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Dynamics of the electron transport in a quantum wire coupled to a quantum-dot array

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Abstract

We study the transport properties of independent carriers through a quantum wire coupled to a quantum-dot array. The electrical conductance at zero temperature can be expressed through a non-linear discrete dynamical system as the number of quantum dots varies. The dynamical system shows a rich behavior, determining a non-trivial conductance dependence on the Fermi energy. The conductance depends smoothly on the Fermi energy far from the site-energy of the quantum dots. At the center of the band the conductance develops a complex pattern due to constructive and destructive interference in the ballistic channel.

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1. Introduction

Recent progress in semiconductor manufacturing makes it feasible to tune the physical properties of quantum dots (QDs) in a controllable way [1]. While single QDs are referred to as *artificial atoms*, an array of coupled QDs can be then considered as an *artificial crystal* [2–4]. Quantum effects in these artificial crystals are potentially useful in nanotechnology since coupling to the continuum states shows an even–odd parity effect in the conductance when the Fermi energy is localized at the center of the energy band [5–8].

In this context, we have recently considered a new quantum device based on a quantum wire (QW) coupled to a QD array [9], acting as scatterer for transmission through the QW. This arrangement provides a unique way to tuning the QW transport properties by virtue of the attached QD array. The conductance at zero temperature through the QW shows a complex behavior as a function of the Fermi energy. This dependence can be accounted for by an equivalent dynamical system, the results being strongly dependent on the number of QDs in the attached array.

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In this Letter we report further progress along the lines indicated above and shed more light on the complex behavior of the conductance of the QW when the number of QDs in the attached array is large. To this end, we carry out an extensive analytical study of the equivalent dynamical system, and focus the attention on the conditions for the existence or absence of fixed points of the non-linear map. In addition, a generalization of the even–odd parity effect to the case of Fermi energy lying out of the center of the band is carried out.

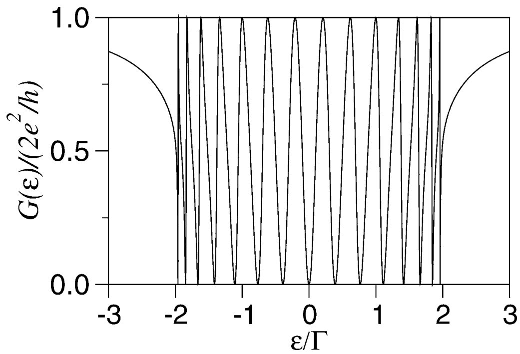

Fig. 1. Conductance, in units of $2e^2/h$, versus Fermi energy, in units of Γ, for $N = 15$ QD array with $V_c = \Gamma$ and $\varepsilon_0 = 0$.

2. Model Hamiltonian and conductance

We model the system by assuming independent carriers. Thus, the system Hamiltonian can be written as $H = H_{\mathrm{QW}} + H_{\mathrm{QD\text{-}QW}} + H_{\mathrm{QD}}^N$, where

$$H_{\mathrm{QW}} = v\sum_i (c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i),$$

$$H_{\mathrm{QD\text{-}QW}} = V_0(d_1^\dagger c_0 + c_0^\dagger d_1),$$

$$H_{\mathrm{QD}}^N = \varepsilon_0 \sum_{l=1}^{N} d_l^\dagger d_l + V_c \sum_{l=1}^{N-1} (d_l^\dagger d_{l+1} + d_{l+1}^\dagger d_l). \quad (1)$$

The operators $c_i^\dagger$ and $d_l^\dagger$ create an electron at sites i and l, respectively. Here v and V_c are the hoppings in the QW and in the array with N QDs, respectively. The number of sites in the QW is taken to be infinity. Finally, ε_0 is the energy level of each QD and V_0 is the hopping between the QW and the QD array.

In the limit of vanishing potential drop across the QW, the conductance G is related to the transmission coefficient at the Fermi energy by the one-channel Landauer formula at zero temperature. After some algebra, the conductance can be cast in the form [9]

$$G_N = \frac{2e^2}{h}\frac{Q_N^2}{Q_N^2 + \Gamma^2}, \quad (2)$$

where Q_N is the continued fraction

$$Q_N = \varepsilon - \varepsilon_0 - \cfrac{V_c^2}{\varepsilon - \varepsilon_0 - \cfrac{\ddots}{\varepsilon - \varepsilon_0 - \cfrac{V_c^2}{\varepsilon - \varepsilon_0}}}, \quad (3)$$

and $\Gamma(\varepsilon) \equiv V_0^2/2v|\sin kd|$. The dispersion relation in the QW is $\varepsilon = 2v\cos kd$, d being the lattice spacing. Notice that $\Gamma \simeq V_0^2/2v$ close to the center of the band. As an example, Fig. 1 shows the conductance for $N = 15$ when $V_c = \Gamma$ and $\varepsilon_0 = 0$. We observe the occurrence of N antiresonances and $N - 1$ resonances in the conductance of the QW (see Ref. [9] for further details).

