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Conductance control in quantum wires by attached quantum dots

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The electronic transport of two quantum dots side-coupled to a quantum wire is studied by means of the two impurity Anderson Hamiltonian. The conductance is found to be a superposition of a Fano and a Breit–Wigner resonances as a function of the Fermi energy, provided the gate voltages of the quantum dots are slightly different.

1 Introduction

Recent progress in nanofabrication of quantum devices enables to study electron transport through quantum dots (QDs) in a very controllable way. QDs are very promising systems due to their physical properties as well as their potential application in electronic devices. These structures are small semiconductor or metal structures in which electrons are confined in all spatial dimensions. As a consequence, discreteness of energy and charge arise. For this reason QDs are often referred as *artificial atoms*. Two or more QDs can be coupled to form an artificial molecule sharing electrons. This analogy opens the way to look for new electronic effects that might resemble quantum optics. In this way, it has been recently demonstrated that coupled QDs shows the electronic counterpart of Fano and Dicke effects that can be controled via a magnetic flux [1].

In this work we study electron transport properties of a double QD side attached to a quantum wire (QW). We examine the linear conductance at zero temperature and obtain the associated density of states when the gate voltages of the QDs are slightly different. The density of states is the sum of two Lorentzians, namely a narrow one and a wide one. Thus, these states can be viewed as long- and short-living states. We have found that these resonant states have marked effects on the electron transport accross the QW, thus allowing for a fine control of the conductance.

2 Model

The system under consideration is formed by two QDs connected to a QW, as shown schematically in Fig. 1. The dots are composed of two atomic sites connected by tunneling coupling. The entire system is modeled by a two impurity Anderson Hamiltonian, that can be written as $H = H_{\mathrm{W}} + H_{\mathrm{D}} + H_{\mathrm{WD}}$ where

$$H_{\mathrm{W}} = -v \sum_{\langle i \neq j \rangle} \sum_{\sigma} (c_{i\sigma}^{\dagger} c_{j\sigma} + c_{i\sigma} c_{j\sigma}^{\dagger}) ,$$

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$$H_D = \sum_{\alpha=1,2} \sum_{\sigma} \varepsilon_\alpha d^\dagger_{\alpha\sigma} d_{\alpha\sigma} + U \sum_{\alpha=1,2} n_{\alpha\uparrow} n_{\alpha\downarrow} , \tag{1}$$

$$H_{WD} = -V_0 \sum_{\alpha=1,2} \sum_{\sigma} (d^\dagger_{\alpha\sigma} c_{0\sigma} + c^\dagger_{0\sigma} d_{\alpha\sigma}) .$$

Here $\sigma = \uparrow, \downarrow$ denotes the spin index. $c^\dagger_{i\sigma}$ and $c_{i\sigma}$ are the electron creation and annihilation operators at site i of the QW with spin σ, respectively. Correspondingly, $d^\dagger_{\alpha\sigma}$ and $d_{\alpha\sigma}$ creates and annihilates an electron at the QD $\alpha = 1, 2$ with spin σ. Moreover, $n_{\alpha\sigma} \equiv d^\dagger_{\alpha\sigma} d_{\alpha\sigma}$ and U is the Hubbard energy. The site-energy at the QW is assumed to be zero and the hopping in the QW is denoted by v, whereas V_0 couples site 0 of the QW to both QDs.

3 Zero temperature conductance

Hereafter we will neglect the Hubbard term U and the spin index, so we will be dealing with a one-electron problem. Therefore, the reduced Hamiltonian

$$H = -v \sum_{\langle i \neq j \rangle} (c^\dagger_i c_j + c_i c^\dagger_j) + \sum_{\alpha=1,2} \varepsilon_\alpha d^\dagger_\alpha d_\alpha - V_0 \sum_{\alpha=1,2} (d^\dagger_\alpha c_0 + c^\dagger_0 d_\alpha) \tag{2}$$

corresponds to the simplified system depicted in Fig. 1.

