The role of topology in the evolution of cooperation

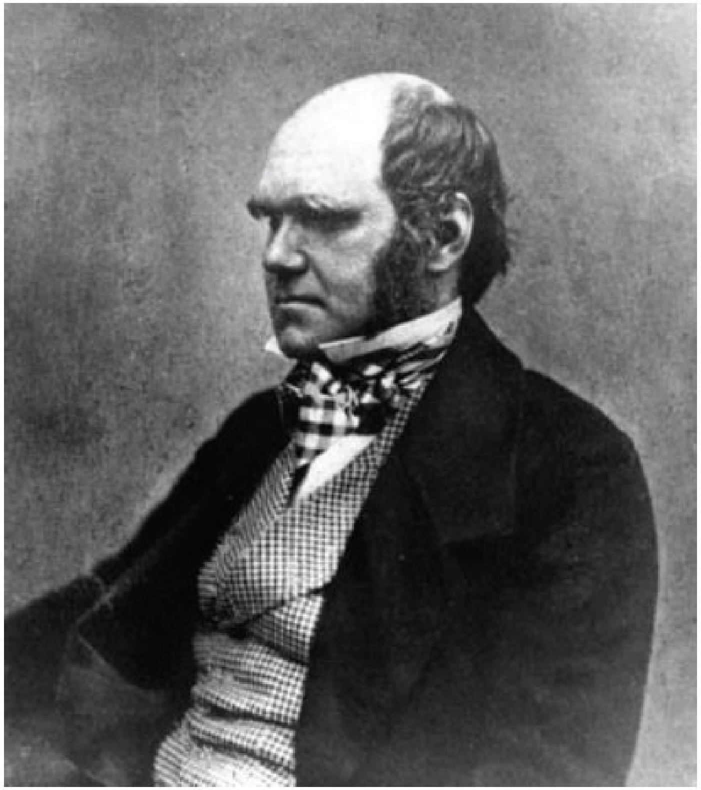
Madrid, February 18th, 2011

GISC annual workshop

Summary

- Introduction
- Social dilemmas and Game Theory
- Well mixed populations
- Lattices and random topologies
- Conclusions

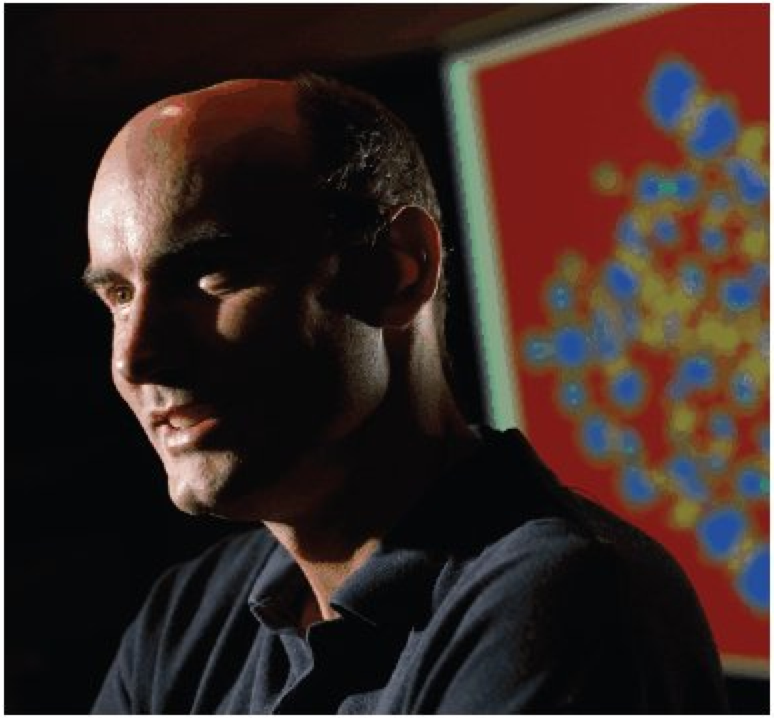
The puzzle of the emergence of cooperation



He who was ready to sacrifice his life (…), rather than betray his comrades, would often leave no offspring to inherit his noble nature… Therefore, it seems scarcely possible (…) that the number of men gifted with such virtues (…) would be increased by natural selection, that is, by the survival of the fittest.

Charles Darwin
(*Descent of Man*, 1871)

The hypothesis of structured population



Martin A. Nowak and Robert M. May,
Nature **359**, 826 (1992)

Spatial structure promotes cooperation in evolutionary game theory (network reciprocity)

