Available online at www.sciencedirect.com
ScienceDirect
Physica E 35 (2006) 126–130
PHYSICA E
www.elsevier.com/locate/physe

Dicke effect in a quantum wire with side-coupled quantum dots

P.A. Orellana[a,*], F. Domínguez-Adame[b], E. Diez[c]

[a]Departamento de Física, Universidad Católica del Norte, Casilla 1280, Antofagasta, Chile
[b]GISC, Departamento de Física de Materiales, Universidad Complutense, E-28040 Madrid, Spain
[c]Departamento de Física Fundamental, Universidad de Salamanca, 37008 Salamanca, Spain

Received 30 May 2006; accepted 27 June 2006
Available online 21 August 2006

Abstract

A system of an array of side-coupled quantum dots attached to a quantum wire is studied theoretically. Transport through the quantum wire is investigated by means of a noninteracting Anderson tunneling Hamiltonian. Analytical expressions of the transmission probability and phase are given. The transmission probability shows an energy spectrum with forbidden and allowed bands that depends on the up–down asymmetry of the system. In up–down symmetry only the gap survives, and in up–down asymmetry an allowed band is formed. We show that the allowed band arises by the indirect coupling between the up and down quantum dots. In addition, the band edges can be controlled by the degree of asymmetry of the quantum dots. We discuss the analogy between this phenomenon with the Dicke effect in optics.
© 2006 Elsevier B.V. All rights reserved.

PACS: 73.21.La; 73.63.Kv; 85.35.Be

Keywords: Dicke effect; Quantum dots; Quantum wire

1. Introduction

Quantum interference effects in quantum wires (QWs) are potentially useful in nanotechnology since coupling to the continuum states shows an even–odd parity effect in the conductance when the Fermi energy is localized at the center of the energy band [1–9]. Consequently, a fine control of the electron transport can be achieved by varying the external parameters of the QW.

In this context, we have recently considered new quantum devices based on an array of quantum dots (QDs) [10], a double QD [11] and nanorings [12] coupled to a QW. The attached device acts as scatterer for electron transmission through the QW and allows to tune its transport properties. It was found that the conductance at zero temperature through the QW shows a complex behavior as a function of the Fermi energy: far from the center of the band the conductance depends smoothly on the Fermi energy, while around the center it develops an oscillating band with resonances and antiresonances due to quantum interference in the ballistic channel. Moreover, the transmission phase of the electron carries information complementary to the transmission probability. This phase has been measured in QDs [13,14] and recently it was reported that [15] the experimental observation of the Fano–Kondo antiresonance in a QW with a side-coupled QD. These experiments proved that transport through the system has a coherent component.

In this work we report further progress along the lines indicated above. In particular, we study theoretically transport properties of a set of side-coupled double QDs attached to a perfect QW. We find analytical expressions for the transmission probability and transmission phase. The transmission probability at the center of the energy spectrum shows an energy spectrum with gap or allowed band depending of the symmetry up–down, to be explained below. In a symmetry up–down, an even–odd parity effect in the transmission phase at the center of the band is demonstrated. Moreover, we show that an allowed band is formed in the asymmetric case and that the width of this band can be controlled by suitable gate voltages. This

*Corresponding author. Tel.: +56 553 55 507; fax: +56 553 55 521.
E-mail address: orellana@ucn.cl (P.A. Orellana).

1386-9477/$ - see front matter © 2006 Elsevier B.V. All rights reserved.
doi:10.1016/j.physe.2006.06.017

phenomenon is in analogy to the Dicke effect in quantum optics, that takes place in the spontaneous emission of two closely lying atoms radiating a photon into the same environment [16]. In the electronic case, however, the decay rates (level broadening) are produced by the indirect coupling of the up–down QDs, giving rise to a fast (*superradiant*) and a slow (*subradiant*) mode. This close analogy opens the way to exploit new electronic effects that usually arise in atomic physics. In this regard, it has been shown that coupled QDs display the electronic counterpart of Fano and Dicke effects that can be controlled via a magnetic flux [17]. Recently, Brandes reviewed the Dicke effect in mesoscopic systems [18].

