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LETTERS AND COMMENTS

Comment on 'Exact solvability of the delta-shell potential in momentum space'

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Abstract. A recent article (Villaroel D 1998 *Eur. J. Phys.* **19** 85–92) discussed the solution of the Dirac equation for the potential $\delta(r - R)$ in momentum space by direct analogy with the non-relativistic case. Here I point out that this procedure is incorrect and that a careful analysis of the boundary conditions at $r = R$ is required even in momentum space.

In a recent paper, Villaroel [1] studied the discrete spectrum of the Dirac equation for the electrostatic-like delta-shell potential $V(r) = v\delta(r - R)$ with $R > 0$. He found an electronic spectrum which is different to previous results [2] and claimed that the latter are incorrect. Furthermore, he stated that the *correct boundary condition at $r = R$ is a rather delicate issue* (sic) but in the abstract it is established that *no explicit analysis of the boundary condition for the wavefunction is necessary at $r = R$* (sic) because calculations are carried out in momentum space. The purpose of this comment is to point out that (i) previous results [2] are correct because correct boundary conditions were considered, and (ii) a careful analysis at $r = R$ is required even in momentum space.

Almost two decades ago, Sutherland and Mattis [3] pointed out that the relativistic δ-function potential presents some ambiguities, since potentials of different shapes approaching the δ-function limit yield different values of the wavefunction at the interaction point. This ambiguity was circumvented more than a decade ago by McKellar and Stephenson [4]. The same arguments become valid in the case of the delta-shell potential [2] since the terms related to the eigenvalues of the operator $\boldsymbol{J}$ (which are absent in one dimension) are non-singular at $r = R > 0$. Therefore, the delta-shell should be regarded as the limit of an electrostatic-like sharply peaked potential provided that the right boundary conditions at $r = R$ are established [2]. However, the results of [1] were obtained with the wrong boundary conditions, in the sense that those boundary conditions do not represent any physically reasonable potential.

It is well known that the formal solution of the Dirac equation in momentum space relies on the following definition [5]:

$$\int_{R-\epsilon}^{R+\epsilon} \delta(r - R)\psi(r)\,\mathrm{d}r = \frac{1}{2}\left[\psi(R + \epsilon) + \psi(R - \epsilon)\right] \tag{1}$$

where ϵ is a small positive quantity. Note that the product of the δ-function and the discontinuous function $\psi(r)$ is not well-defined in the strict distribution theory sense. Clearly, similar problems arise when passing to momentum space since the Fourier transform of the product $\delta(r - R)\theta(r - R)$ is ill-defined, θ being the Heaviside step function. Equation (1) provides ambiguous results from a physical point of view [4]. Once again, we stress that the correct boundary conditions were given in [4] in the one-dimensional case and in [2] in the case of the delta-shell potential.

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In conclusion, different boundary conditions give different bound states and spectra. Since the delta-shell potential is highly idealized, one should take care about which boundary conditions are correct from a physical point of view. Those boundary conditions are given in [2], whereas (1), obtained from a formal manipulation of the δ-function potential in momentum space, was proved to present some ambiguities. Finally, an explicit analysis of the boundary condition at $r = R$ for the wavefunction is *indeed* necessary even in momentum space.

References

[1] Villaroel D 1998 Exact solvability of the delta-shell potential in momentum space *Eur. J. Phys.* **19** 85–92

[2] Domínguez-Adame F 1990 Exact solutions of the Dirac equation with surface delta interactions *J. Phys. A: Math. Gen.* **23** 1993–9

[3] Sutherland B and Mattis D C 1981 Ambiguities with the relativistic δ-function potential *Phys. Rev.* A **24** 1194–7

[4] McKellar B H J and Stephenson G 1987 Klein paradox and the Dirac–Kronig–Penney model *Phys. Rev.* A **36** 2566–9

[5] Domínguez-Adame F 1987 Relativistic and nonrelativistic Kronig–Penney models *Am. J. Phys.* **55** 1003–6