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Complex behavior of the conductance of quantum wires with a long quantum-dot array

F. Domínguez-Adame[a,*], I. Gómez[a], P.A. Orellana[b], M.L. Ladrón de Guevara[b]

[a]GISC, Departamento de Física de Materiales, Facultad de Física, Universidad Complutense, E-28040 Madrid, Spain
[b]Departamento de Física, Universidad Católica del Norte, Casilla 1280, Antofagasta, Chile

Abstract

We consider electron transport through a quantum wire with an attached quantum-dot array, when the number of dots is large. To this end, we use a noninteracting Anderson Hamiltonian. The conductance at zero temperature shows a complex behavior as a function of the Fermi energy. In particular, two well-defined energy regions are observed. Far from the site-energy of the quantum dots, the conductance depends smoothly on the Fermi energy. On the contrary, at the center of the band the conductance develops an oscillating pattern with resonances and antiresonances due to constructive and destructive interference in the ballistic channel, respectively. We discuss analytically in detail the physical origin of this complex behavior.

Keywords: Quantum wires; Quantum-dot array; Fermi energy

1. Introduction

Latest advances in nanofabrication of quantum devices make it possible to obtain quantum dots (QDs) in a controllable way [1]. We have recently proposed a new quantum device based on a quantum wire (QW) with an attached QD array [2]. In this case the QD array acts as scattering center for transmission through the QW. This configuration can be regarded as a quantum wave guide with side-stub structures, similar to those reported in Ref. [3]. The conductance at zero temperature through the QW shows a complex behavior as a function of the Fermi energy, being strongly dependent on the number of QDs in the attached array. For a uniform QD array, we found that the conductance develops an oscillating pattern with resonances (perfect transmission) and antiresonances (perfect reflection). In addition, we found an odd–even symmetry related to the number of QDs in the array, namely perfect transmission takes place if this number is even ($G=2e^2/h$) but perfect reflection arises for an odd number ($G=0$). These results indicate the feasibility of tuning the QW transport properties through the QD array. In this work we report further progress along the lines indicated above. In particular, we study in detail the complex behavior of the conductance of the QW when the number of QDs in the attached array is large.

2. Model Hamiltonian and conductance

We model the system by using a noninteracting Anderson tunneling Hamiltonian that can be written as $H = H_{\mathrm{QW}} + H_{\mathrm{QD}}^N + H_{\mathrm{QD-QW}}$, where

$$H_{\mathrm{QW}} = v\sum_i (c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i), \tag{1}$$

$$H_{\mathrm{QD}}^N = \varepsilon_0 \sum_{l=1}^{N} d_l^\dagger d_l + V_c \sum_{l=1}^{N-1} (d_l^\dagger d_{l+1} + d_{l+1}^\dagger d_l),$$

$$H_{\mathrm{QD-QW}} = V_0(d_1^\dagger c_0 + c_0^\dagger d_1).$$

The operators $c_i^\dagger$ and $d_l^\dagger$ create an electron at sites i and l, respectively. Here v and V_c are the hoppings in the QW and in the array with N QDs, respectively. V_0 is the hopping between the QW and the amay. Finally, ε_0 is the energy level of each QD. Notice that we are assuming uniform hopping and identical QDs in the array, although this is not an essential requirement of the model since more general situations can be handled [2]. Fig. 1 shows a schematic view of the system.

The experimentally accessible quantity is the linear conductance G, which is related to the transmission

* Corresponding author. Tel.: +34-91-394-4488; fax: +34-91-394-4547.
E-mail address: adame@fis.ucm.es (F. Domínguez-Adame).

