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Effects of mixed dynamics in spin systems with conserved magnetization

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SUMMARY

- Introduction and motivations

- The model: conserved and non-conserved magnetization

- First numerical results

- First theoretical considerations

- Open questions

Introduction I: the voter model (VM)

N spins; at each elementary time step one individual is picked up and imitates a neighbour's spin;

The average magnetization is always conserved;

The dynamics depends on topology: *e.g*. in 1D lattice consensus is reached following a power law, in WS systems ends up in a stationary steady state (therm. limit), *etc.*;

Introduction II: Coordination Game (CG)

- Individuals receive a positive payoff if they have the same strategy of their neighbours;
- A typical coordination game can be described by this payoff matrix

$$\hat{M} = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$$

- If *a* = *b*, average magnetization is conserved as in the VM;
- Consider Unconditional Imitation (UI) update rule: at the end of each round, individuals imitate the strategy of their best performing neighbour (if there is at least one with higher payoff);
- With UI, on lattices the frozen state is compound of several domains separated by still bonds (no consensus reached), differently from VM dynamics.

Mixing VM and CG: why?

1 – Both models admit consensus as stable equilibrium and, if $a = b$, conserve magnetization;

2 – CG represents a “rational” interaction, whilst VM an “irrational” one;

3 – Both dynamics can be seen as noise inserted into the other one;

4 – What are the consequences of such mixing?

Mixing VM and CG: implementation

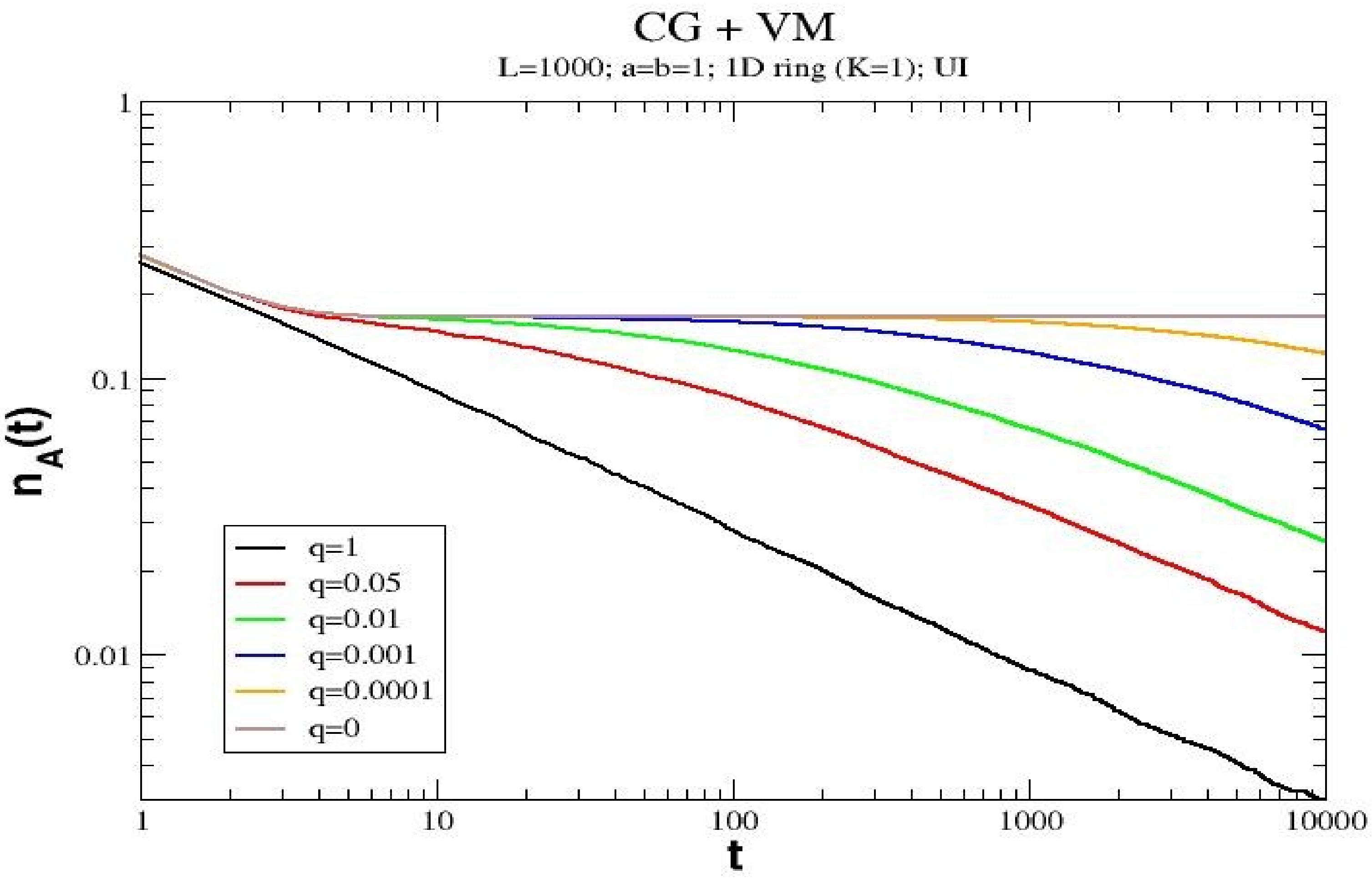
1 – The algorithm is asynchronous;

