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# Unusual behaviour of the conductance in Gaussian superlattices

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#### Abstract

We study the conductance at finite temperature of pass-band GaAs-Al<sub>x</sub>Ga<sub>1-x</sub>As superlattices with Gaussian modulated Al mole fraction. Such structures present bands of almost unscattered electronic states and high peak-to-valley ratio in the j-V characteristic. We found a critical point, indicating the onset for the transition from the conducting to the nonconducting regime, by tuning the chemical potential in the vicinity of the band of unscattered states. Remarkably, the conductance of the Gaussian superlattice remains finite around the critical point even at zero temperature. © 2004 Elsevier B.V. All rights reserved.

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### 1. Introduction

During the last two decades the progress of heterostructure growth technology has made possible the birth of low-dimensional physics, giving us unforeseeable discoveries and optening a new generation of electronics and optoelectronics devices. These devices, based on the concept of *bandgap engineering*, have modified people's every-day life. Among the devices that can be obtained with a semiconductor superlattice (SL), a desirable one would be a high-performance band-pass filter capable of transmitting electrons lying within a given energy band. A band-pass filter based on a GaAs-Al<sub>x</sub>Ga<sub>1-x</sub>As SL was first proposed by Tung and

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Lee [1], but was never fabricated because the current state of the art of molecular beam epitaxy does not allow to grow it with a low number of defects.

Recently, Gómez et al. [2–4] proposed and grew a new electron band-pass filter design, also based on GaAs-Al<sub>x</sub>Ga<sub>1-x</sub>As heterostructures and referred to as Gaussian SL (GSL). The improved design allowed to be built with a limited number of defects and keeping much better performance than uniform SL. The interest of GSL is twofold. First, the transmission probability  $\tau$  is almost equal to unity for energies within the allowed bands, in contrast to the oscillatory behaviour of  $\tau$  as a function of energy in uniform SLs. Second, the transition from the gaps to the allowed bands is extremely sharp, thus leading to j-V characteristic presenting peak-to-valley ratios much greater than uniform SL.

The aim of this article is to study electron transport properties of the GSL in the vicinity of the

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transition from the nonconducting regime to the conducting regime. Our main goal is to report the existence of a saturation stage in this conducting–nonconducting transition. To this end, first we compare the transmission probability for both the GSL and the uniform SL. Next, we analyze the DC-conductance of the GSL as a function of the chemical potential for several temperatures to find the transition energy. Then, we describe the dependence of the DC-conductance on the temperature for values of the chemical potential located above and below the transition energy. We show that a saturation regime appears and survives even at zero temperature. Finally, we develop a model for a better understanding of the origin of the saturation regime.

#### 2. Transmission through Gaussian superlattices

The GSL is a quantum well based GaAs-Al<sub>x</sub>Ga<sub>1-x</sub> As SL, where only the barrier heights are modulated by a proper choice of the Al mole fraction x, according to the modulating function  $V_0 \exp[-(z_h^n)^2/\sigma^2]$ , where  $z_b^n$  is the coordinate along the growth direction of the *n*th barrier midpoint and  $V_0$  is the maximum height of the potential barrier entering the heterostructure. The origin of coordinates is set at the centre of the GSL. In particular, we have considered a GSL with 15 barriers and  $V_0 = 350$  meV, corresponding to the samples used in Ref. [2]. The width of the barriers is 1.5 nm, the width of the wells is 6.2 nm, and the parameter  $\sigma$  is 28.875 nm. For comparison we also considered an uniform SL with the same widths for barriers and wells, respectively, 1.5 and 6.2 nm. Fig. 1 depictes the conduction-band edge profiles of the uniform SL and GSL. For the uniform SL the Al fraction, x, is constant and the barrier height is  $V_0 = 350$  meV for all the 15 barriers. This uniform SL will help us to compare the features of the GSL to those found in conventional uniform SLs.

For our present purposes, it is enough to focus on electron states close to the GaAs gap and use the one-band effective mass framework to find the envelope functions. We calculate the transmission probability  $\tau$  for the GSL and for the uniform SL with the parameters detailed above by means of the standard transfer-matrix method (see Ref. [2] for details). In Fig. 2 the calculated transmission probability is plot-



Fig. 1. Schematic diagram of the conduction-band edge profiles of (a) uniform and (b) Gaussian SLs.



