Ecosystem assembly for responsive strategies

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Aim

Systematic study of emergence and domination of pure and mixed responsive strategies via an evolutionary invasion process

Outline

- Games
- Responsive strategies: direct reciprocity
- Invasion scheme
- Results
- Summary and Conclusions

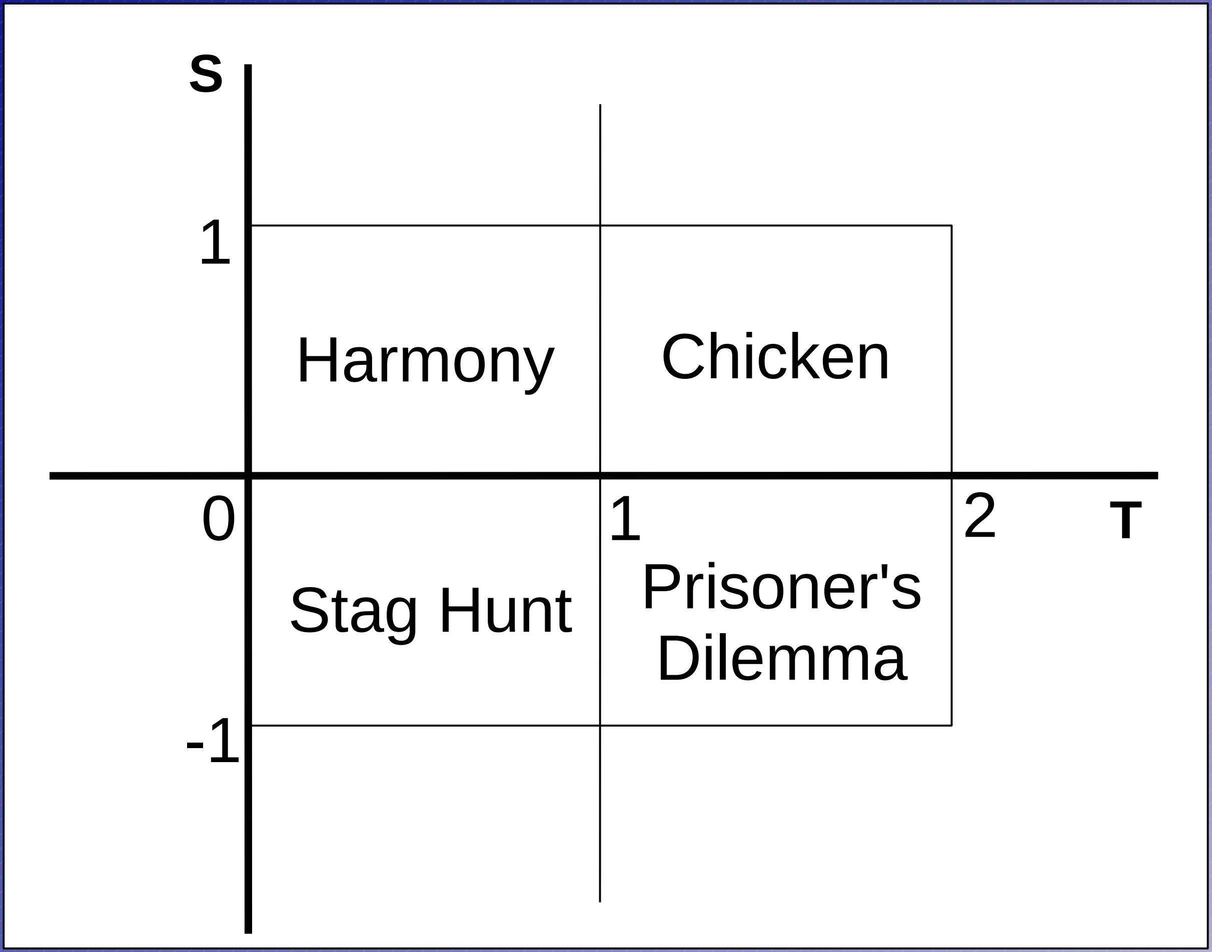
Games

Games

- Two players
- Symetric games
- Two possible actions: cooperate or defect
- Simultaneous decisions