2x2 Symmetric social dilemmas

- 2 players
- 2 strategies: Cooperate or Defect

	C	D
C	1	S
D	T	0

$T > 1$: temptation to defect

$S < 0$: risk in cooperation

2x2 Symmetric social dilemmas

	C	D
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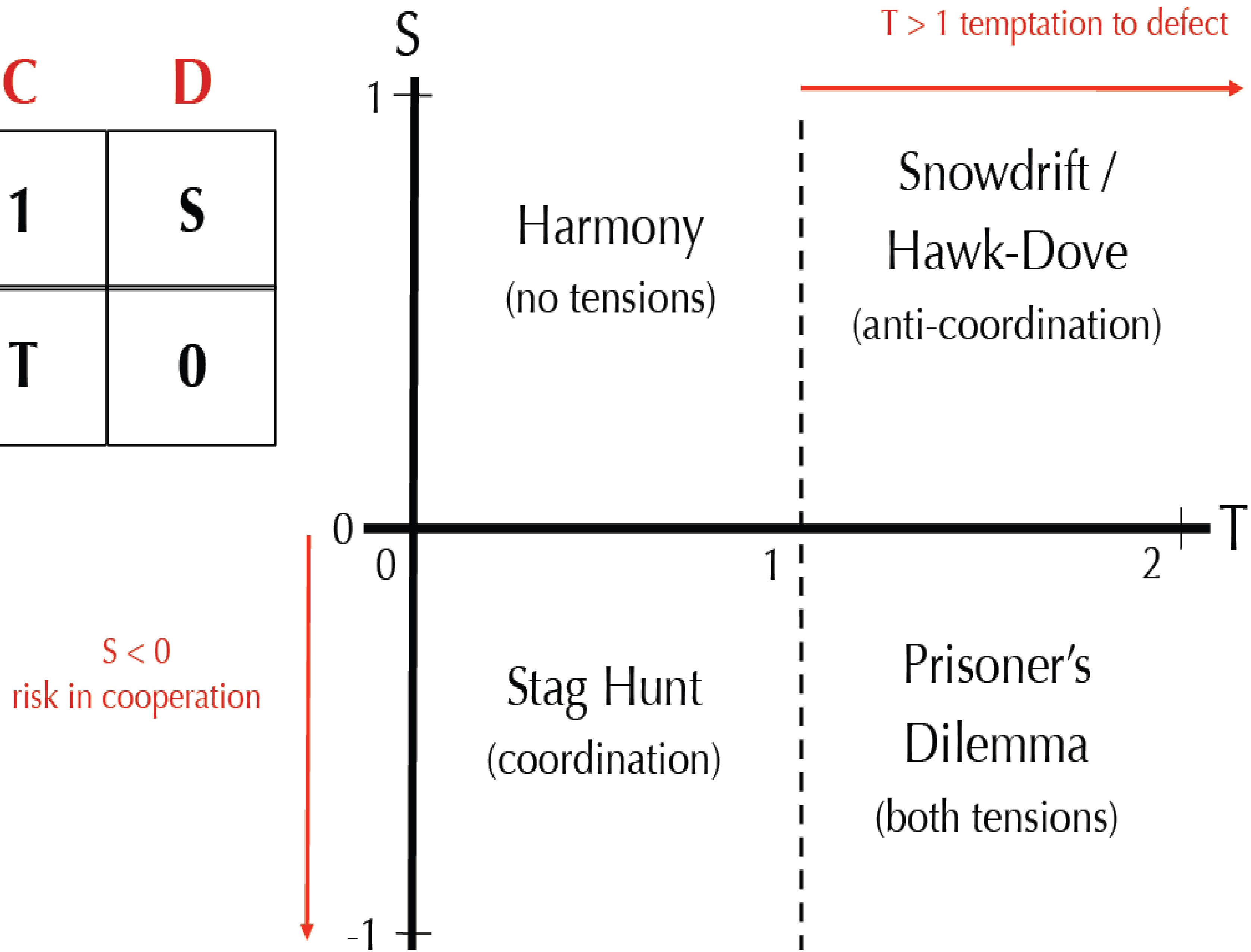


The framework of evolutionary game theory

Well mixed population:

Individuals reproduce according to payoff accumulated after a round of games with everybody else

Replicator equation

$$\frac{\dot{x}_c}{x_c} = f_c - \bar{f} \quad \text{or} \quad \dot{x}_c = x_c(1 - x_c)(f_c - f_d)$$

Theorem: Evolutionarily stable states (ESS) are Nash equilibria; strict Nash equilibria are ESS

The Prisoner's Dilemma Game

- $T>1$, $0<S<1$: cooperation riskful and defection temptating;
- Nash equilibrium: (D,D) for two players, $x_c=0$ in well-mixed populations;
- But global cooperation would be more convenient!
- For the previous reason, PDG is maybe the most useful model in order to investigate the emergence of cooperation in a population.

Numerical simulations: update rules

- Replicator dynamics (REP): choose a neighbour at random, and adopt its strategy (if larger than yours) with probability proportional to the difference between the payoffs;
- Unconditional imitation (UI): choose the strategy of the neighbour with the largest payoff (if larger than yours);
- Moran's rule (MOR):

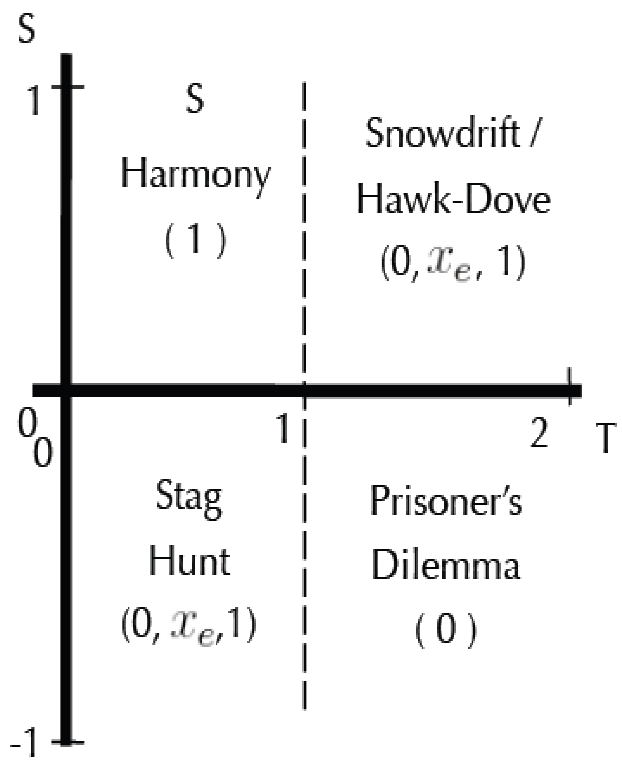
$$\mathcal{P}\{s_i^{t+1} \leftarrow s_j^t\} = \frac{\pi_j^t - \Psi}{\sum_{k \in N_i^*} (\pi_k^t - \Psi)}$$