2. Model

The system under consideration is shown in Fig. 1. The QW is attached to a N side-coupled double QDs. The system, assumed in equilibrium, is modeled by a noninteracting Anderson tunneling Hamiltonian [3] that can be written as $H = H_{\mathrm{QW}} + H_{\mathrm{QD}} + H_{\mathrm{QW-QD}}$, where

$$H_{\mathrm{QW}} = -v\sum_i (c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i), \tag{1a}$$

describes the dynamics of the QW, v being the hopping between neighbor sites of the QW, and $c_i^\dagger$ (c_i) creates (annihilates) an electron at site i.

On the other side, H_{D}, given by

$$H_{\mathrm{QD}} = \sum_{j=1}^{N}\sum_{\alpha=u,d} \varepsilon_\alpha d_{j\alpha}^\dagger d_{j\alpha} \tag{1b}$$

is the Hamiltonian for the N side-coupled double QDs, where $d_{j\alpha}$ ($d_{j\alpha}^\dagger$) is the annihilation (creation) operator of an electron in the QD (j,α), ε_α is the corresponding single energy level. Here the index α refers to the up (u) and down (d) QD, attached at site j of the QW. The coupling between the two subsystems (QW and QDs) is described by the Hamiltonian

$$H_{\mathrm{QW-QD}} = -V_0\sum_{j=1}^{N}\sum_{\alpha=u,d}(d_{j\alpha}^\dagger c_j + c_j^\dagger d_{j\alpha}), \tag{1c}$$

where V_0 is the coupling between the QW and one of the QDs.

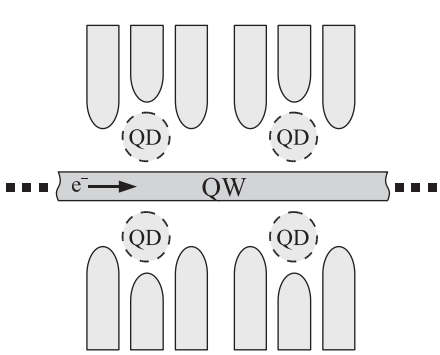

Fig. 1. N side-coupled double QDs attached to a perfect QW.

The Hamiltonian for the QW, H_{QW}, corresponds to a free-particle Hamiltonian on a lattice with spacing unity and whose eigenfunctions are expressed as Bloch solutions

$$|k\rangle = \sum_{j=-\infty}^{\infty} \mathrm{e}^{\mathrm{i}kj}|j\rangle, \tag{2}$$

where $|k\rangle$ is the momentum eigenstate and $|j\rangle$ is a Wannier state localized at site j. The dispersion relation associated with these Bloch states reads

$$\omega = -2v\cos k. \tag{3}$$

Consequently, the Hamiltonian supports an energy band from $-2v$ to $+2v$ and the first Brillouin zone expands the interval $[-\pi,\pi]$. The stationary states of the entire Hamiltonian H can be written as

$$|\psi_k\rangle = \sum_{j=-\infty}^{\infty} a_j^{(k)}|j\rangle + \sum_{j=1}^{N}\sum_{\alpha=u,d} b_{j\alpha}^{(k)}|j,\alpha\rangle, \tag{4}$$

where the coefficient $a_j^{(k)}$ ($b_{j\alpha}^{(k)}$) is the probability amplitude to find the electron at site j of the QW [at the QD (j,α)] in the state k, that is, $a_j^{(k)} = \langle j|\psi_k\rangle$ and $b_{j\alpha}^{(k)} = \langle j,\alpha|\psi_k\rangle$.

The amplitudes $a_j^{(k)}$ obey the following linear difference equation:

$$\omega a_j^{(k)} = v\left(a_{j+1}^{(k)} + a_{j-1}^{(k)}\right), \quad j\leqslant 0 \text{ and } j>N, \tag{5a}$$

$$\omega a_j^{(k)} = v\left(a_{j+1}^{(k)} + a_{j-1}^{(k)}\right) - V_0\left(b_{j,u}^{(k)} + b_{j,d}^{(k)}\right), \quad j=1,\ldots,N, \tag{5b}$$

$$(\omega-\varepsilon_\alpha)b_{j\alpha}^{(k)} = -V_0 a_j^{(k)}, \quad j=1,\ldots,N, \ \alpha=u,d. \tag{5c}$$

The amplitudes $b_{j,\alpha}^{(k)}$ can be expressed in terms of $a_j^{(k)}$ as follows:

$$b_{j\alpha}^{(k)} = -\frac{V_0}{\omega-\varepsilon_\alpha}\, a_j^{(k)}, \tag{6}$$