		Player 2	
		C	D
Player 1	C	1	**S**
	D	**T**	0

Games



Player 2

Player 1

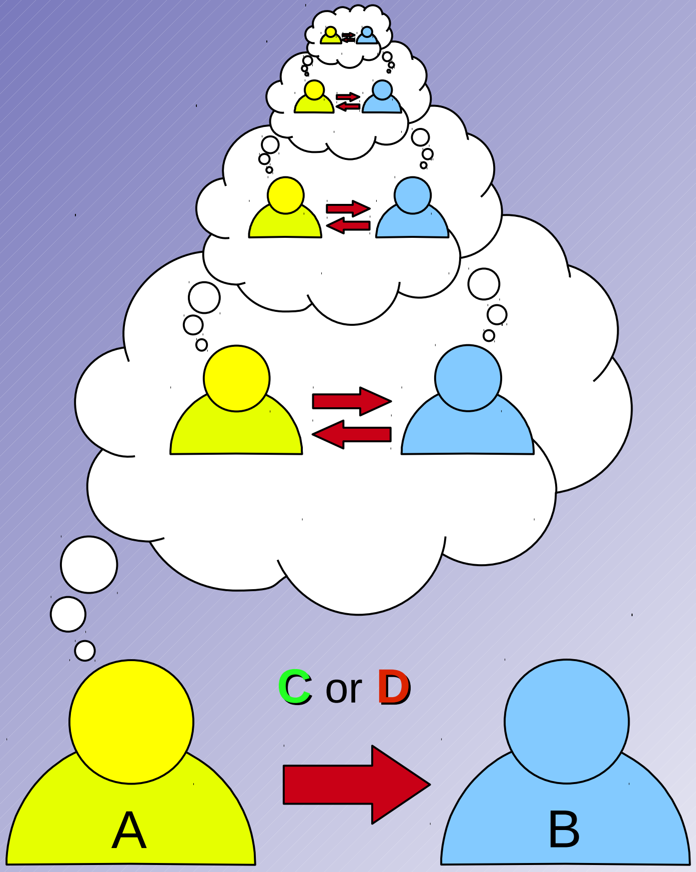
	C	D
C	1	**S**
D	**T**	0

Each game is determined by the values of *S* and *T*

Responsive strategies: direct reciprocity

Direct reciprocity

Players follow responsive strategies in which **only her and her opponent's previous action** are taken into account.



Direct reciprocity

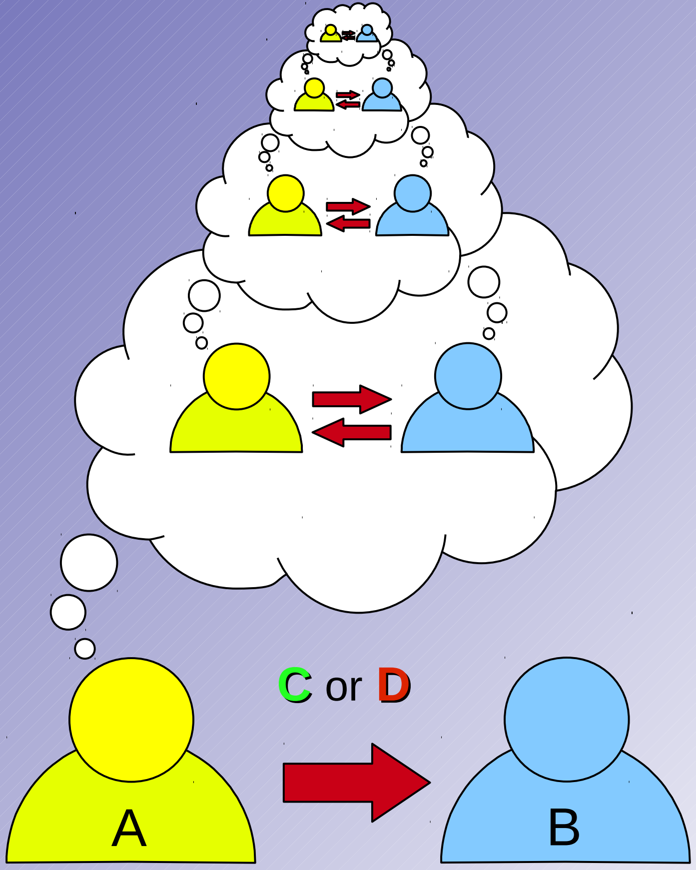
Strategies

A strategy is given by four probabilities:

$$p_{CC} \quad p_{CD} \quad p_{DC} \quad p_{DD}$$

$$p = \begin{cases} C: & 1-\varepsilon \\ D: & \varepsilon \end{cases} \qquad \varepsilon = 0.01$$