It is worth to mention that Γ can be regarded as the width of the antiresonance when $N = 1$ and $Q_1 = \varepsilon - \varepsilon_0$, as can bee seen from (2). On increasing N, the conductance displays N antiresonances. Since the entire energy range spanned by the antiresonances, namely $|\varepsilon - \varepsilon_0|/\Gamma < 2$, is independent of N (see Fig. 1 and discussions in the next section), the average width Γ_N of the antiresonance must scale as $1/N$ on increasing the number of QDs in the array. By analogy with the single antiresonance case, we define an *effective* coupling V_{eff} through the relationship $\Gamma_N \equiv V_{\mathrm{eff}}^2/2v$. Therefore, the effective coupling between the QW and the QD array scales as $V_{\mathrm{eff}} \sim V_0/\sqrt{N}$. This scaling results from the fact that the eigenfunction amplitude in the QD array scales as $1/\sqrt{N}$ at the edges, i.e., at the site directly coupled to the QW.

3. Equivalent dynamical system

When the number of attached QDs is large, a rich phenomenology appears for different values of the Fermi energy. When the Fermi energy lies far from the center of the QW band ($|\varepsilon - \varepsilon_0| > 2V_c$), the conductance presents regular and smooth behavior.

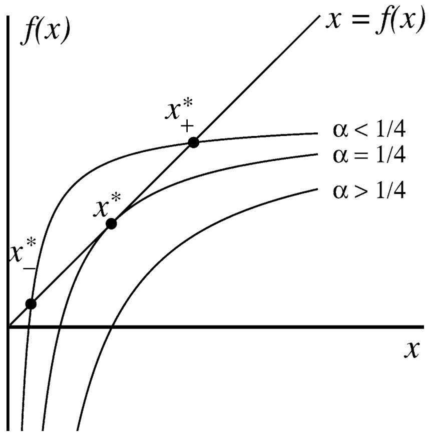

Fig. 2. Mapping of the non-linear map (4) showing the fixed points.

However, the conductance strongly fluctuates close to the center of the QW band for minute variations of the Fermi energy ($|\varepsilon - \varepsilon_0| < 2V_c$). In order to shed light onto this complex behavior, the continued fraction Q_N in Eq. (3) is written as $Q_N = (\varepsilon - \varepsilon_0)x_N$, where x_N satisfies the following non-linear map,

$$x_{N+1} = f(x_N) = 1 - \frac{\alpha}{x_N}, \quad N = 1, 2, 3, \ldots \tag{4}$$

with $x_1 = 1$ and $\alpha \equiv V_c^2/(\varepsilon - \varepsilon_0)^2$ for $\varepsilon \neq \varepsilon_0$. Thus, we are faced to a one-dimensional map (4) with control parameter α.

The equivalent non-linear map has two fixed points at

$$x_\pm^* = \frac{1}{2}(1 \pm \sqrt{1 - 4\alpha}\,), \tag{5}$$

when $\alpha < 1/4$, namely $|\varepsilon - \varepsilon_0| > 2V_c$, as shown in Fig. 2. The fixed point x_+^* (x_-^*) is stable (unstable). This result explains the smooth tails seen in Fig. 1 when $|\varepsilon - \varepsilon_0|/\Gamma > 2$.

The non-linear map (4) undergoes a saddle-node bifurcation at $\alpha = \alpha_{\mathrm{bif}} \equiv 1/4$, that is $|\varepsilon - \varepsilon_0| = 2V_c$, with a single fixed point at $x^* = 1$. There are not fixed points when $\alpha > \alpha_{\mathrm{bif}}$, namely $|\varepsilon - \varepsilon_0| < 2V_c$ (see Fig. 2). Consequently, minute variations of the Fermi energy result in a dramatic change in the conductance of the QW, as it can be concluded from Fig. 1.

4. Periodic orbits

As stated in Ref. [9], when the Fermi energy matches the energy of the QDs ($\varepsilon = \varepsilon_0$) the conductance takes on two values, $G_N = 0$ (perfect reflection) for N odd and $G_N = 2e^2/h$ (perfect transmission) for N even. From the viewpoint of the equivalent dynamical system ($\alpha \to \infty$) this symmetry corresponds to a periodic orbit of period 2. Let us try to generalize this result to get all the possible finite sets of values of the conductance associated to periodic orbits of the system.