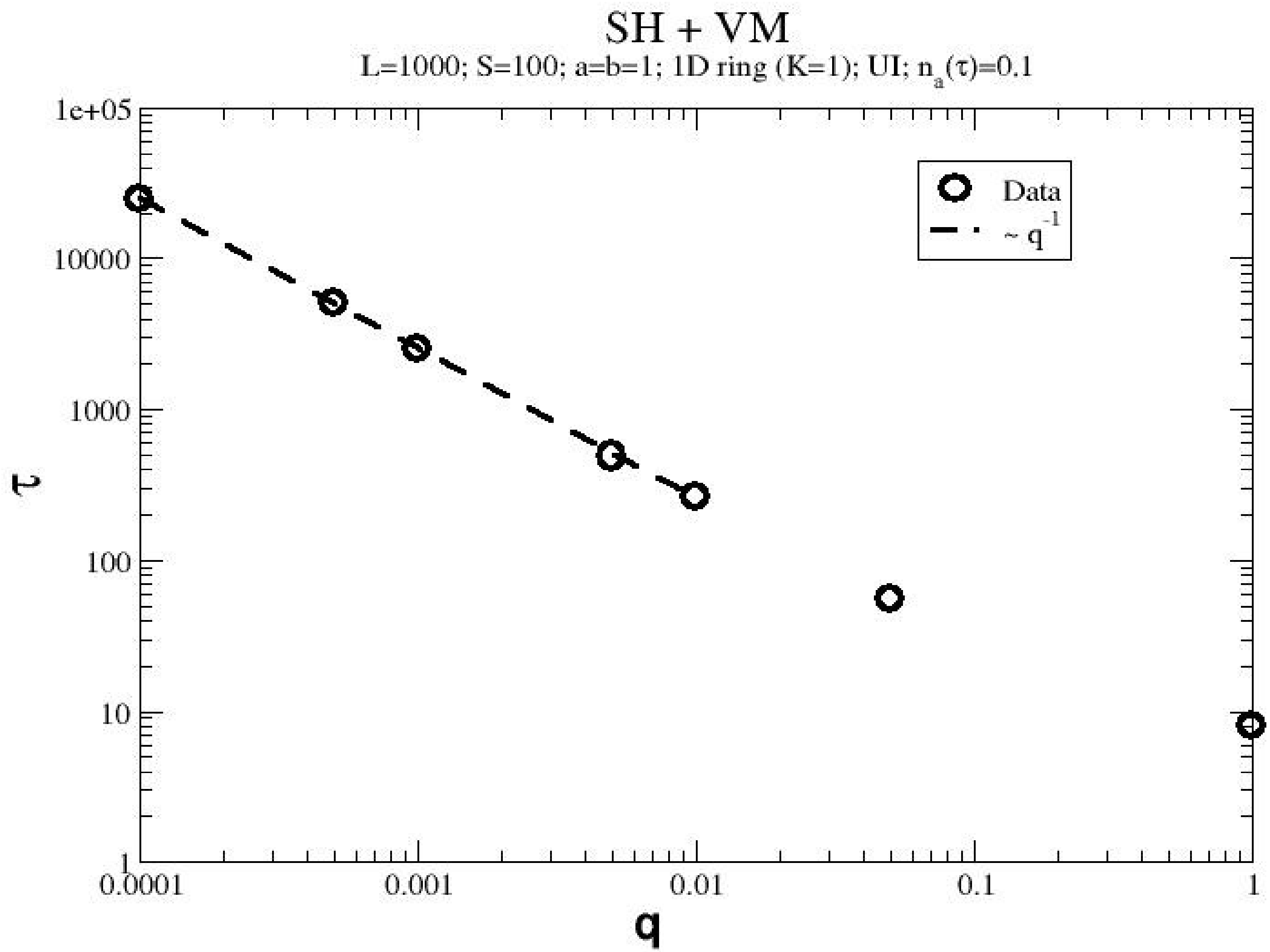
2 – The spin "up" is assumed as the C strategy, "down" as D;

3 – At each elementary time step an agent is picked at random: with probability *q* it interacts as VM, with *1-q* as CG;