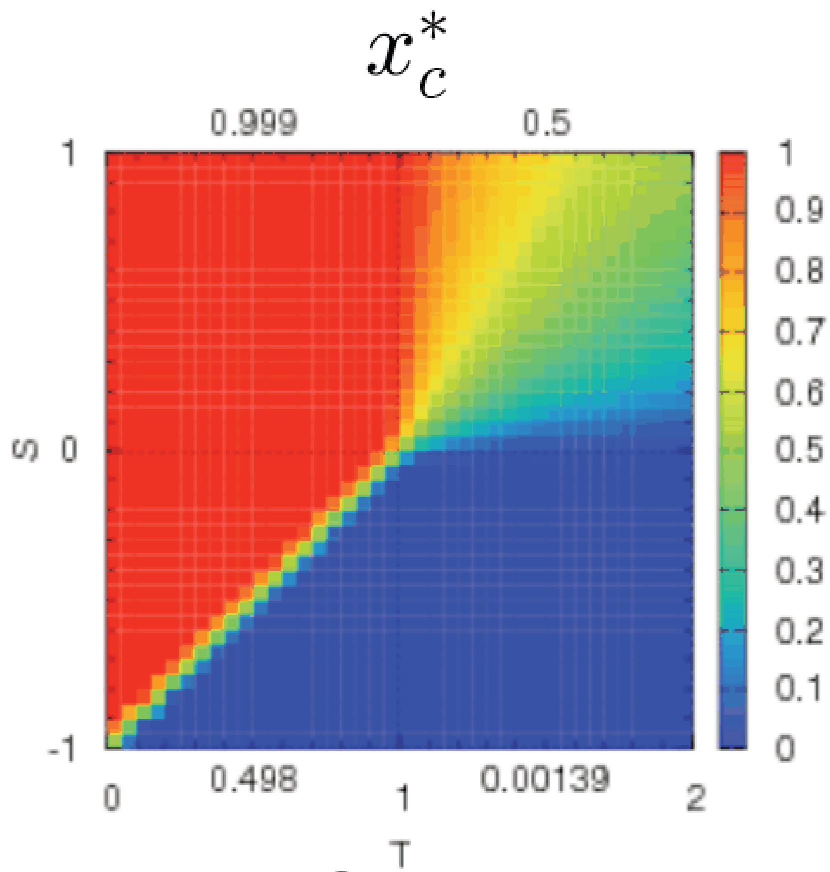
Well-mixed populations

Standard reference: replicator dynamics on a complete network

	C	D
C	**1**	**S**
D	**T**	**0**

$$x_e = \frac{S}{S+T-1}$$



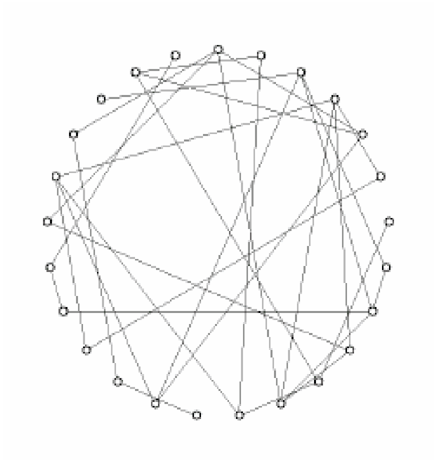


$$x^0 = 0.5$$

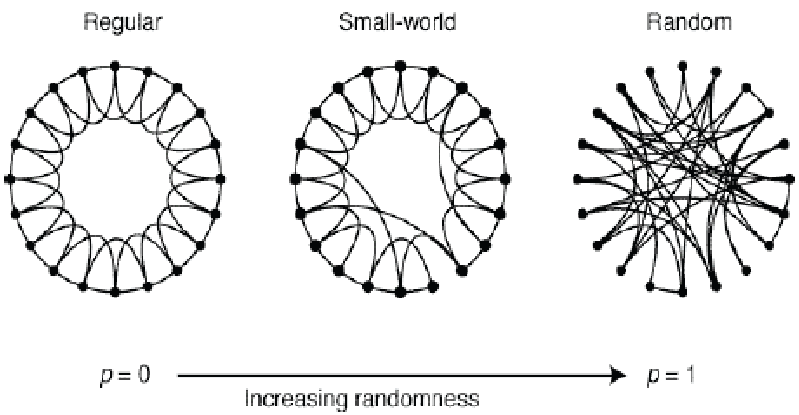
$$\dot{x}_c = x_c(1-x_c)(f_c - f_d)$$

Random Topologies

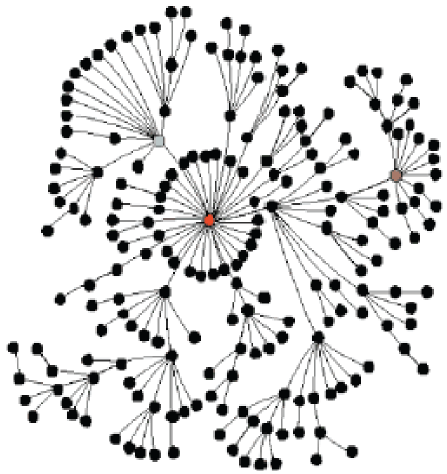
Types of model graphs



Random
(Erdös-Renyi)



Small world
(Watts-Strogatz)



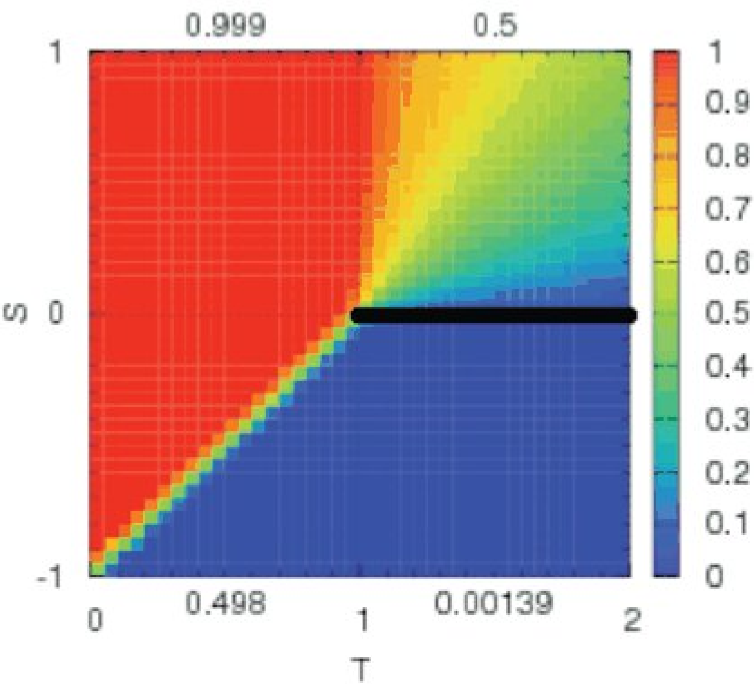
Scale free
(Barabási-Albert)