16 possible strategies



Direct reciprocity

Strategies

	1	S	T	0
	CC	CD	DC	DD
All-D	0	0	0	0
All-C	1	1	1	1
Tit For Tat	1	0	1	0
Win Stay, Lose Shift (WSLS) - Pavlov	1	0	0	1

Prisoner's Dilemma:

$T>1>0>S$

Direct reciprocity

Payoffs

Iterative game – Markov Chain:

$$
M_{AB}=\begin{array}{c}
\overbrace{\begin{array}{cccc}\quad\textbf{CC}\quad & \qquad\qquad\textbf{CD}\qquad\qquad & \qquad\qquad\textbf{DC}\qquad\qquad & \qquad\qquad\textbf{DD}\qquad\end{array}}^{\textbf{Next round}} \\
\begin{pmatrix}
p^{A}_{CC}\,p^{B}_{CC} & p^{A}_{CC}\left(1-p^{B}_{CC}\right) & \left(1-p^{A}_{CC}\right)p^{B}_{CC} & \left(1-p^{A}_{CC}\right)\left(1-p^{B}_{CC}\right) \\
p^{A}_{CD}\,p^{B}_{DC} & p^{A}_{CD}\left(1-p^{B}_{DC}\right) & \left(1-p^{A}_{CD}\right)p^{B}_{DC} & \left(1-p^{A}_{CD}\right)\left(1-p^{B}_{DC}\right) \\
p^{A}_{DC}\,p^{B}_{CD} & p^{A}_{DC}\left(1-p^{B}_{CD}\right) & \left(1-p^{A}_{DC}\right)p^{B}_{CD} & \left(1-p^{A}_{DC}\right)\left(1-p^{B}_{CD}\right) \\
p^{A}_{DD}\,p^{B}_{DD} & p^{A}_{DD}\left(1-p^{B}_{DD}\right) & \left(1-p^{A}_{DD}\right)p^{B}_{DD} & \left(1-p^{A}_{DD}\right)\left(1-p^{B}_{DD}\right)
\end{pmatrix}
\end{array}
\begin{array}{c} \\ \textbf{CC} \\ \textbf{CD} \\ \textbf{DC} \\ \textbf{DD} \end{array}
\left.\begin{array}{c} \\ \\ \\ \\ \\ \end{array}\right\}
\begin{array}{c}\textbf{Previous} \\ \textbf{round}\end{array}
$$

Stationary probability vector:

$$\pi_{AB}=\pi_{AB}M_{AB}$$

Average payoff (A vs B):

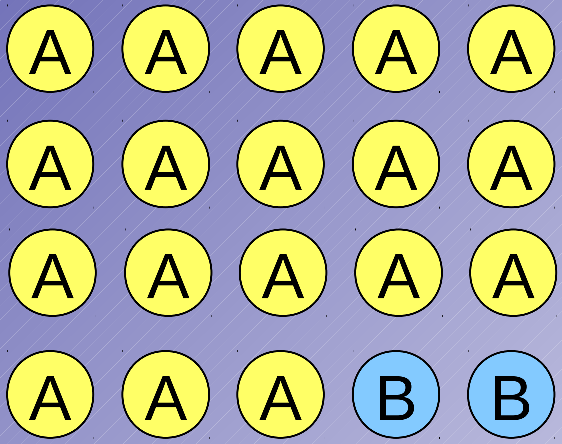
$$W_{AB}=\pi_{AB}\cdot\left(1,S,T,0\right)$$