From Eq. (6) above, Eq. (5c) becomes

$$(\omega-\tilde{\varepsilon})a_j^{(k)} = v\left(a_{j+1}^{(k)} + a_{j-1}^{(k)}\right), \quad j=1,\ldots,N, \tag{7}$$

where the site energy $\tilde{\varepsilon} \equiv V_0^2/[(\omega-\varepsilon_u) + 1/(\omega-\varepsilon_d)]$ depends on the electron energy ω. Thus, the problem reduces to a linear chain of N sites of effective energies $\tilde{\varepsilon}$. In order to study the solutions of Eq. (7), we assume that the electrons are described by a plane wave incident from the far left with unity amplitude and a reflection amplitude r, and at the far right by a transmission amplitude t. That is,

$$a_j^{(k)} = \mathrm{e}^{\mathrm{i}kj} + r\mathrm{e}^{-\mathrm{i}kj}, \quad j<1,$$

$$a_j^{(k)} = t\mathrm{e}^{\mathrm{i}kj}, \quad j>N. \tag{8}$$

The solution in the region $j=1,\ldots,N$, can be written as

$$a_j^{(k)} = A\mathrm{e}^{\mathrm{i}qj} + B\mathrm{e}^{-\mathrm{i}qj} \text{ if } |(\omega-\tilde{\varepsilon})/2v|\leqslant 1,$$

$$q = -\cos^{-1}[-(\omega-\tilde{\varepsilon})/2v],$$

$$a_j^{(k)} = Ce^{\kappa j} + De^{-\kappa j} \text{ if } |(\omega - \tilde{\varepsilon})/2v| > 1,$$
$$\kappa = -\cosh^{-1}[-(\omega - \tilde{\varepsilon})/2v]. \tag{9}$$

Inserting Eqs. (8) and (9) into Eq. (7), we get a inhomogeneous system of linear equations for A, B, C, D, t and r, leading to the following result: if $|(\varepsilon - \tilde{\varepsilon})/2v| \leqslant 1$,

$$t = (-2ie^{ikN}/\Delta)\sin k, \tag{10a}$$

with Δ given by

$$\Delta = e^{-ik}\frac{\sin(N+1)q}{\sin q} - 2\frac{\sin Nq}{\sin q} + e^{ik}\frac{\sin(N-1)q}{\sin q}. \tag{10b}$$

On the contrary, when $|(\varepsilon - \tilde{\varepsilon})/2v| > 1$

$$t = (2ie^{-ikN}/\Delta)\sin k, \tag{10c}$$

with Δ given by

$$\Delta = e^{-ik}\frac{\sinh(N+1)\kappa}{\sinh\kappa} - 2\frac{\sinh N\kappa}{\sinh\kappa} + e^{ik}\frac{\sinh(N-1)\kappa}{\sinh\kappa}. \tag{10d}$$

The transmission probability is given by $T = |t|^2$, and it is related to the linear conductance at the Fermi energy ε_F by the one-channel Landauer formula at zero temperature, $G = (2e^2/h)T(\omega = \varepsilon_F)$ [19]. We also can obtain the transmission phase as $\phi_t = \tan^{-1}(\mathrm{Im}\, t/\mathrm{Re}\, t)$.

3. Results

To uncover the main features of the electron transport through the QW and the effects of the attached QDs, we now consider several physical situations. If $|(\omega - \tilde{\varepsilon})/2v| \leqslant 1$, the transmission probability and transmission phase reduces to

$$T = \frac{1}{\cos^2(Nq) + (\sin(Nq)\cot(k/2)/\sin q)^2}, \tag{11a}$$

$$\phi_t = \arctan\left\{\frac{\alpha\cos k\cos Nk + \beta\sin k\sin Nk}{\alpha\cos k\sin Nk - \beta\sin k\cos Nk}\right\}, \tag{11b}$$

with $\alpha = \sin(N+1)q - 2\sin Nq + \sin(N-1)q$ and $\beta = \sin(N+1)q - \sin(N-1)q$. We note that the transmission probability oscillates as a function of both N and q. On the other hand, when $|(\varepsilon - \tilde{\varepsilon})/2v| > 1$ we get

$$T = \frac{1}{\cosh^2(N\kappa) + (\sinh(N\kappa)\cot(k/2)/\sinh\kappa)^2}, \tag{11c}$$

$$\phi_t = \arctan\left\{\frac{\delta\cos k\cos Nk + \eta\sin k\sin Nk}{\delta\cos k\sin Nk - \eta\sin k\cos Nk}\right\}, \tag{11d}$$

where $\delta = \sinh(N+1)\kappa - 2\sinh N\kappa + \sinh(N-1)\kappa$ and $\eta = \sinh(N+1)\kappa - \sinh(N-1)\kappa$. Therefore, in this energy region N tends exponentially to zero when N is large, namely $T \sim e^{-2N\kappa}$.