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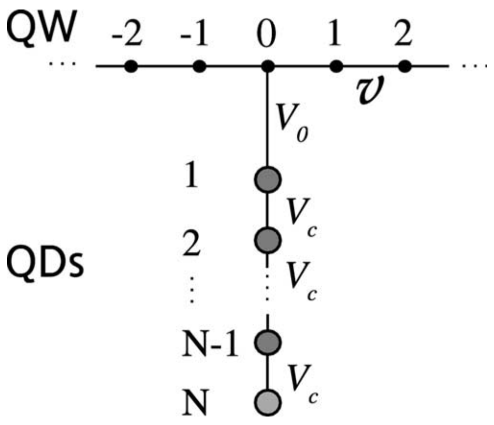

Fig. 1. Quantum dot array attached to a perfect quantum wire.

coefficient at the Fermi energy by the one-channel Landauer formula at zero temperature. It is a matter of algebra to obtain [2]

$$G = \frac{2e^2}{h}\frac{Q_N^2}{Q_N^2 + \Gamma^2}. \tag{2}$$

where Q_N is the continued fraction

$$Q_N = \varepsilon - \varepsilon_0 - \cfrac{V_c^2}{\varepsilon - \varepsilon_0 - \cfrac{\ddots}{\varepsilon - \varepsilon_0 - \cfrac{V_c^2}{\varepsilon - \varepsilon_0}}}, \tag{3}$$

where $\Gamma(\varepsilon) \equiv V_0^2/2v \sin(kd)$ can be regarded as the level broadening, and the dispersion relation in the QW is $\varepsilon = 2v \cos(kd)$, d being the lattice spacing. Notice that the level broadening can be fairly well approximated by $\Gamma \simeq V_0^2/2v$ close to the center of the band. It is worth mentioning that the spectrum (zeroes of Q_N) depends only on the hopping in the QD array (V_c) while Γ is only function of V_0^2/v. Consequently, both magnitudes can be controlled independently in an actual experiment. This is one of the main advantages of the present set-up.

3. Results

To evaluate the conductance at zero temperature when the number N of attached QDs is large, we must rely on numerical calculations. Fig. 2 shows the results for $N = 15$ and 16 when $V_c = \Gamma$ and $\varepsilon_0 = 0$. We observe the occurrence of N antiresonances and $N - 1$ resonances in the conductance of the QW. The positions of the antiresonances correspond exactly to the electronic spectrum of the isolated QD array. This property could be used to measure the energy spectrum of the N QD array. It should be stressed that the particular set-up we suggested allows us to control the energy and the width of the antiresonances in an independent fashion. On further increasing N,

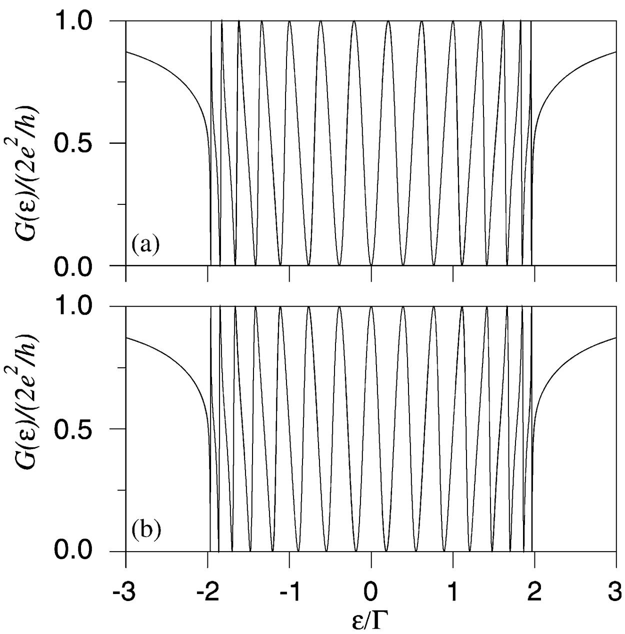

Fig. 2. Conductance, in units of $2e^2/h$, versus Fermi energy, in units of Γ, for (a) $N = 15$ and (b) $N = 16$ QD array with $V_c = \Gamma$ and $\varepsilon_0 = 0$.

the antiresonances never merge into a single stop-band, as one would naively expect; this statement can be rigorously demonstrated [2].

Let us consider the case when the Fermi energy is pinned at the value of the energy level of the QD. From Fig. 1 we notice that $G(0) = 0$ for $N = 15$ and $G(0) = 2e^2/h$ for $N = 16$. This suggests the occurrence of an odd–even parity. In fact, it is straightforward to prove the existence of this odd–even parity for arbitrary N [2]. This symmetry arises from the fact that the energy level of the QDs ε_0 is always in the electronic spectrum of the isolated QD array, provided the number of the QDs is odd.