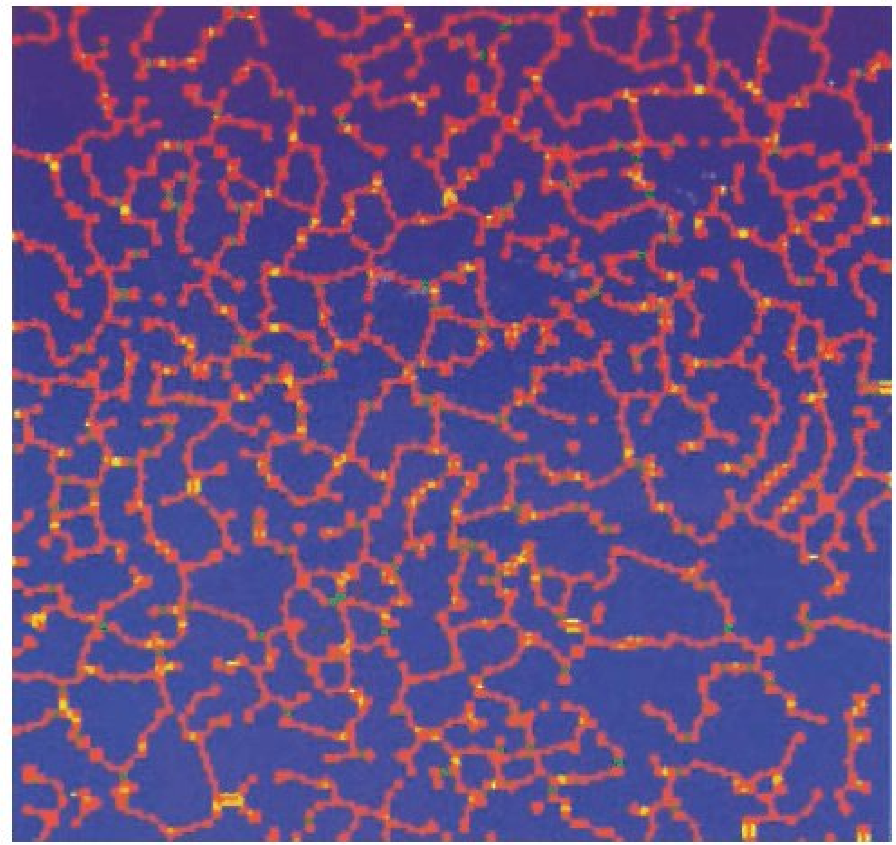
Seminal result on spatial structure

M. A. Nowak & R. M. May, *Nature* **359**, 826 (1992)

$$\begin{array}{cc} & \begin{array}{cc} \mathrm{C} & \mathrm{D} \end{array} \\ \begin{array}{c} \mathrm{C} \\ \mathrm{D} \end{array} & \begin{pmatrix} 1 & 0 \\ T & \epsilon \end{pmatrix} \end{array}$$

$$1 \leq T \leq 2 \qquad \epsilon \lesssim 0$$





$$1.75 < T < 1.8$$

$$0.7 < x_c^* < 0.95$$

- In fact, the effect of spacial structure on cooperation is more complex: **<u>no universality</u>**;
- From Roca-Cuesta-Sánchez, *Phys. of Life Rev.* **6**, 208-249 (2009):