Fig. 2. Transmission probability  $\tau$  as a function of energy *E* for (a) Gaussian and (b) uniform SLs with the same number of barriers and layers widths. Dashed lines indicated the energy of the highest barriers. Notice that  $\tau$  is almost constant and equal to unity for the GSL while the usual oscillatory behaviour is observed in the uniform SL.

ted as a function of the incident energy *E* for (a) the GSL and (b) the uniform SL. Notice the occurrence of two bands below the highest barrier (dashed lines at 0.35 eV). As we discussed before the value of  $\tau$  for the GSL is almost constant and equal to unity for

energies within these two bands in contrast to the standard oscillating behaviour for the uniform SL.

#### 3. Conductance

Once we have computed the transmission coefficient, the finite-temperature four-probe DC conductance G can be obtained in the Landauer–Büttiker formalism [5] through the following expression, earlier discussed in detail by Engquist and Anderson [6]

$$G(T,\mu) = \frac{2e^2}{h} \frac{\int (-\partial f/\partial E)\tau(E) \,\mathrm{d}E}{\int (-\partial f/\partial E)[1-\tau(E)] \,\mathrm{d}E},\tag{1}$$

where integrations extend over the allowed bands,  $f(\mu, \beta)$  is the Fermi–Dirac distribution,  $\mu$  denotes the chemical potential of the sample and  $\beta = 1/k_{\rm B}T$ , where  $k_{\rm B}$  is the Boltzman constant. From the above formula the zero temperature limit can also be obtained straightforwardly, giving the well-known Landauer formula [7] which describes four probe DC conductance measurements at zero temperature

$$G(0,\mu) = \frac{2e^2}{h} \frac{\tau(\mu)}{1 - \tau(\mu)}.$$
(2)

We now proceed to compute the DC conductance for the GSL. In the upper panel of Fig. 3 we plotted G as a function of the chemical potential  $\mu$  for several values of the temperature. Notice the crossing zone that allows us to define a critical energy  $E_{\rm c} = 207.3$  meV, in the transition from the gap to the allowed band. In the lower panel of Fig. 3 we show the dependence of G on temperature for values of the chemical potential above and below the critical energy  $E_{\rm c}$ . For energies far from the critical point the behaviour of G is explained by the scaling theory. In the gaps we deal with spatially localized states, therefore we expect G to vanish when temperature approaches zero. On the other hand, for energies within the allowed bands the states spread over the whole sample (extended) and G should approach infinity with temperature going down to zero. However, for energies in the vicinity of the critical energy the situation is completely different. The conductance G exhibits a plateau for low enough temperature below and above the critical energy. When the chemical potential lies far apart from the critical point the size of the plateau reduces



Fig. 3. Upper panel; Conductance of the GSL as a function of the chemical potential  $\mu$  for several values of the temperature between T = 240 and 30 K. The circle marks the crossing point that allows us to define a critical energy  $E_c = 207.3$  meV. The lower panel shows the conductance G as a function of temperature for values of  $\mu$  above and below  $E_c$  (between  $\mu = 209$  and 198 meV). Notice the plateaus in the vicinity of  $E_c$ .

until it disappears. The transmission coefficient in the vicinity of the critical energy [see Fig. 2(a)] is a very sharp monotonic function but is not a step function. Therefore, the value of *G* could not be neither zero or infinity even at zero temperature, as is straightforwardly deduced from Eq. (2). The presence of clean plateaus on GSL is remarkable and we will trace back to this point later. It can be guessed that they appear due to the unusual *flat profile* of  $\tau$  in the conducting regime and the fact that  $\tau$  grows sharply and monotonically in the vicinity of  $E_c$ , as we will show later.