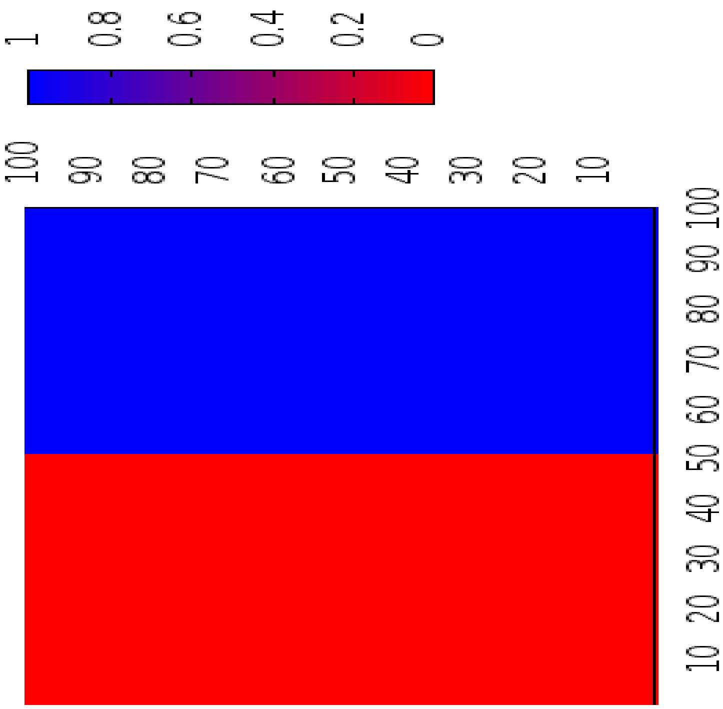
4 – Time is measured in montecarlo steps.

Results: euclidean lattices, *a* = *b*

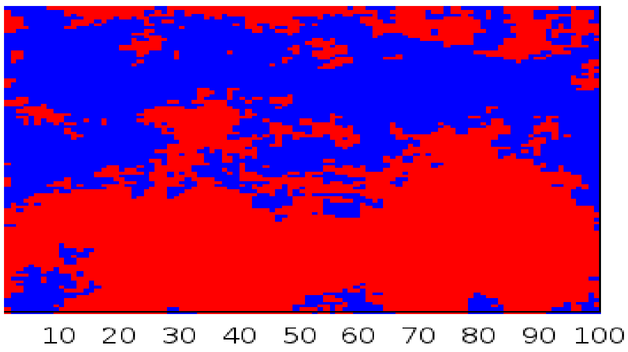



SH + VM
L=1000; S=100; a=b=1; 1D ring (K=1); UI; n_a(τ)=0.1
1e+05
10000
1000
100
10
1
0.0001
0.001
0.01
0.1
1
q
τ
Data
~ q^-1