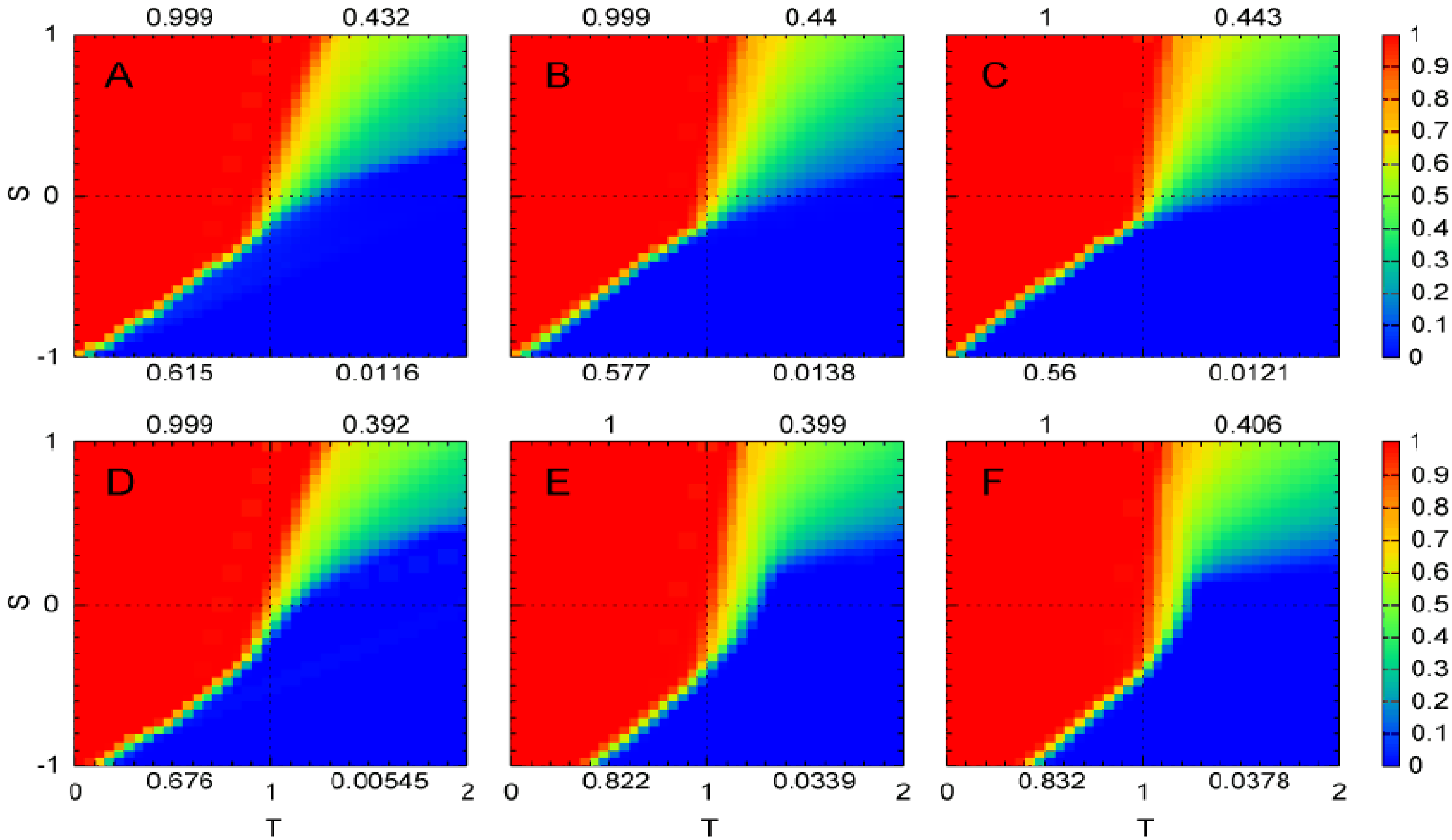


Figure 13: Asymptotic density of cooperators x^* in homogeneous random networks (upper row, A to C) compared to regular lattices (lower row, D to F), with degrees $k = 4$ (A, D), 6 (B, E) and 8 (C, F). The update rule is the replicator rule and the initial density of cooperators is $x^0 = 0.5$. The plots show that the main influence occurs in Stag Hunt and Snowdrift games, specially for regular lattices with large clustering coefficient, $k = 6$ and 8 (see main text).