The linear conductance can be obtained from Landauer formula at zero temperature [2]

$$\mathcal{G} = \frac{2e^2}{h} T(\omega = \varepsilon_F) , \tag{3}$$

where ε_F is the Fermi energy and $T(\omega)$ is the transmission probability, given by

$$T(\omega) = \frac{2\Gamma_L(\omega)\, \Gamma_R(\omega)}{\Gamma_L(\omega) + \Gamma_R(\omega)} \,\mathrm{Im}\,[G^W_0] . \tag{4}$$

Here G^W_0 is the Green's function at site 0 of the QW (see Fig. 2), $\Gamma_{L(R)}$ is the coupling of this site to the left (right) side of the QW.

By using a Dyson equation we calculate the Green's function at site 0 of the QW coupled to the QDs, obtaining the following expression

$$G^W_0 = \frac{i}{2v \sin k} \frac{1}{1 - i\gamma(g_1 + g_2)} , \tag{5}$$

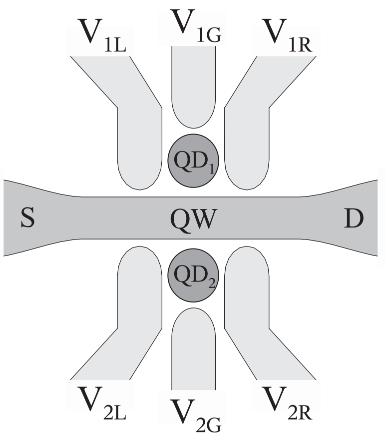

Fig. 1 Schematic view of the two quantum dots attached to quantum wire. Current passing from the source (S) to the drain (D) is controled by the gate voltages V^1_G and V^2_G.

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where $\gamma = \pi V_0^2/2v \sin k$ with $k = \arccos(-\omega/2v)$. g_α is given by the following equation:

$$g_\alpha = \frac{1}{\omega - \varepsilon_\alpha}, \qquad \alpha = 1, 2. \tag{6}$$

By considering symmetric couplings $\Gamma_\mathrm{L} = \Gamma_\mathrm{R} = \Gamma(\omega) = 2v \sin k$ we can obtain the linear conductance at zero temperature

$$\begin{aligned} \mathcal{G} &= \frac{2e^2}{h} \Gamma(\omega) \operatorname{Im}[G_0^W(\omega)]|_{\omega=\varepsilon_\mathrm{F}} \\ &= \frac{2e^2}{h} \frac{[(\omega-\varepsilon_1)(\omega-\varepsilon_2)]^2}{[(\omega-\varepsilon_1)(\omega-\varepsilon_2)]^2 + \gamma^2[2\omega-(\varepsilon_1+\varepsilon_2)]^2} |_{\omega=\varepsilon_F}. \end{aligned} \tag{7}$$

4 Density of states

The density of states (DOS) can give us a better understanding of the transport properties of the system. To obtain it, we calculate the diagonal elements of the Green's functions of the QDs, G^α with $\alpha = 1, 2$,

$$G_\alpha = g_\alpha + \frac{i\gamma |g_\alpha|^2}{1 - i\gamma(g_1 + g_2)}. \tag{8}$$

First we require the local density of states at each QD, ρ_α, from the imaginary part of G_α, and then we obtain the DOS summing over α,

$$\rho = \frac{\gamma}{\pi} \sum_{\alpha=1,2} \frac{(\omega-\varepsilon_\alpha)^2}{[(\omega-\varepsilon_1)(\omega-\varepsilon_2)]^2 + \gamma^2[2\omega-(\varepsilon_1+\varepsilon_2)]^2}. \tag{9}$$

Setting the sites energies as $\varepsilon_1 = \varepsilon_0 + \Delta V$ and $\varepsilon_2 = \varepsilon_0 - \Delta V$ and taking $\Delta V \ll \gamma$, the DOS reduces to

$$\rho \approx \frac{1}{\pi} \frac{2\gamma}{(\omega-\varepsilon_0)^2 + 4\gamma^2} + \frac{1}{\pi} \frac{\Delta V^2/2\gamma}{(\omega-\varepsilon_0)^2 + (\Delta V^2/2\gamma)^2}. \tag{10}$$