2D, *a* = *b*: roughness



q=0.01; t=500



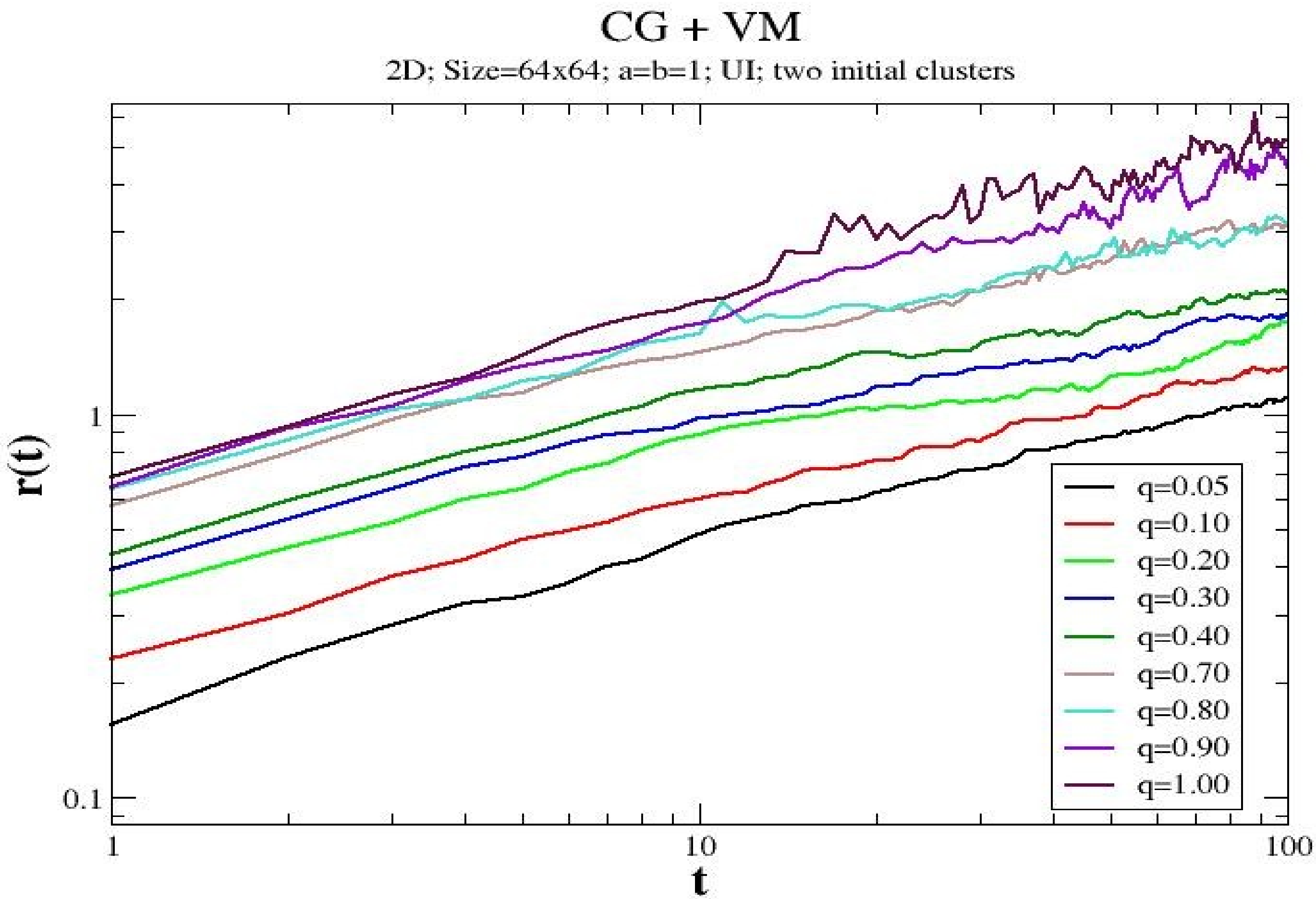
q=0.99; t=500

t=0 ∀ q

q=0 ∀ t

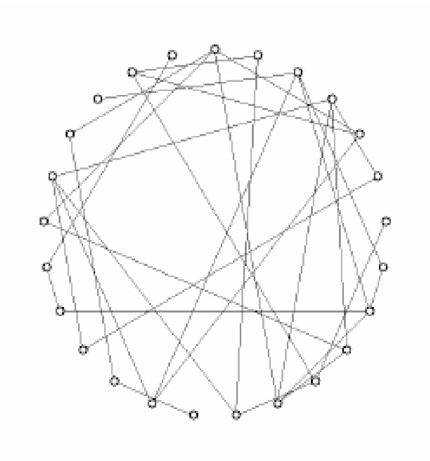
$$r=\sqrt{\langle h^2\rangle-\langle h\rangle^2}$$

Roughness of the interface

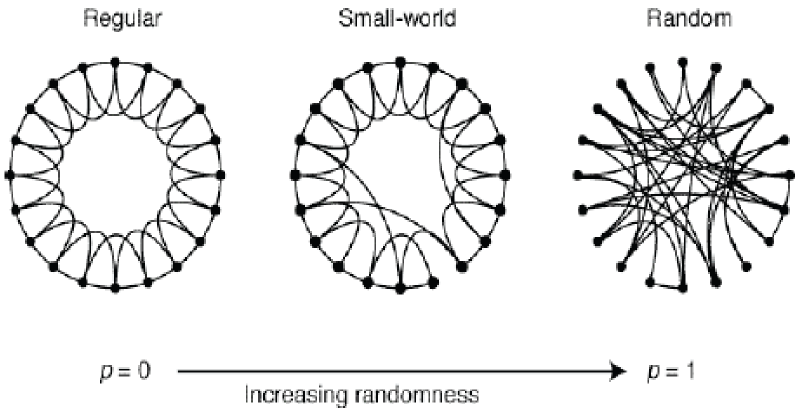


Random topologies

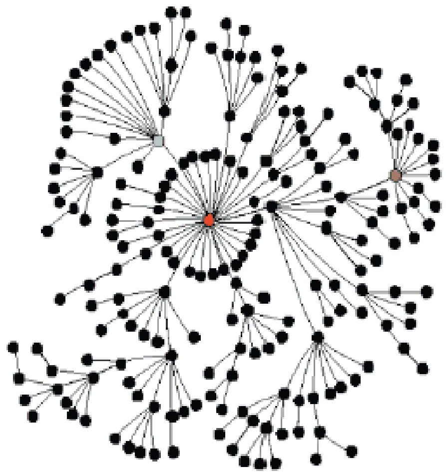
Types of model graphs



Random
(Erdös-Renyi)

