Control of electron transport through a quantum wire by side-attached nanowires

P. A. Orellana[*, 1], **M. L. Ladrón de Guevara**[1], **F. Domínguez-Adame**[2], and **I. Gómez**[2]

[1] Departamento de Física, Universidad Católica del Norte, Casilla 1280, Antofagasta, Chile
[2] Departamento de Física de Materiales, Universidad Complutense, 28040 Madrid, Spain

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A system of arrays of nanowires side-coupled to a quantum wire is studied. Transport through the quantum wire is investigated by using a noninteracting-electron Anderson tunneling Hamiltonian. An analytical expression of the conductance at zero temperature is given, showing a band with alternating forbidden and allowed minibands due to the discrete structure of the nanowires. The conductance is found to exhibit a forbidden miniband in the center of the band for an odd number of sites in the nanowires, while shows an allowed band for an even number.

1 Introduction

Quantum effects in these nanowires (NWs) are potentially useful in nanotechnology, since coupling to the continuum states shows an even-odd parity effect in the conductance when the Fermi energy is pinned at the center of the energy band [1, 2]. In this context, we have recently considered a new quantum device based on a quantum wire (QW) coupled to a NW [3], and to a nanoring [4] which acts as scatterer for electron transmission through the QW. In this work we report further progress along the lines indicated above. In particular, we study theoretically the transport properties of a set of side-coupled NWs attached to a perfect QW. We find an analytic expression for the conductance at zero temperature, which shows a band with alternating forbidden and allowed energy intervals (for short referred to as minibands hereafter). We also find a general even-odd parity effect when the Fermi energy is located at the center of the band.

2 Model

The system consists of a QW connected to a number N of side-attached NWs with M sites of one energy level (see Fig. 1). The system, assumed in equilibrium, is modeled by a noninteracting-electron Anderson tunneling Hamiltonian that can be written as $H = H_{\mathrm{QW}} + H_{\mathrm{NW}} + H_{\mathrm{QW-NW}}$, where

$$H_{\mathrm{QW}} = v \sum_i \left(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i\right), \tag{1a}$$

describes the dynamics of the QW, v being the hopping between neighbor sites of the QW, and $c_i^\dagger$ (c_i) creates (annihilates) an electron at the ith site. H_{NW}, given by

$$H_{\mathrm{NW}} = \sum_{j=1}^{N} \sum_{l=1}^{M} \varepsilon_{j,l} d_{j,l}^\dagger d_{j,l} + V_c \sum_{j=1}^{N} \sum_{l=1}^{M-1} \left(d_{j,l}^\dagger d_{j,l+1} + \mathrm{h.c.}\right), \tag{1b}$$

[*] Corresponding author: e-mail: orellana@ucn.cl

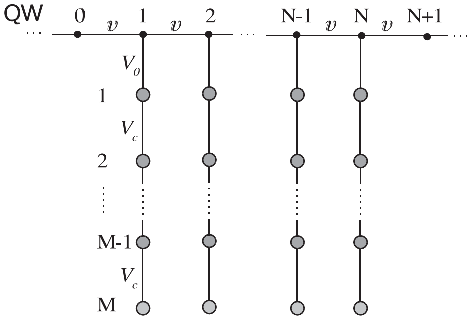

Fig. 1 Schematic view of the QW with the side-attached NWs.

is the Hamiltonian for the N side-attached NWs, where $d_{j,l}$ ($d_{j,l}^\dagger$) is the annihilation (creation) operator of an electron in the quantum dot l of the jth NW, $\varepsilon_{j,l}$ is the corresponding single level energy, and V_c the tunneling coupling between sites in the NWs, assumed all equal. The coupling of the QW with the side-attached NWs is described by the Hamiltonian

$$H_{\mathrm{QW-NW}} = V_0 \sum_{j=1}^{N} \left(d_{j,1}^\dagger c_j + c_j^\dagger d_{j,1} \right) , \tag{1c}$$

where V_0 is the hopping between the QW and the NWs.

The Hamiltonian for the QW, H_{QW}, corresponds to the free-particle Hamiltonian on a lattice with spacing unity, whose dispersion relation is $\varepsilon = 2v \cos k$. Consequently, the Hamiltonian supports an energy band from $-2v$ to $+2v$. We assume that the electrons are described by a plane wave incident from the far left with unity amplitude and a reflection amplitude r, and at the far right by a transmission amplitude t. We then obtain the following expression

$$t = 2i\, \mathrm{e}^{-iNk} \left(\mathrm{e}^{-ik} \varDelta_N + 2\varDelta_{N-1} + \mathrm{e}^{ik} \varDelta_{N-2} \right)^{-1} \sin k , \tag{2a}$$

where

$$\varDelta_N = \begin{cases} \dfrac{\sin\,[(N+1)q]}{\sin q} , & |(\varepsilon - \tilde{\varepsilon})/2v| \le 1 , \\[2ex] \dfrac{\sin\,[(N+1)\kappa]}{\sin \kappa} , & \text{otherwise.} \end{cases} \tag{2b}$$

For the sake of simplicity we have defined $\cos q = (\varepsilon - \tilde{\varepsilon})/2v$ and $\cosh \kappa = |(\varepsilon - \tilde{\varepsilon})/2v|$, where $\tilde{\varepsilon} = (V_0^2/V_c)\, \sin\,[(M+1)\theta]/\sin M\theta$ and $\cos\theta = (\varepsilon - \varepsilon_0)/2V_c$.

