψ_{++} and ψ_{--} are negligible. At relativistic energies, however, the components are of the same order of magnitude, as occurs in the one-body Dirac equation.

Before concluding, let us comment that, unlike the case of the square-well potential discussed by Coutinho *et al.* (12), the abnormal states of the two-body Dirac oscillator are not strongly localized. For a shallow square well, the spread of the wave function outside the well may be roughly estimated as m^{-1}, while the two-body Dirac oscillator is localized in a distance of the order of $(m\omega)^{-1/2}$, becoming very large when the coupling is weak.

5. Concluding remarks

It is hard to find in the literature solvable potentials for two interacting Dirac particles in (1 + 1) dimensions. Some works deal with the δ interaction (9) and the square-well potential (12), for which the energy levels can be explicitly written down. In this paper we have introduced a linear interaction in the two-body Dirac equation, which allows us to find analytical solutions. All the states obtained are bound; there exists no critical value for the coupling constant to bind particles. As a consequence, the Klein paradox has been overcome, avoiding the *leakage* of particles to infinity. Eigenfunctions are simply given in terms of Hermite polynomials times the usual exponential factor.

Allowing one of the masses to be infinite, we have shown that the eigenfunction of the light particle satisfies the Dirac oscillator equation, previously introduced by Moshinsky and Szczepaniak (4). More interesting results appear in the case of two interacting particles of equal masses. The effective frequency of the oscillator obeys a relativistic Doppler law, although the starting equation is not fully covariant. For given values of the coupling constant and the quantum number, there exist three possible values of the energy, corresponding to particle–particle, antiparticle–antiparticle, and particle–antiparticle systems. In the weak-coupling limit, the energy of the system approaches the sum of the constituent particles ($2m$, $-2m$, and zero, respectively). For particle–particle and antiparticle–antiparticle systems, a depression of energy levels appears in relation to the nonrelativistic predictions, when the interaction is strong. We have demonstrated the existence of an infinite degeneracy of levels accumulating just below $E = 0$ for the particle–antiparticle system, as the coupling constant becomes vanishingly small. On the contrary, in the case of very large coupling constant, the binding energy of this system has been found to be independent of this parameter.

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