To avoid the profusion of free parameters, for the sake of clarity we set the energies of the up and down QDs as, $\varepsilon_u = \Delta V$ and $\varepsilon_d = -\Delta V$, hereafter. We first consider the case $\Delta V = 0$. In this case the transmission exhibits a forbidden band (gap). Fig. 2 shows the transmission probability versus ω for different values of N. It is apparent that T tends to zero within a range $[-2\gamma, 2\gamma]$, with $\gamma = V_0^2/2v$, and the system shows a gap of width 4γ.

Inside the energy region $[-2\gamma, 2\gamma]$ the conductance does not present any feature that depends on N. However, at the center of the band ($\omega = 0$, $k = \pi/2$, $\kappa \to \infty$) the transmission phase takes on the following values, depending on the parity of N: When N is odd, $\phi_t \to \pm\pi/2$ for $\omega \to \mp 0$, while for N even, $\phi_t = 0$ for $\omega = 0$. We can understand this result as follows. Each time that the electron passes near a double QDs, it undergoes a phase change equal to $\pi/2$ due to the destructive interference between the discrete levels in the double QD and the continuum states of the QW.

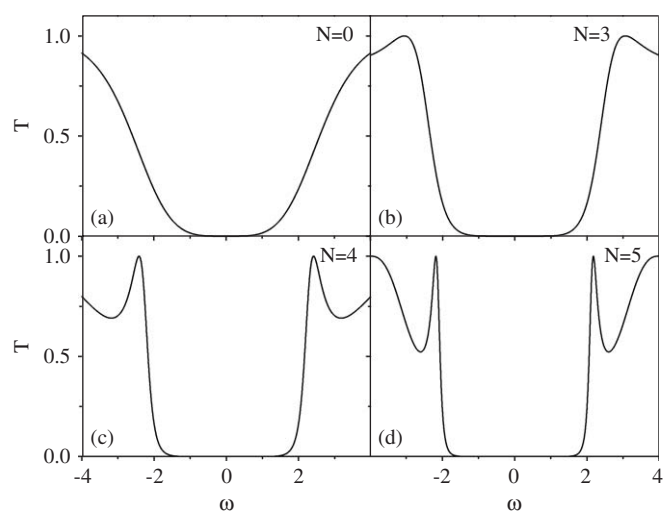

Fig. 2. Transmission probability as a function of energy, in units of $\gamma = V_0^2/2v$, for $\Delta V = 0$ and (a) $N = 2$, (b) $N = 3$, (c) $N = 4$ and (d) $N = 5$.

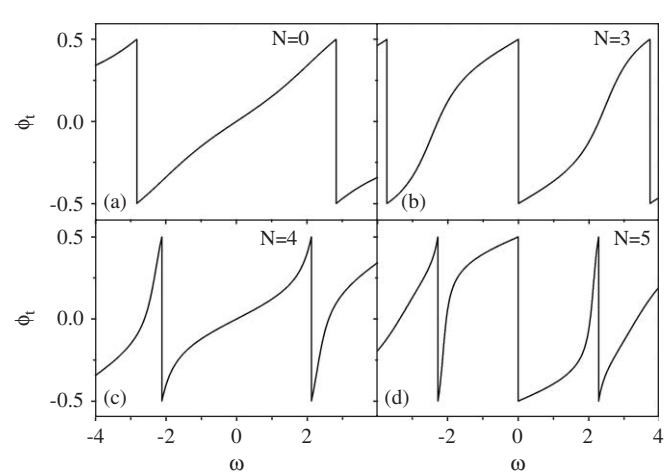

Fig. 3. Transmission phase versus energy, in units of $\gamma = V_0^2/2v$, for $\Delta V = 0$ and (a) $N = 2$, (b) $N = 3$, (c) $N = 4$ and (d) $N = 5$.