When the number of attached QDs is large, a rich phenomenology appears for different values of the Fermi energy. When the Fermi energy lies far from the center of the QW band ($|\varepsilon - \varepsilon_0| > 2V_c$), the conductance presents regular and smooth behavior. However, the conductance strongly fluctuates close to the center of the QW band for minute variations of the Fermi energy ($|\varepsilon - \varepsilon_0| < 2V_c$). In order to shed light onto this complex behavior, the continued fraction Q_N in Eq. (3) is written as $Q_N = (\varepsilon - \varepsilon_0)x_N$, where x_N satisfies the following recursive equation,

$$x_{N+1} = f(x_{N-1}) = 1 - \frac{\alpha}{x_N}, \quad N = 1, 2, 3, \ldots \tag{4}$$

with $x_1 = 1$ and $\alpha \equiv V_c^2/(\varepsilon - \varepsilon_0)^2$ for $\varepsilon \neq \varepsilon_0$. Thus, we are faced to a one-dimensional map (4) with control parameter α. This map has two fixed points at

$$x_{\pm}^* = \frac{1}{2}(1 \pm \sqrt{1 - 4\alpha}), \tag{5}$$

when $\alpha < 1/4$, namely $|\varepsilon - \varepsilon_0| > 2V_c$, as shown in Fig. 3. The fixed point $x_+^*(x_-^*)$ is stable (unstable). The conductance

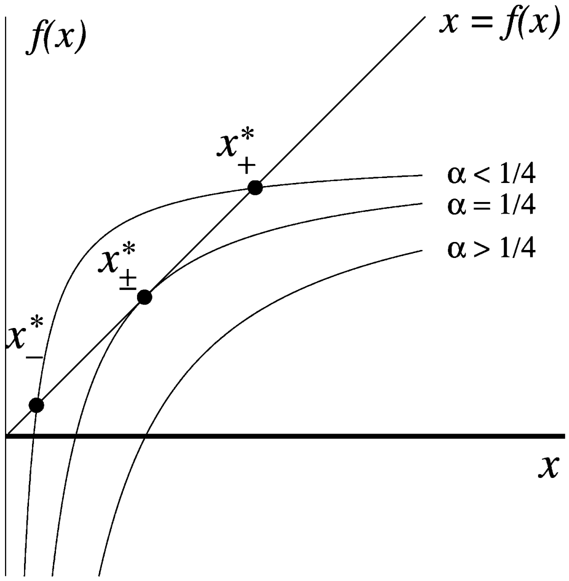

Fig. 3. Mapping of the nonlinear map (4) showing the fixed points.

for $N \to \infty$ is,

$$G_\infty = \frac{2e^2}{h} \frac{(|\varepsilon - \varepsilon_0| + \sqrt{(\varepsilon - \varepsilon_0)^2 - 4V_c^2})^2}{(|\varepsilon - \varepsilon_0| + \sqrt{(\varepsilon - \varepsilon_0)^2 - 4V_c^2})^2 + \Gamma^2}, \quad (6)$$

for $|\varepsilon - \varepsilon_0| > 2V_c$. This result explains the smooth tails seen in Fig. 2 when $|\varepsilon - \varepsilon_0|/\Gamma > 2$.

The nonlinear map (4) undergoes a bifurcation at $\alpha = 1/4(|\varepsilon - \varepsilon_0| = 2V_c)$, and there are not fixed points when $\alpha > 1/4$, namely $|\varepsilon - \varepsilon_0| < 2V_c$ (Fig. 3). Consequently, minute variations of the Fermi energy result in a dramatic change in the conductance of the QW, as it can be concluded from Fig. 2.

4. Summary

In summary we studied a noninteracting QD array side-coupled to a quantum wire. We found that conductance at zero temperature develops an oscillating band with resonances and antiresonances due to constructive and destructive interference in the ballistic channel, respectively. We show that this band is related to the electronic properties of the isolated QD array. The complex pattern of the conductance as a function of the Fermi energy has been explained from the occurrence or absence of fixed points of a one-dimensional nonlinear map.

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