Invasion scheme

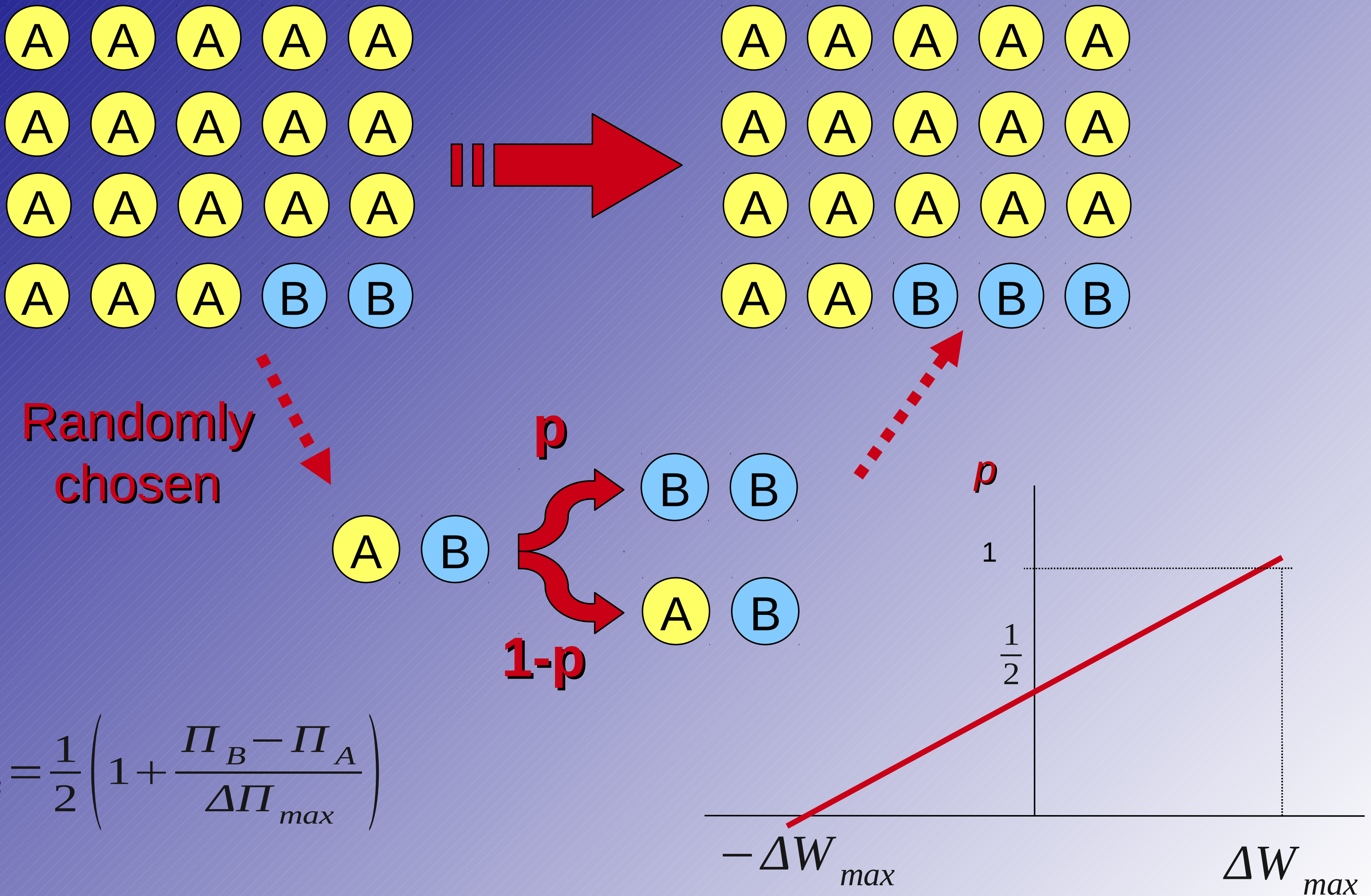
Invasion process

A ***resident*** strategy is fixed. A ***mutant*** strategy tries to invade it.