Consider now the situation with $\Delta V \neq 0$. Fig. 3 shows the transmission probability for different values of ΔV, for $N = 4$ and 5. We note that an allowed band develops at the center of the gap. It is straightforward to show that for $\Delta V \ll \gamma$ the width of this allowed band is $\Delta V^2/2\gamma$. Moreover, the transmission probability becomes always unity at the center of the allowed band, independently of the value of N.

Fig. 4 displays the transmission probability for several values of ΔV when $N = 4$ (solid line) and $N = 5$ (dashed line). We note that the shape of the allowed band is independent of the value of ΔV. Additionally, Fig. 5 shows the transmission phase for a similar set of parameters in the region of the allowed band. We note a series of discontinuities of the transmission phase at some values of the energy ω. It is worth to note that the drops in the transmission phase do not imply vanishing transmission probability. At the energies of the drops of the transmission phase, the real part of the transmission amplitude vanishes and at the same time its imaginary part changes its sign. Additionally from Eq. (11b), we obtain that the transmission phase is zero (or $m\pi$, m integer) at the center of the allowed band, independent of the value of N and ΔV. At the same time, the transmission probability becomes unity.

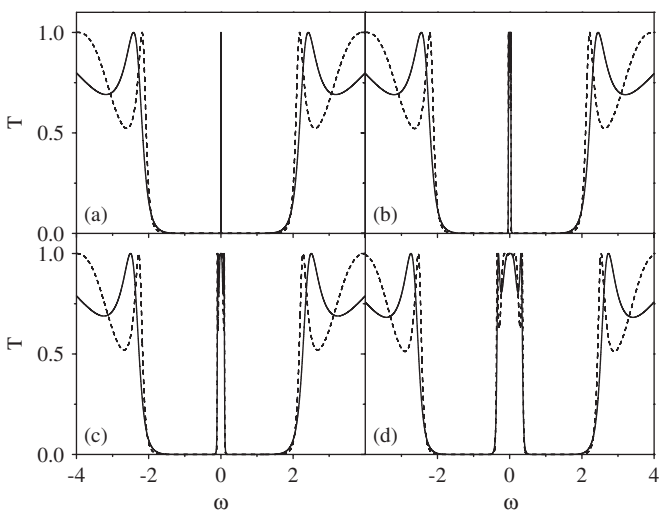

Fig. 4. Transmission probability versus energy, in units of γ for $N = 4$ (solid line) and $N = 5$ (dashed line) for (a) $\Delta V = 0.1\gamma$, (b) $\Delta V = 0.3\gamma$, (c) $\Delta V = 0.5\gamma$ and (d) $\Delta V = 1.0\gamma$.

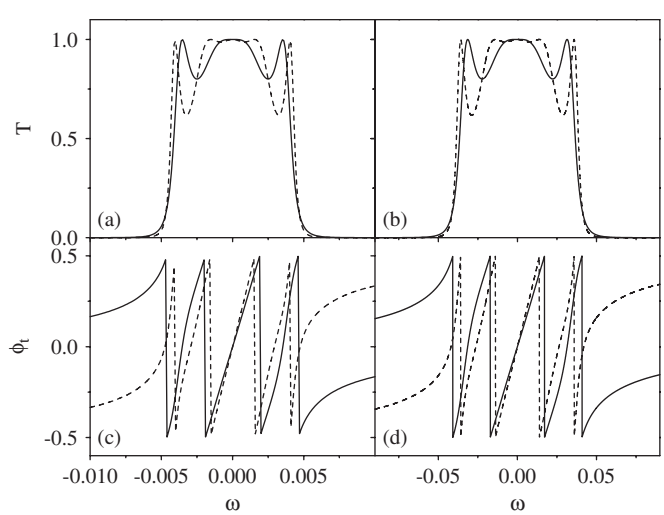

Fig. 5. Enlarged views of the transmission probability and transmission phase versus energy, in units of γ, for $N = 4$ (solid line) and $N = 5$ (dashed line) for $\Delta V = 0.1\gamma$ (left panels) and $\Delta V = 0.3\gamma$ (right panels).