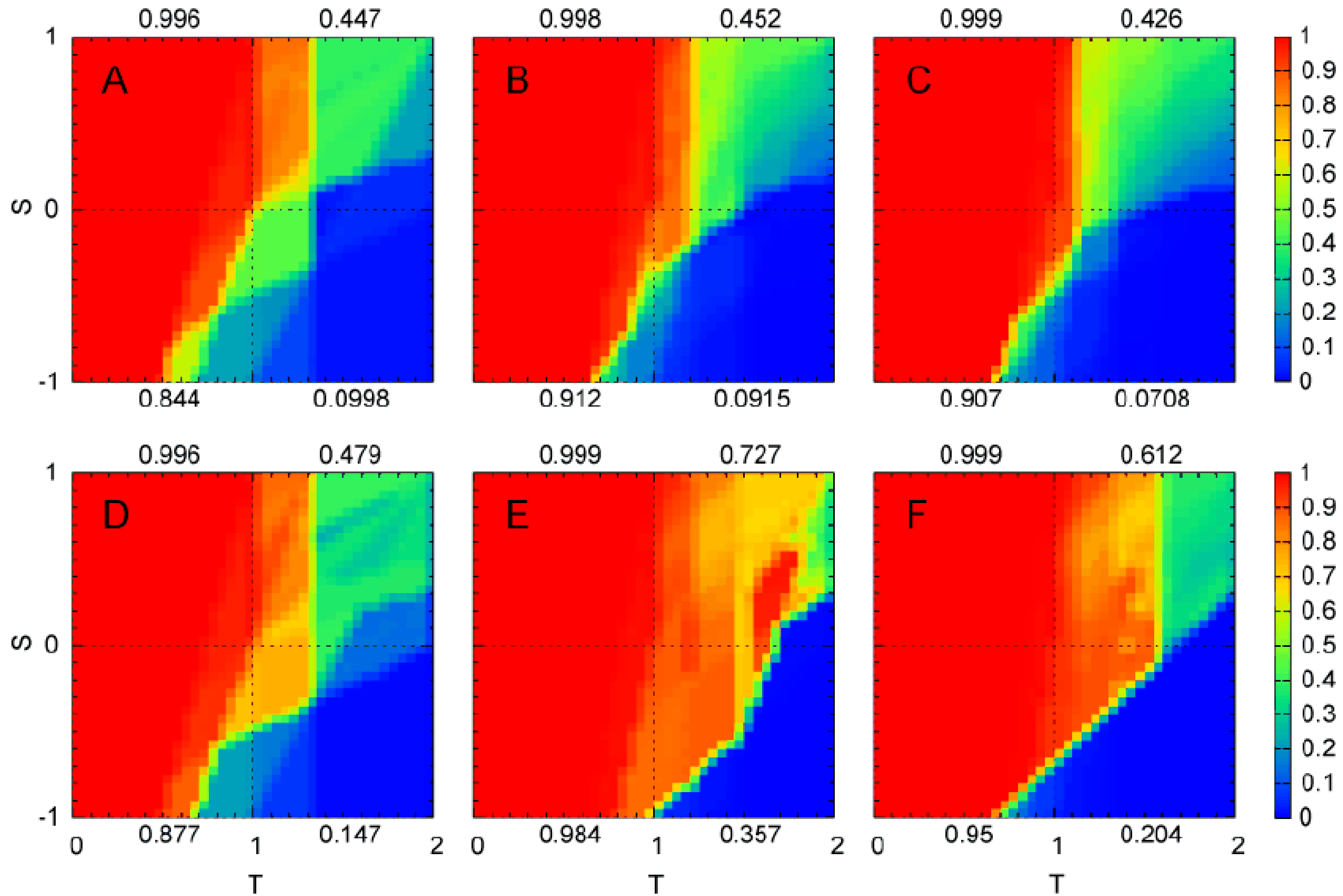


Figure 15: Asymptotic density of cooperators x^* in homogeneous random networks (upper row, A to C) compared to regular lattices (lower row, D to F), with degrees $k = 4$ (A, D), 6 (B, E) and 8 (C, F). The update rule is unconditional imitation and the initial density of cooperators is $x^0 = 0.5$. Again as in Fig. 13, spatial lattices have greater influence than random networks when the clustering coefficient is high ($k = 6$ and 8). In this case, however, the beneficial effect for cooperation goes well into Snowdrift and Prisoner's Dilemma quadrants.

Mixing update rules

- What happens if in a population there are individuals with <u>different</u> update rules?
- When is the cooperation enhanced in this case?
- And what happens if also the update rules can evolve (i.e., if an agent imitates a neighbour's strategy, it copies its update rule, too)?
- In the last case, which is the favoured update rule?

Fixed and evolving update rules

Cardillo, Gómez-Gardeñes, Vilone, Sánchez, ***NJP***, 103034 (2010)