We compute now the DC conductance for the uniform SL. In the upper panel of Fig. 4 we have plotted *G* as a function of the chemical potential  $\mu$  for several values of the temperature. The lower panel of Fig. 4 displays the conductance as a function of temperature for several values of  $\mu$ . Notice that, though some plateus are still observed, they seem to disappear for some values of the chemical potential *even near* the critical energy. Thus, it seems that



Fig. 4. Conductance *G* of the uniform SL as a function of the chemical potential  $\mu$  (upper panel) for several values of the temperature (between T = 240 and 30 K) and as a function of temperature (lower panel) for several values of  $\mu$  (between  $\mu = 217$  and 175 meV). Notice the completely different behaviour as compared to Fig. 3 and the absence of plateaus.

conductance plateaus are not so cleanly observed in uniform SLs as in GSLs.

## 4. A simple model

Aiming to understand the origin of the plateaus, we propose a toy model to elucidate the main features of the unusual behaviour of *G* for the GSL. The dependence of  $\tau$  with the chemical potential observed in Fig. 2 looks like a step function but with a region of finite width *W* between the two regimes, namely nonconducting ( $\tau = 0$  in the gap) and conducting ( $\tau = 1$  in the allowed band). We now study the behaviour of *G* for an ideal system with an "ad hoc" step function transmission probability. Needless to say that this idealized system should behave like the GSL as far as the "ad hoc"  $\tau$  keeps the main features similar to the computed values of  $\tau$  for the GSL.

In the inset of Fig. 5 we have plotted the ad hoc transmission coefficient  $\tau$  that mimics the behaviour of the GSL. The single parameter W measures the width of the region of the transition between the conducting



Fig. 5. Conductance G as a function of the chemical potential  $\mu$  for several values of the temperature (T between 30 and 240 K from top to bottom of the upper panel) and as a function of temperature for several values of  $\mu$  ( $\mu$  between 92 and 110 meV from bottom to top of the lower panel). The inset shows the ad hoc  $\tau$  function used to compute G.

and nonconducting regimes. For simplicity we set a linear dependence between the two regimes but any smooth function would provide similar results. We take the values  $E_c = 100.0$  meV and W = 1 meV. The upper panel of Fig. 5 shows the DC conductance *G* as a function of the chemical potential for several values of temperature, calculated from (1) for the "ad hoc" transmission coefficient  $\tau$ . The lower panel displays the DC conductance *G* as a function of the temperature for several values of the  $\mu$  above and below  $E_c$ . Both results are very similar qualitatively and quantitatively to those showed in Fig. 3.

The dependence of the plateaus with the value of W is shown in Fig. 6, where we plotted the conductance G as a function of temperature for several values of the width W. The value of the chemical potential is  $\mu = 0.10005$  meV. It can be demonstrated that the conductance for  $\beta^{-1} \ll W$  for a given value of the chemical potential  $\mu$  is given by

$$G(T,\mu) = \frac{2e^2}{h} \frac{\tau(\mu)}{1 - \tau(\mu)} + \mathcal{O}(T^2).$$
 (3)



Fig. 6. Conductance G as function of temperature for several values of the width W of the ad hoc  $\tau$ , shown in the inset of Fig. 5. The value of  $\mu$  in (c) is  $\mu = 0.10005$  meV and we checked that the results are not modified for any value of  $\mu$  in the vicinity of the transition. The case W = 0 corresponds to the step function. Notice that the smaller the width W the smaller the size of the plateaus.

Furthermore, we will find a plateau below temperatures satisfying the following condition  $\beta^{-1} \approx W$ . The qualitative behaviour of the results does not depend on  $\mu$  in the vicinity of the transition. The case W = 0 corresponds to the step function. Notice that the smaller the width W the smaller the size of the plateaus, approaching an ideal M–I transition for the case of the step-function with W = 0. It should be mentioned that it is not easy to change the value of W in a real GSL, but we have enlarged the scope of our results showing that the plateaus exhibited by the GSL will appear for any system with a  $\tau$  close to a step function. We would like to stress again that the particular behaviour of  $\tau$  between the conducting and non-conducting regimes is not important provided we keep the smooth shape and monotonic increase of the transmission coefficient, as occurs in the GSL.

#### 5. Conclusions

To conclude, we have reported the unusual behaviour of the conductance G for a GSL, showing the existence of a saturation regime in the conducting– nonconducting transition at any temperature. We also mimic the system by using an "ad hoc" transmission coefficient, similar to a step function but with a finite width of the transition region. This naive model allows us to better understand the nature of the conducting–nonconducting transition.

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