This phenomenon resembles the Dicke effect in optics, which takes place in the spontaneous emission of a pair of atoms radiating a photon with a wave length much larger than the separation between them [16]. The luminescence spectrum is characterized by a narrow and a broad peak, associated with long- and short-lived states, respectively. The former state, weakly coupled to the electromagnetic field, is called *subradiant*, and the latter, strongly coupled, *superradiant* state. In the present case this effect is due to the indirect coupling between up–down QDs through the QW. The states strongly coupled to the QW yield a forbidden band with width 4γ and the states weakly coupled to the QW give an allowed Dicke band with width $\Delta V^2/2\gamma$.

4. Summary

In this work we studied the transmission probability and transmission phase through a QW with an array of side-coupled double QDs. We found that the transmission probability displays a gap or a band at the center of the energy spectrum, due to destructive and constructive interference in the ballistic channel, respectively. For an array with symmetry up–down ($\Delta V = 0$) only a gap develops at the center of the band. For $\Delta V \neq 0$ an allowed band arises at the center of the energy spectrum independently of the value of ΔV. This phenomenon is in analogy to the Dicke effect in optics. The set of side-coupled double QDs seems a suitable system to study the Dicke effect in experiments on quantum transport. This effect could be used to develop nanodevices where an extremely fine control of electron transport could be achieved just by varying the gate potential of the QDs.

Acknowledgments

P.A.O. would like to thank financial support from Millennium Science Nucleus Condensed Matter Physics and FONDECYT under Grants nos. 1060952 and 7020269. Work in Madrid was supported by MCyT (MAT2003-01533). E. Diez acknowledge support by Junta de Castilla y León (SA007B05) and MEC (Ramón y Cajal and FIS2005-01375). Additionally, P. A. O. would like to thank the hospitality of Departamento de Física de Materiales (UCM) and Departamento de Física Fundamental (USAL) during his visits.

References

[1] A.W. Holleitner, C.R. Decker, H. Qin, K. Eberl, R.H. Blick, Phys. Rev. Lett. 87 (2001) 256802.
[2] A.W. Holleitner, R.H. Blick, A.K. Huttel, K. Eber, J.P. Kotthaus, Science 297 (2002) 70.
[3] W.Z. Shangguan, T.C. Au Yeung, Y.B. Yu, C.H. Kam, Phys. Rev. B 63 (2001) 235323.
[4] A.I. Yanson, G. Rubio-Bollinger, H.E. van den Brom, N. Agraít, J.M. van Ruitenbeek, Nature (London) 395 (1998) 780.
[5] A.T. Tilke, F.C. Simmel, H. Lorenz, R.H. Blick, J.P. Kotthaus, Phys. Rev. B. 68 (2003) 075311.
[6] R.H.M. Smit, C. Untiedt, G. Rubio-Bollinger, R.C. Segers, J.M. van Ruitenbeek, Phys. Rev. Lett. 91 (2003) 076805.
[7] A. Oguri, Phys. Rev. B 63 (2001) 115305.
[8] Z.Y. Zeng, F. Claro, Phys. Rev. B 65 (2002) 193405.
[9] T.-S. Kim, S. Hershfield, Phys. Rev. B 65 (2002) 214526.
[10] P.A. Orellana, F. Domínguez-Adame, I. Gómez, M.L. Ladrón de Guevara, Phys. Rev. B 67 (2003) 085321.
[11] P.A. Orellana, F. Domínguez-Adame, Phys. Status. Solidi. (a) 203 (2006) 1178.
[12] P.A. Orellana, M.L. Ladrón de Guevara, M. Pacheco, A. Latgé, Phys. Rev. B 68 (2003) 195321;
P.A. Orellana, M. Pacheco, Phys. Rev. B 71 (2005) 235330.
[13] A. Yacoby, M. Heiblum, D. Mahalu, H. Shtrikman, Phys. Rev. Lett. 74 (1995) 4047.
[14] R. Schuster, E. Buks, M. Heiblum, D. Mahalu, V. Umansky, H. Shtrikman, Nature 385 (1997) 417.
[15] M. Sato, H. Aikawa, K. Kobayashi, S. Katsumoto, Y. Iye, Phys. Rev. Lett. 95 (2005) 066801.
[16] R.H. Dicke, Phys. Rev. 89 (1953) 472.
[17] P.A. Orellana, M.L. Ladrón de Guevara, F. Claro, Phys. Rev. B 70 (2005) 233315.
[18] T. Brandes, Phys. 408 (2005) 315.
[19] S. Datta, Electronic Transport in Mesoscopic Systems, Cambridge University Press, Cambridge, 1997.