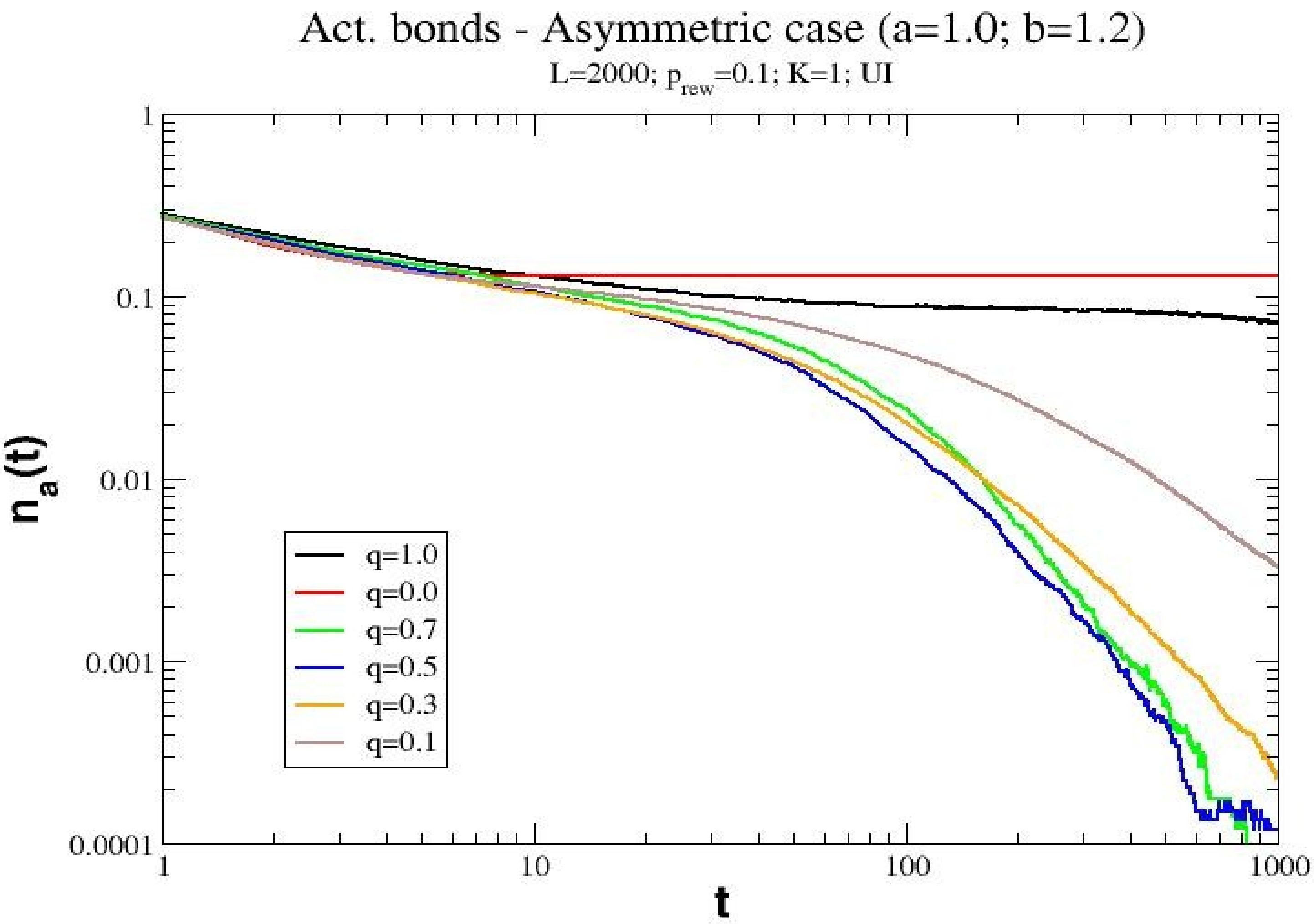


Small world
(Watts-Strogatz)