Q_N can be expressed in the form [9]: $Q_N = D_N/D_{N-1}$ where $D_N = \det(H_{\mathrm{QD}}^N - \varepsilon I)$. In this way, the eigenenergies of the QD array are zeroes of Q_N, which are also zeroes of G_N. The eigenenergies of a QD array of size N are given by

$$E_{k,N} = \varepsilon_0 + V_c \cos\left(\frac{\pi k}{N+1}\right), \quad k = 1, \ldots, N. \tag{6}$$

Let us study the behavior of the conductance (2) when the Fermi energy ε matches one of the values given in Eq. (6) other than ε_0. It is clear that $Q_N(E_{k,N}) = (\varepsilon - \varepsilon_0)x_N = 0$. As $\varepsilon \neq \varepsilon_0$ it follows that $x_N = 0$. Introducing this result in Eq. (4) we get

$$\begin{aligned} x_{N+1} &= f(x_N) = f(0) = \infty, \\ x_{N+2} &= f(x_{N+1}) = f(\infty) = 1 = x_1, \end{aligned} \tag{7}$$

so we have found a periodic orbit of period $N+1$ for the map (4) that, in terms of the conductance (2), yields

$$\begin{aligned} G_N(E_{k,N}) &= 0, \\ G_{N+1}(E_{k,N}) &= \frac{2e^2}{h}, \\ G_{N+2}(E_{k,N}) &= \frac{2e^2}{h}\frac{(E_{k,N} - \varepsilon_0)^2}{(E_{k,N} - \varepsilon_0)^2 + \Gamma^2} \\ &= G_1(E_{k,N}). \end{aligned} \tag{8}$$

This means that we have only $N+1$ different values for the conductance when $\varepsilon = E_{k,N}$. Thus transport properties of the QW may be used to measure the size of the attached QD array. As an example, Table 1 shows the values of x_N, Q_N and G_N for an orbit of period 4 corresponding to the Fermi energy $\varepsilon = E_{1,3} = \varepsilon_0 + \sqrt{2}\,\Gamma$ ($\alpha = 1/2$).

Table 1
Periodic orbit of period 4

N	$x_N = 1 - 1/2x_{N-1}$	$Q_N(E_{1,3}) = \sqrt{2}\,\Gamma x_N$	$\frac{h}{2e^2} G_N(E_{1,3}) = 2x_N^2/(2x_N^2+1)$
1	1	$\sqrt{2}\,\Gamma$	2/3
2	1/2	$(\sqrt{2}/2)\Gamma$	1/3
3	0	0	0
4	∞	∞	1

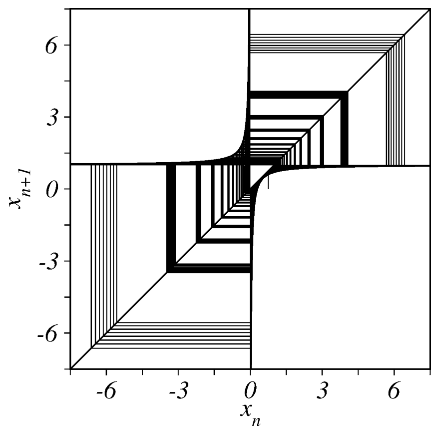

Fig. 3. Quasiperiodic orbit for $\alpha = 0.3$.

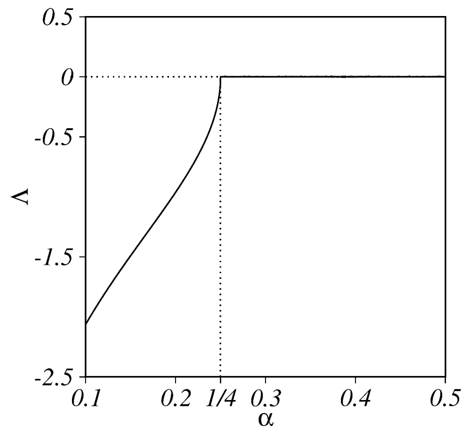

Fig. 4. Lyapunov exponent as a function of α.

5. Quasiperiodic orbits and absence of chaos

Let us now study the behavior of the conductance when the Fermi energy ε belongs to the QW band ($|\varepsilon - \varepsilon_0| < 2V_c$) but is not of the form given in Eq. (6). In this case strong fluctuations of the conductance are observed. Orbits of the non-linear map are not periodic but quasiperiodic, as shown in Fig. 3 for $\alpha = 0.3$.

Finally, we focus the attention on the possible transition to chaotic behavior of the system. In order to survey the existence of chaotic orbits, we calculate the Lyapunov exponent $\Lambda(\alpha)$ as a function of α. As shown in Fig. 4 $\Lambda(\alpha) < 0$ for $\alpha < \alpha_{\mathrm{bif}}$ and $\Lambda(\alpha) = 0$ for $\alpha \geqslant \alpha_{\mathrm{bif}}$ thus ruling out chaotic orbits for any value of α.

6. Summary

In summary, we have studied the transport properties of independent carriers through a QW side-coupled to a QD array. Based on an equivalent dynamical system, we have carried out an analysis of the previously reported complex behavior of the conductance at zero temperature. When the Fermi energy lies far from the level of the QD, the conductance depends smoothly on both the energy and the number of QDs. We have related this feature with the existence of fixed points of the equivalent dynamical system. On the contrary, when the Fermi energy is close to the level of the QD, the conductance develops resonances and antiresonances due to constructive and destructive interference in the ballistic channel, respectively. In this case there are not fixed points. The previously found even–odd symmetry when the Fermi energy matches the level of the QD has been extended to the rest of the eigenenergies of the QD array for any size N, based on the notion of periodic orbits of the non-linear map.

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