3 Results

The linear dimensionless conductance g at the Fermi energy ε is given by the one-channel Landauer formula at zero temperature, $g(\varepsilon) = |t|^2$. If $|\varepsilon - \tilde{\varepsilon}| \le 2v$, we get

$$g = \frac{1}{\cos^2(Nq) + \left[\sin(Nq)(1 + \cos q \cos k)/(\sin q \sin k) \right]^2} , \tag{3a}$$

that is, g oscillates as a function of N and q. On the contrary, when $|\varepsilon - \tilde{\varepsilon}| > 2v$ we get

$$g = \frac{1}{\cosh^2(N\kappa) + \left[\sinh(N\kappa)\,(1 + \cosh\kappa \cos k)/(\sinh\kappa \sin k) \right]^2} . \tag{3b}$$

Thus, g vanishes exponentially when N is large, as a function of the product $N\kappa$, $g \sim e^{-2N\kappa}$.

The conductance is found to exhibit forbidden minibands (minigaps) that depend on the number of sites of the attached NWs. To illustrate this behavior, let us consider first the simplest cases with $M = 1$ and $M = 2$, that is, one and two sites in the side-attached arrays, respectively. Figure 2 shows

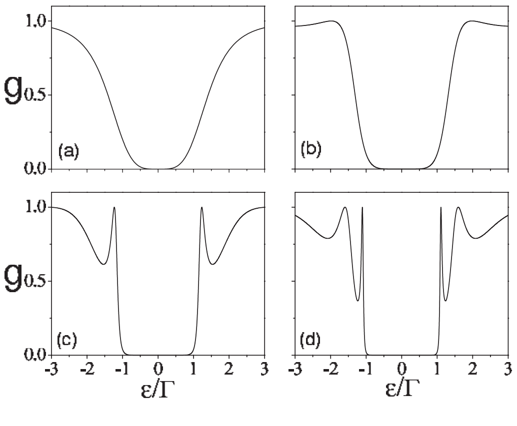

Fig. 2 Dimensionless conductance versus Fermi energy, in units of Γ, for $M = 1$ and a) $N = 2$, b) $N = 3$, c) $N = 5$ and d) $N = 7$ when $V_c = \Gamma$ and $\varepsilon_0 = 0$.

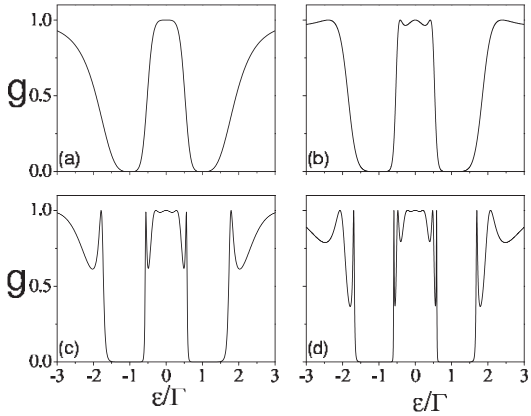

Fig. 3 Dimensionless conductance versus Fermi energy, in units of Γ, for $M = 2$, a) $N = 1$, b) $N = 3$, c) $N = 5$, and d) $N = 7$ NWs, with $V_c = \Gamma$ and $\varepsilon_0 = 0$.

the conductance versus ε for $M = 1$ and different values of the number of arrays N. For N sufficiently large, g vanishes within a range $[-\Gamma, \Gamma]$, with $\Gamma = V_0^2/2v$, and the system shows a minigap of width 2Γ. Figure 3 displays the conductance for $M = 2$. Now the minigaps take place around the bonding ($\varepsilon_- = -V_c$) and the antibonding ($\varepsilon_+ = V_c$) energies of the attached NWs. Moreover, an allowed miniband develops around the center of the band.

For larger M and fixed N, the system develops a set of alternating forbidden and allowed minibands in the range $[-2V_c, 2V_c]$. It is found that the number of forbidden minibands matches exactly the number of sites in the attached NWs, M, and the number of the allowed bands equals $M - 1$. Furthermore, the minigaps open around the energies in the spectrum of the isolated NW. In fact, from Eq. (3b) we can conclude that the conductance vanishes when $\kappa \to \infty$, i.e., $|(\varepsilon - \tilde{\varepsilon})/2v| \to \infty$. This condition is satisfied if $\theta = m\pi/M$ with $m = 1, \ldots, M$ and the respective energies are $\varepsilon = \varepsilon_0 + 2V_c \cos[m\pi/(M+1)]$. These energies correspond to the spectrum of an isolated NW. On the other hand, it follows from Eq. (3a) that, within each allowed miniband, the condition of maximum transmission is reached when $\sin(Nq) = 0$, i.e, $q = n\pi/N$ with $n = 1 \ldots N$. Then, each allowed miniband of the conductance has N maxima. Additionally, an interesting property arises in relation to the single attached NW, namely the odd-even parity effect [3]. If the number M is odd, a forbidden band develops around center of the band while an allowed one arise for M even.

4 Summary

In this work we studied the conductance at zero temperature through a QW with a set of arrays of side-attached NWs. We found that the conductance at zero temperature displays an oscillating pattern with forbidden and allowed minibands, due to constructive and destructive interference in the ballistic channel, respectively. For uniform NW arrays of M sites, M minigaps and $M - 1$ allowed minibands arise. The minigaps develop around the electronic level of an isolate NW. It should be stressed that the particular setup we suggested allows us to control the energy and the width of the minibands in an independent fashion. Moreover, the system shows an odd-even parity behavior of the conductance when the Fermi energy lies at the center of the band. If the number of sites in the NWs is even, an allowed miniband is developed. On the contrary, a minigap is formed when this number is odd. This property arises from the intrinsic electronic properties of the NWs.

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