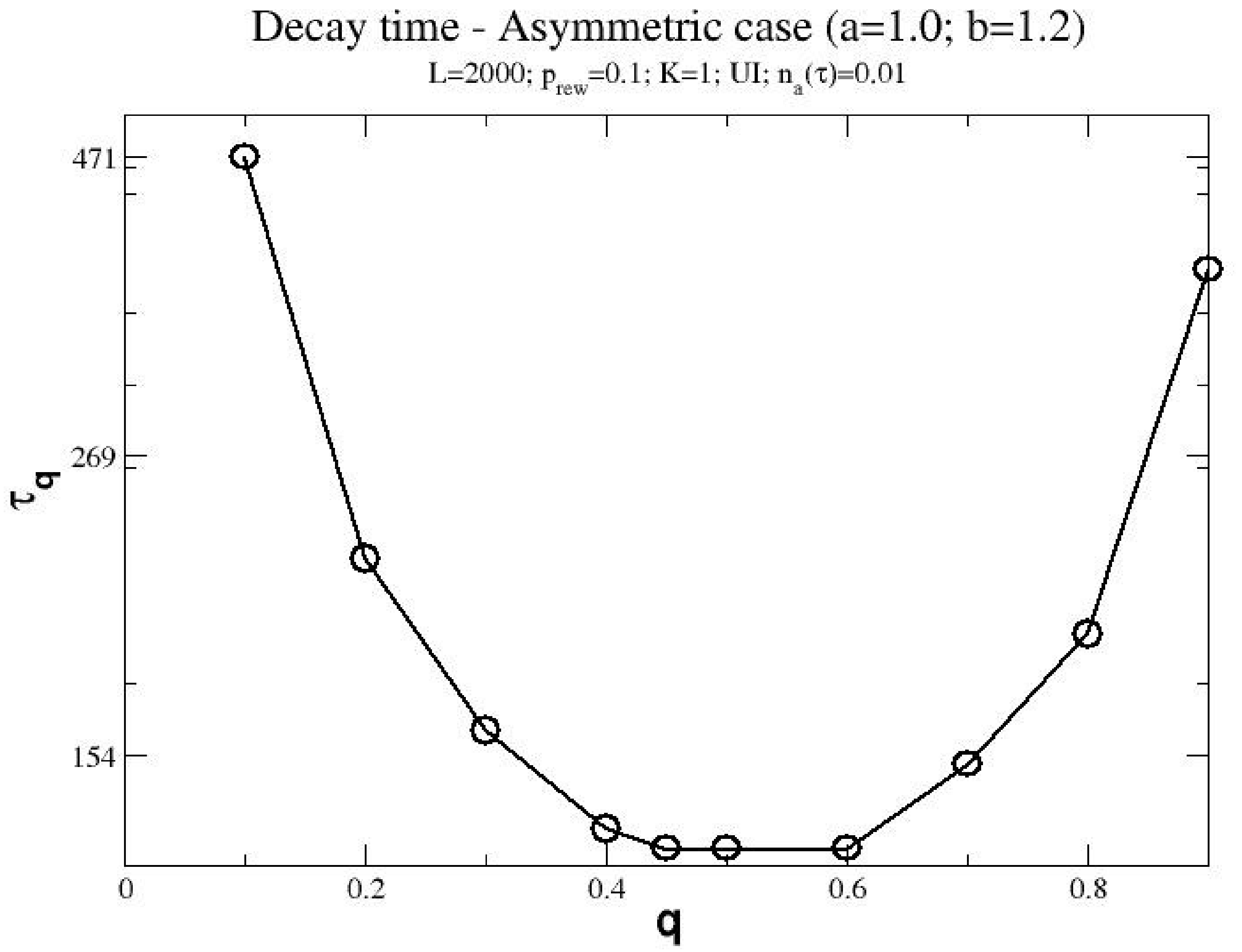


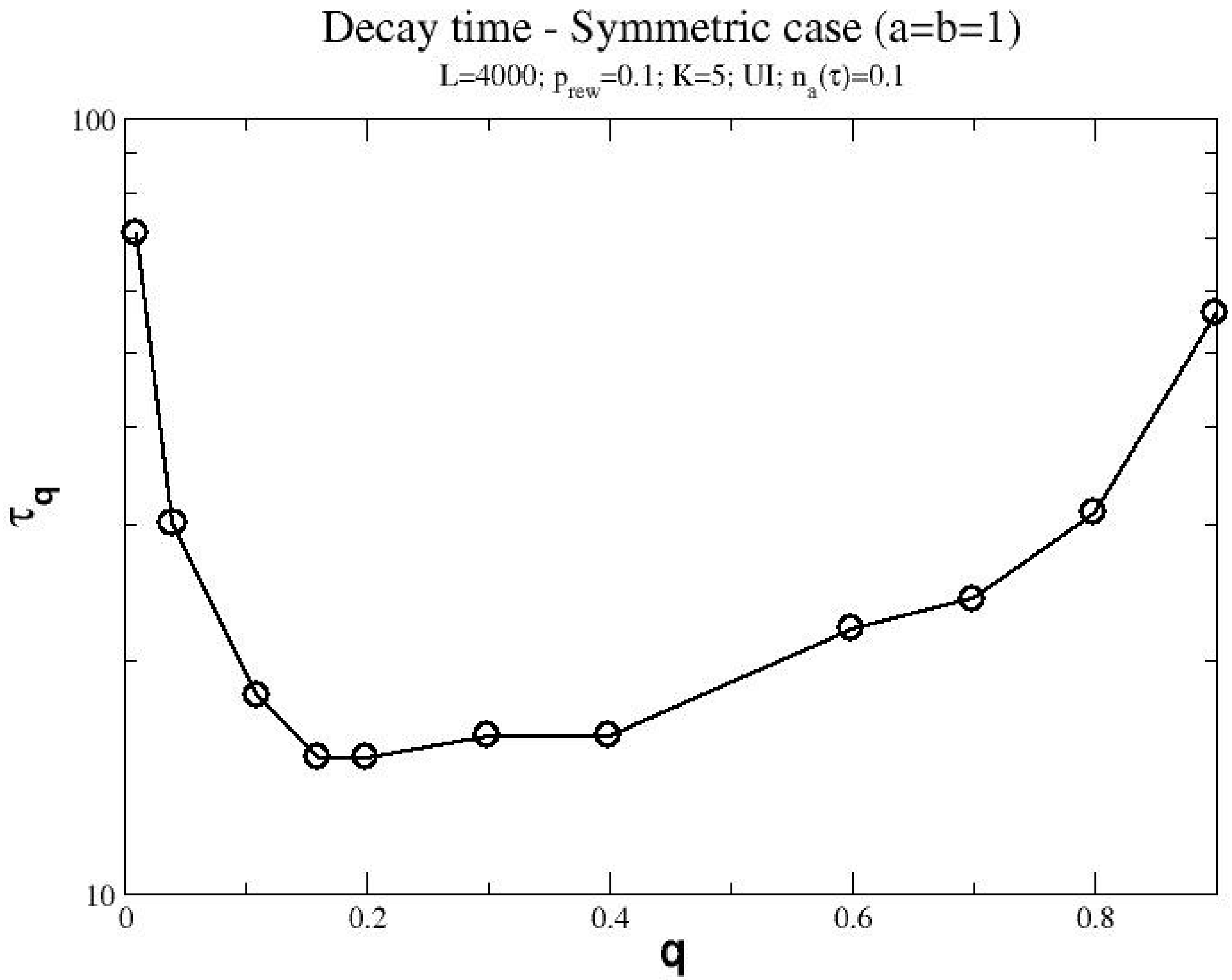
Scale free
(Barabási-Albert)