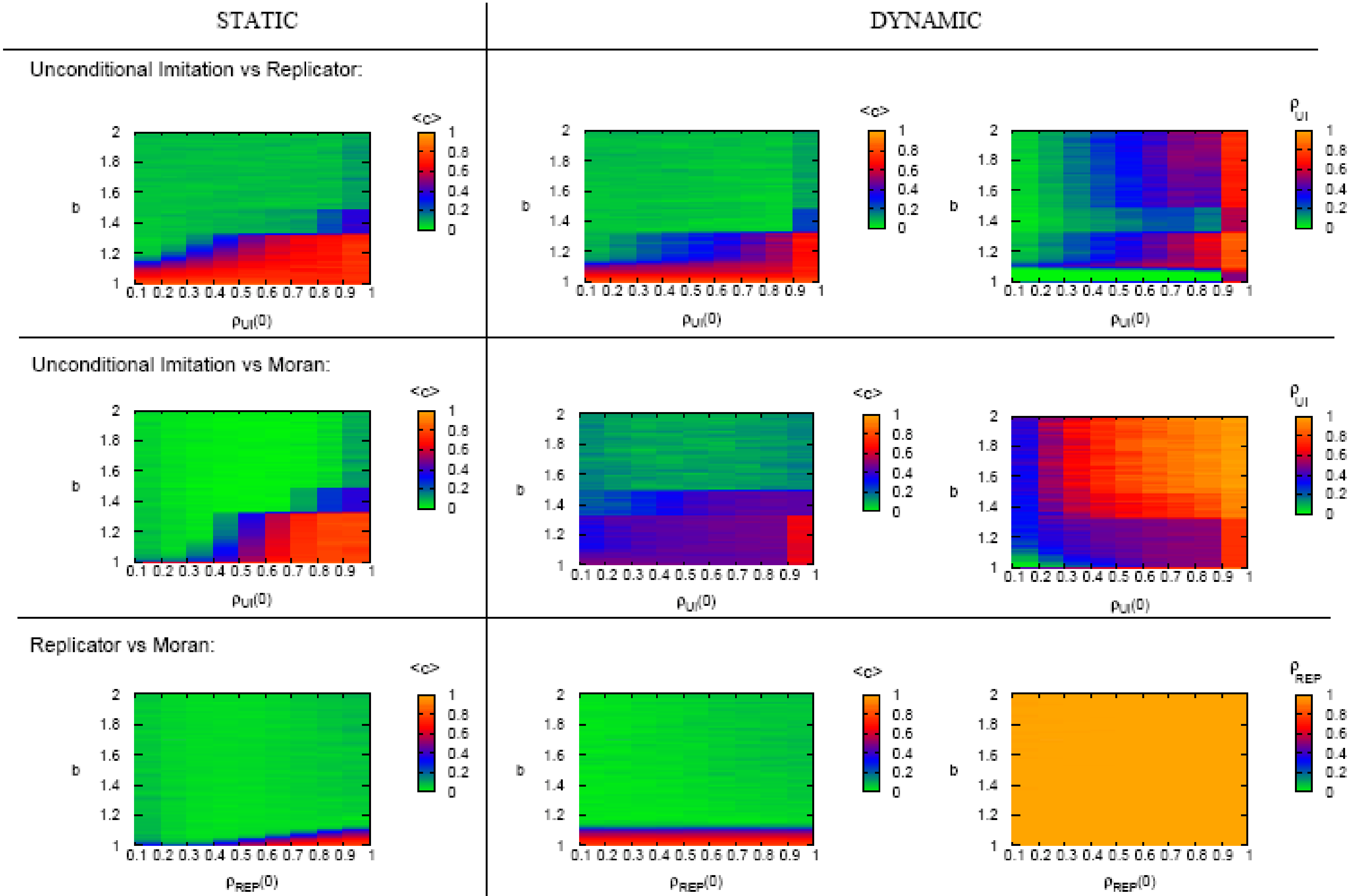


FIG. 5: Lattice

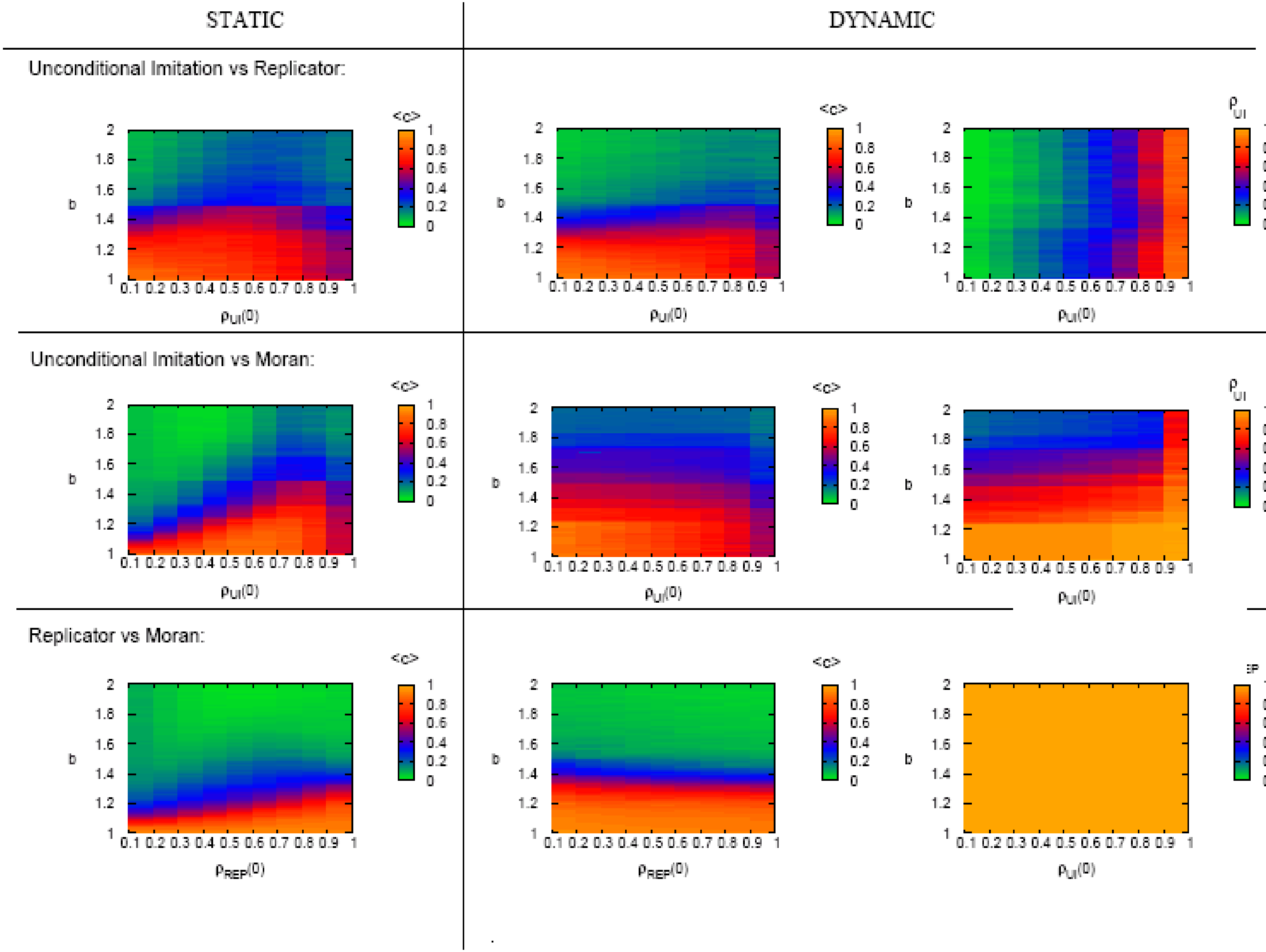


FIG. 6: Erdős-Rényi

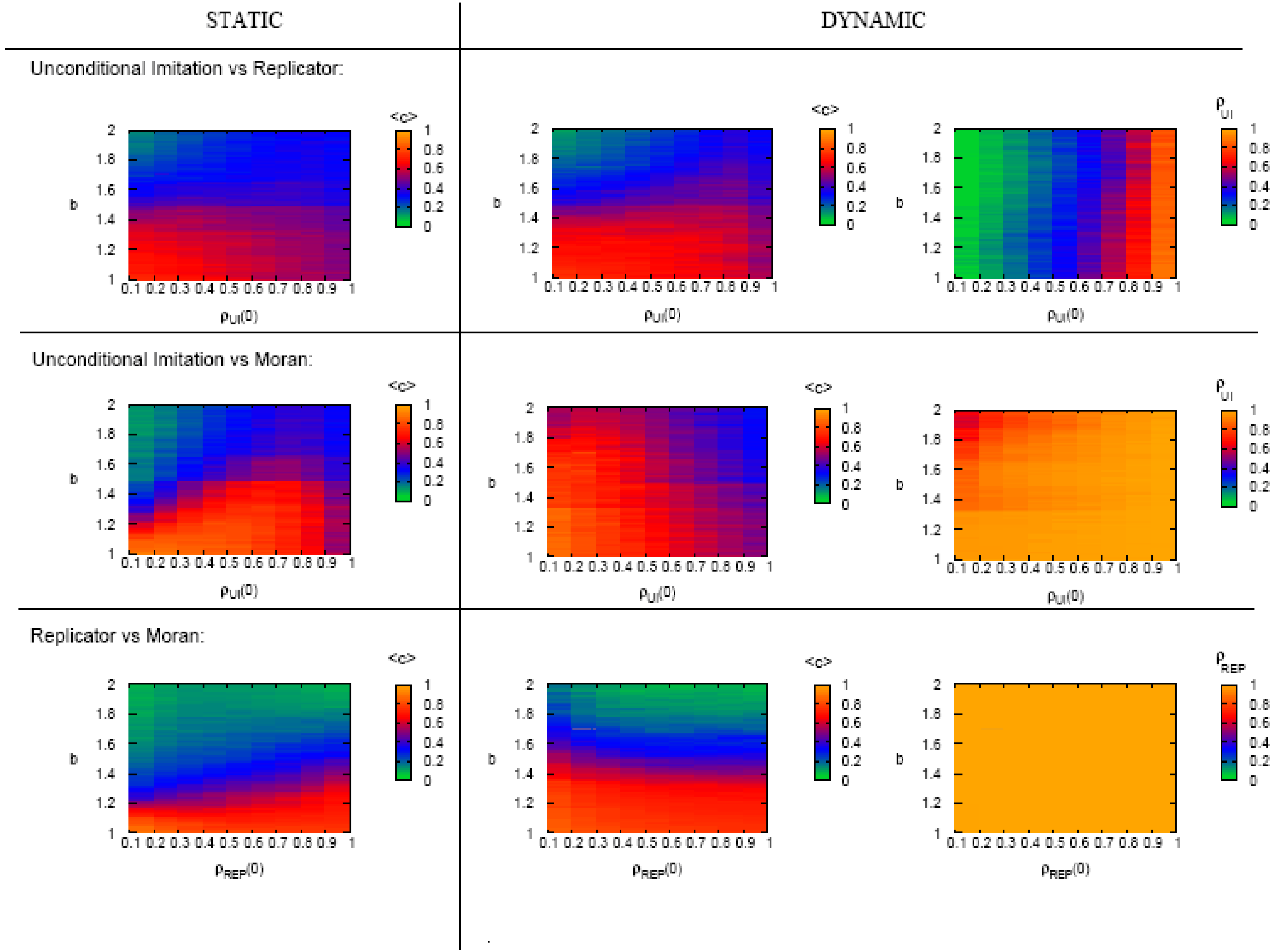
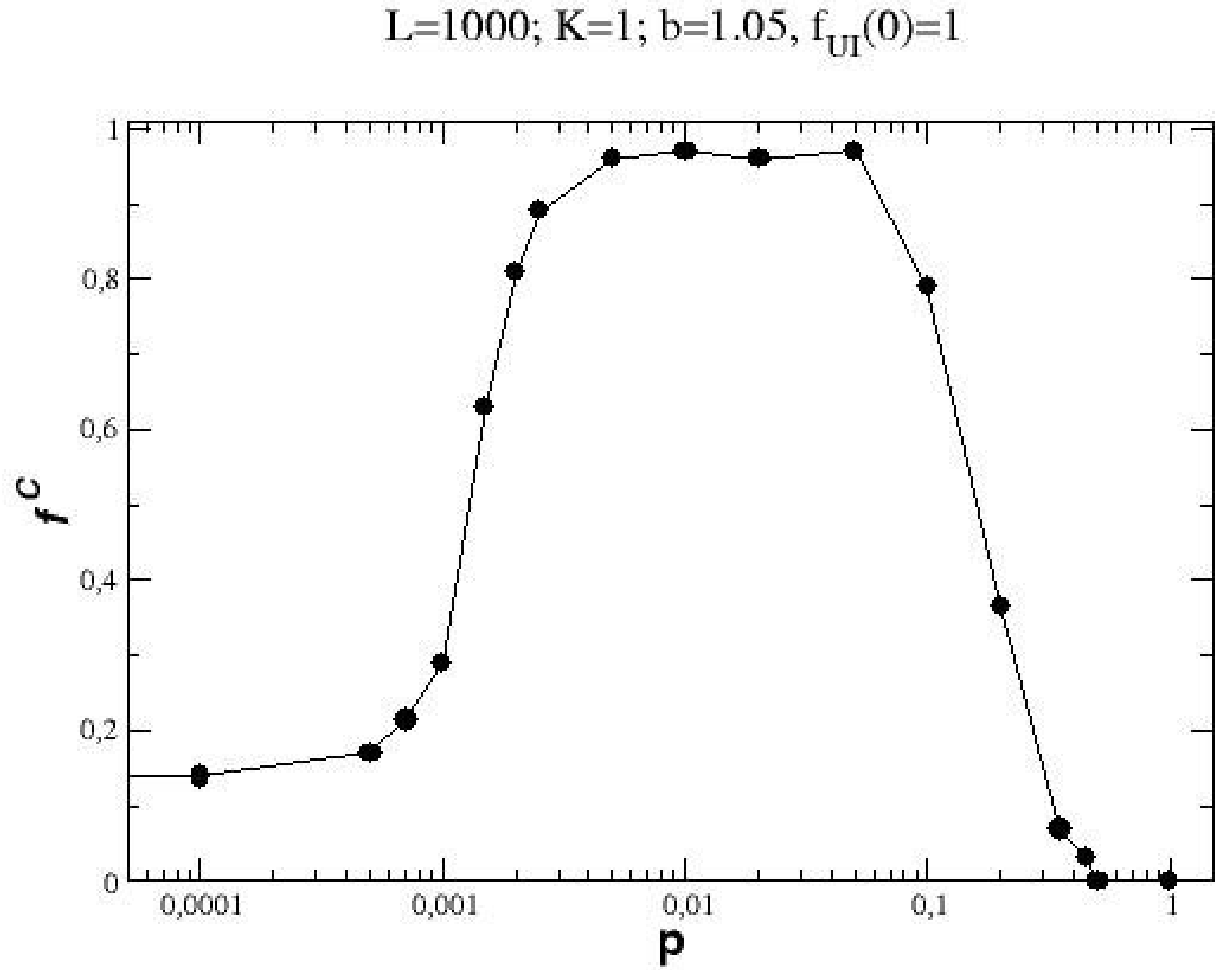


FIG. 7: Scale-free

Small-world topology (not WS)



Unconditional imitation, link-adding model

[Vilone, Sánchez, Gómez-Gardeñes, arXiv: 1010.3547, submitted to ***JSTAT***]

Conclusion and perspectives

- Topology has a great influence in the evolution of cooperation;
- But its role is more complex than expected initially;
- There are other factors influencing the dynamics of the cooperation, as for example the update rule;
- In general, UI and REP seems to be the update rule which most enhance the cooperation;
- SW (with added links) shows a not trivial and very interesting effect (enhancing cooperation with no clustering).

THE END