The DOS is found to be the sum of two Lorentzian with widths $\Gamma_+ = 2\gamma$ and $\Gamma_- = \Delta V^2/2\gamma$, as shown in Fig. 3. This behavior resembles the Dicke effect in optics, which takes place in the spontaneous emission of a pair of atoms radiating a photon with a wave length much larger than the separation between them [3]. The luminescence spectrum is characterized by a narrow and a broad peak, associated with long and short-lived states, respectively. The former state, weakly coupled to the electromagnetic field, is called *subradiant*, and the latter, strongly coupled, *superradiant* state. Our results indicate that the analogy between the electron device and the optical system could be ultimately exploited to uncover new effects in quantum electronics.

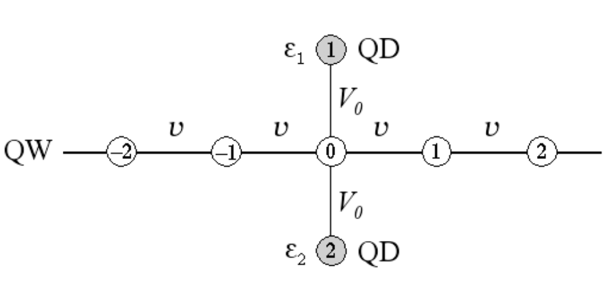

Fig. 2 Simplified model of the quantum dots attached to a quantum wire shown in Fig. 1.

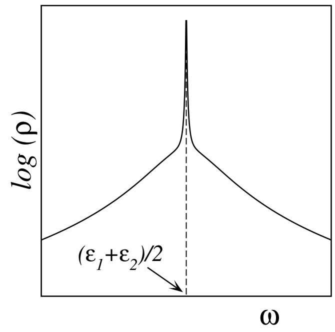

Fig. 3 Sketch of the density of states as a function of energy, obtained as a sum of two Lorentzian functions of very different widths.

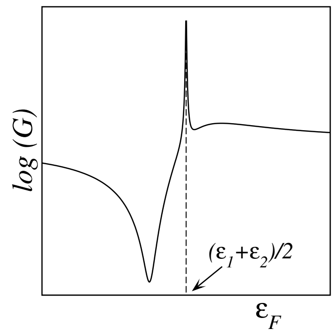

Fig. 4 Sketch of the conductance as a function of the Fermi energy, obtained as a sum of a Fano and a Breit–Wigner line shapes.

On the other hand, from Eq. (7) the conductance can be written as

$$\mathcal{G} = \frac{2e^2}{h}\left(\frac{\omega^2}{(\omega-\varepsilon_0)^2+4\gamma^2} + \frac{(\Delta V^2/2\gamma)^2}{(\omega-\varepsilon_0)^2+(\Delta V^2/2\gamma)^2}\right)\Bigg|_{\omega=\varepsilon_F}. \tag{11}$$

The conductance is the superposition of a Fano line shape and a Breit–Wigner line shape, as depicted in Fig. 3.

In the limit $\Delta V \to 0$ a bound state arises at an energy ε_0,

$$\rho = \frac{1}{\pi}\,\frac{2\gamma}{(\omega-\varepsilon_0)^2+4\gamma^2} + \delta(\omega-\varepsilon_0)\,. \tag{12}$$

Correspondingly, the conductance is reduced to a Fano line shape

$$\mathcal{G} = \frac{2e^2}{h}\,\frac{\omega^2}{(\omega-\varepsilon_0)^2+4\gamma^2}\Bigg|_{\omega=\varepsilon_F}. \tag{13}$$

The appearance of the bound state is due to that both QDs are coupled indirectly through the QW, yielding a level mixing of both QDs and forming collective states.

5 Conclusion

Two QDs with slightly different gate voltages and attached to a QW allow for the control of the conductance. By using a two impurity Anderson Hamiltonian we have found closed analytical expresions for both the conductance and the density of states. The conductance is the superposition of a Fano line shape and a Breit–Wigner line shape. In addition, the density of states is the sum of two Lorentzian of different widths. It resembles the Dicke effect in optics, which opens the possibility of new quantum electron devices based on physical effects that are usually encountered in quantum optics.

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