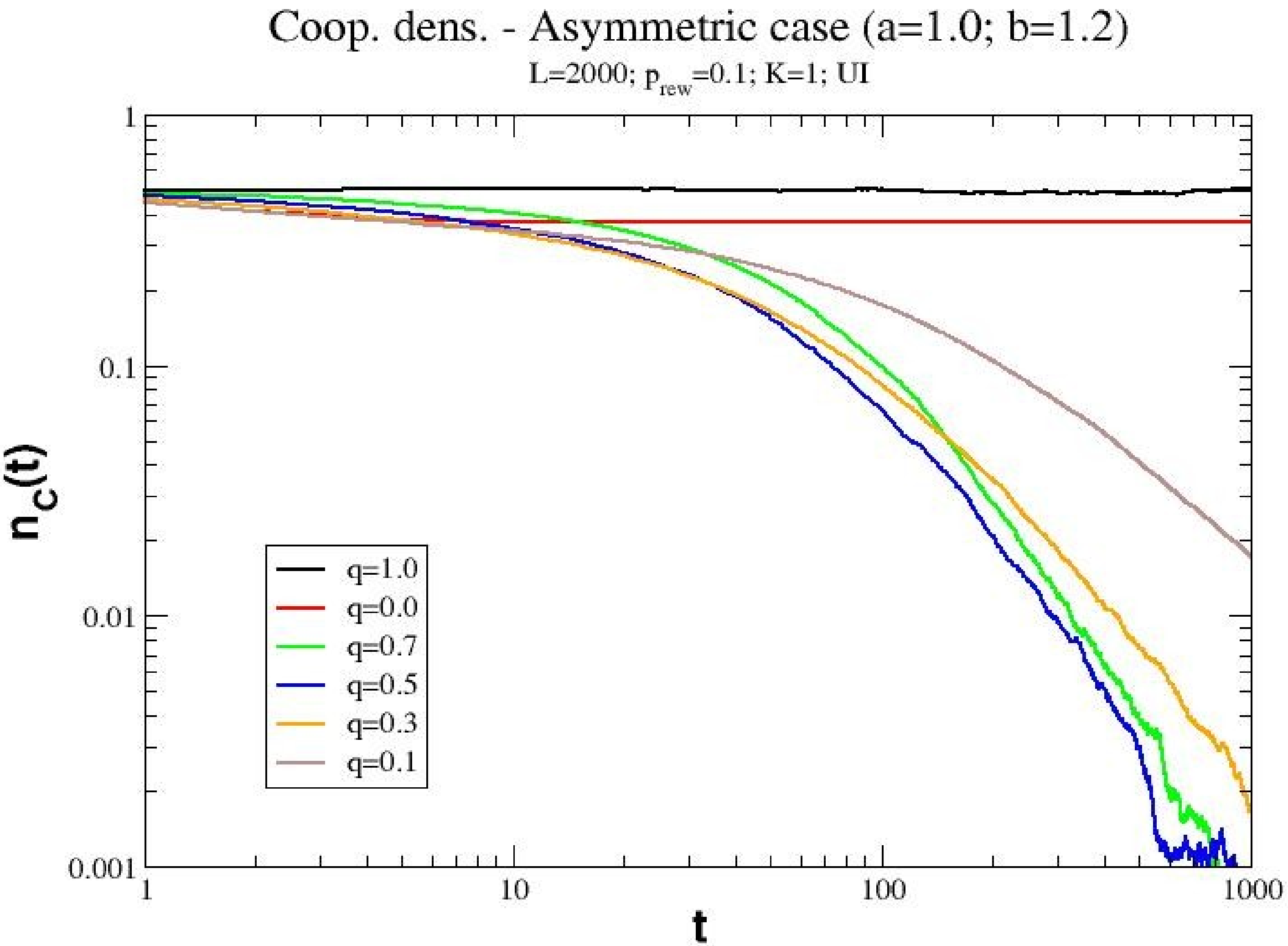
WS and ER: similar behaviour!



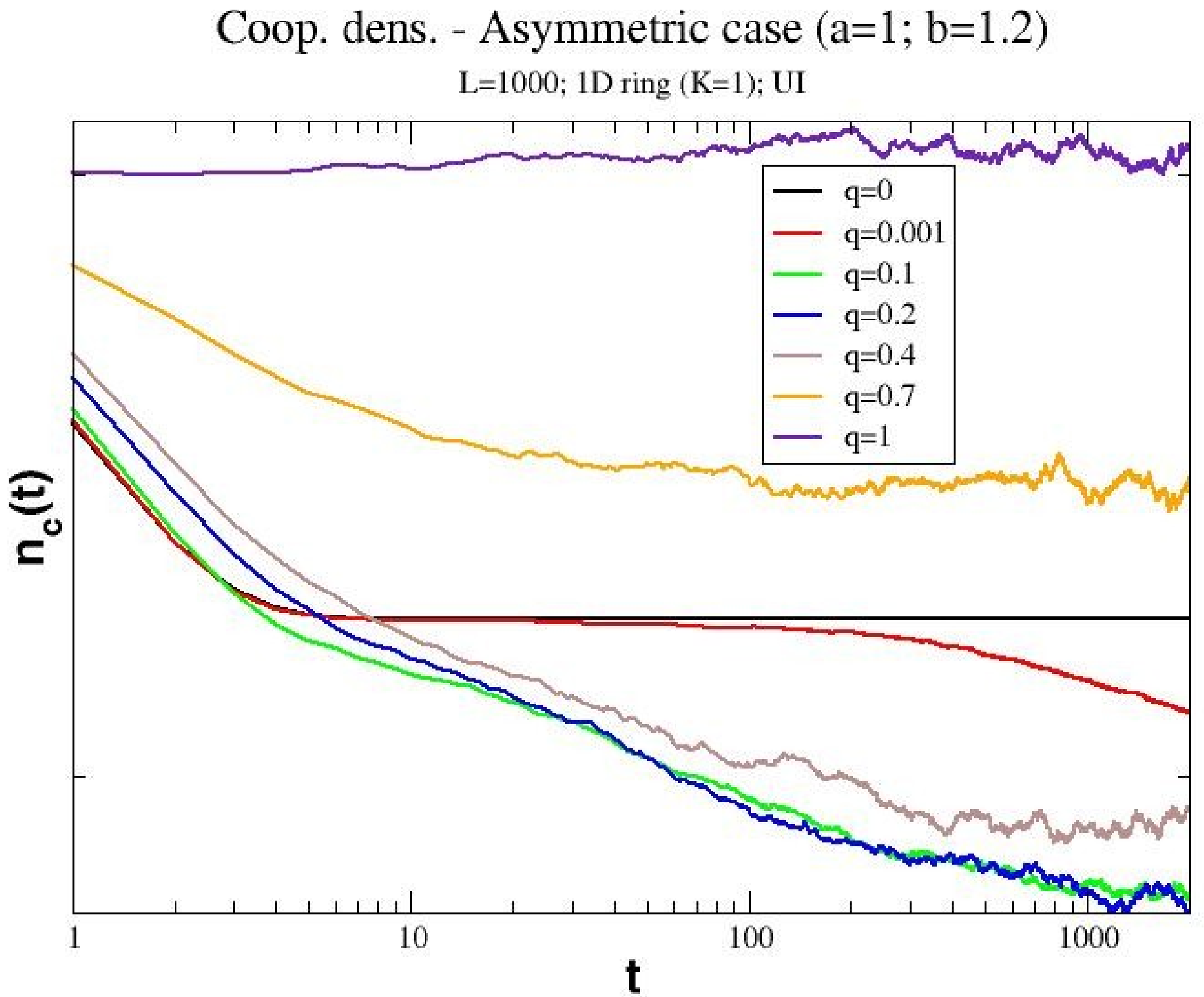
Active bonds: qualitatively the same behaviour if a=b


Decay time - Asymmetric case (a=1.0; b=1.2)
L=2000; p_rew=0.1; K=1; UI; n_a(τ)=0.01
471
269
154
τ_q
0
0.2
0.4
0.6
0.8
q


Decay time - Symmetric case (a=b=1)
L=4000; p_rew=0.1; K=5; UI; n_a(τ)=0.1
100
10
τ_q
0
0.2
0.4
0.6
0.8
q



Magnetization: with a=b is always conserved


Coop. dens. - Asymmetric case (a=1; b=1.2)
L=1000; 1D ring (K=1); UI
q=0
q=0.001
q=0.1
q=0.2
q=0.4
q=0.7
q=1
$n_c(t)$
1
10
100
1000
t

Summarizing...

Euclidean lattices, *a=b*:

Magnetization conserved: $\rho_C(t) = \rho_c(0)\, \forall\, t, q$

Active bonds: smooth cross-over

WS and ER systems, *a=b*:

Magnetization conserved: $\rho_C(t) = \rho_c(0)\, \forall\, t, q$

Active bonds: no monotonic final state!

Summarizing - II

WS and ER systems, *a≠b*:

$$\text{Magnetization:} \begin{cases} \rho_C(t \to \infty) = 0 & q > 0 \\ \rho_C(t \to \infty) = f(b-a) & q = 0 \end{cases}$$

Active bonds: no monotonic final state!

Euclidean lattices, *a≠b*:

Magnetization : no monotonic final state!

Active bonds: smooth cross-over

(Very) first theoretical considerations

1 – The topology has more influence than the symmetry of the payoffs;

2 – We see crossovers more than transitions;

3 – In random topologies, VM and CG act as noise to each other;

4 – Since there is no qualitative difference in the behaviour between 1D and 2D, and between WS and ER, it seems that it is the diameter to count more than the clustering coefficient.

Perspectives (no conclusions yet...)

1 – How do the domains grow?

2 – What are the inner mechanisms so that in random topologies when $a \neq b$ system always reach consensus if $0<q<1$?

3 – What happens in SF networks? And in random regular networks?

4 – Are analytical calculations possible?

A lot of work still to be done...

THE END

MANY THANKS!!