Simulations with 1000 individuals. Initially 10 of them are replaced by mutants

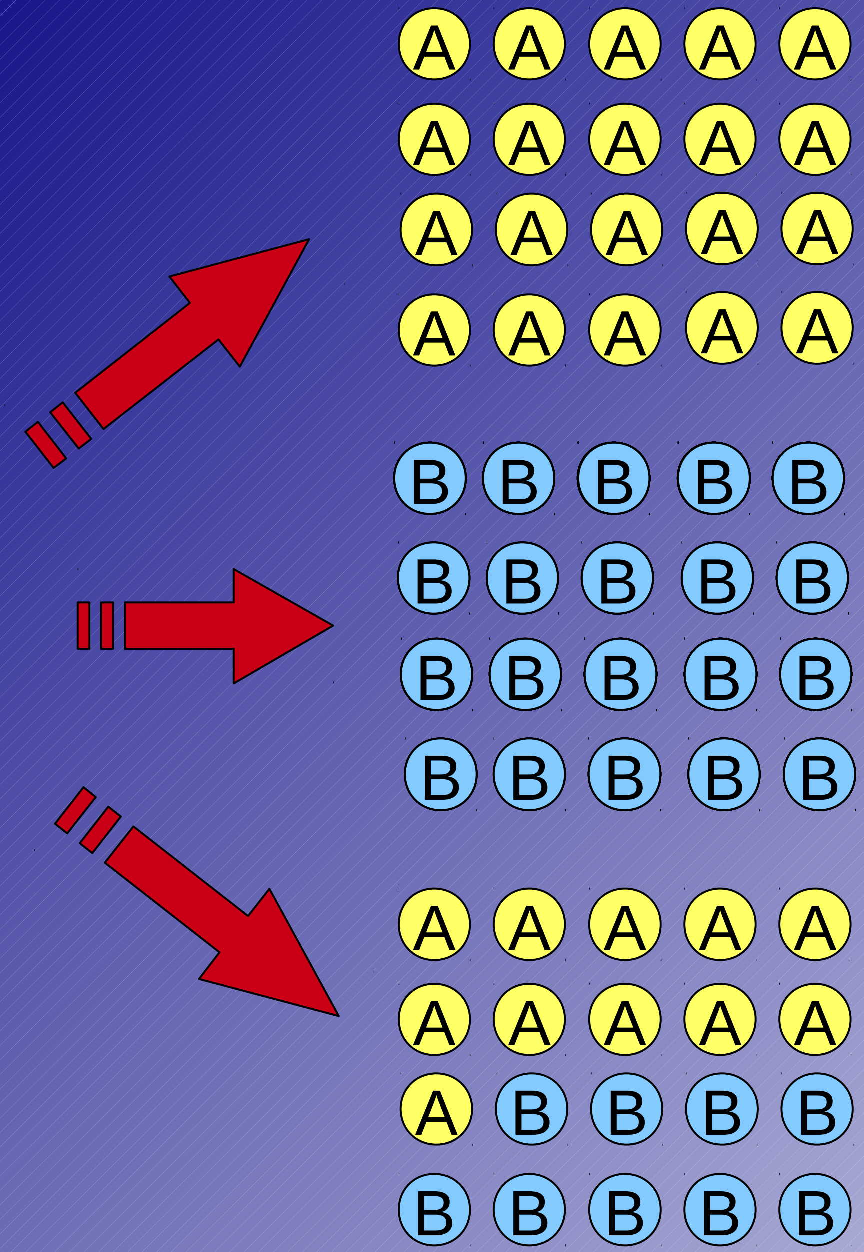


Invasion process



$$p_{A \to B} = \frac{1}{2}\left(1 + \frac{\Pi_B - \Pi_A}{\Delta\Pi_{max}}\right)$$

Invasion process



Mutant is expelled

Mutant dominates

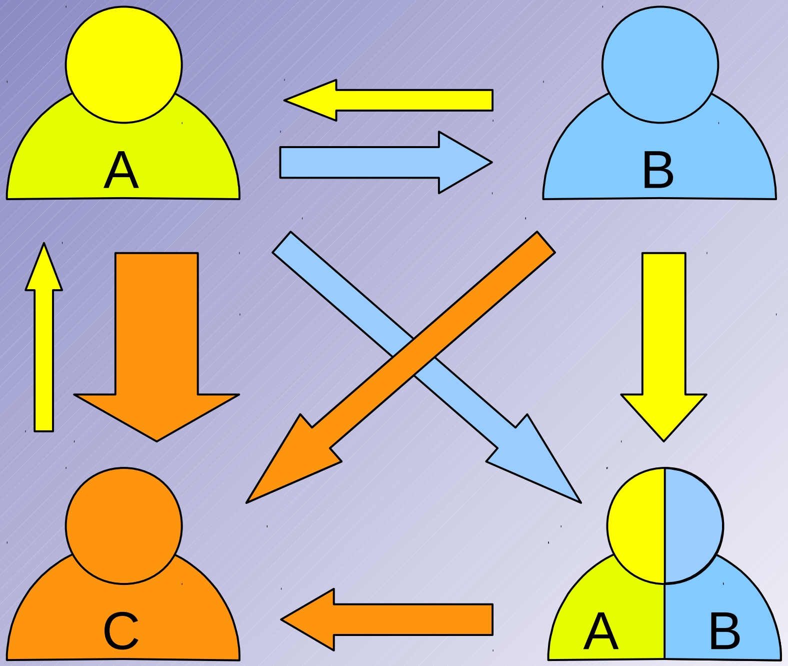
Mixed equilibrium

Invasion process

A total of 100 simulations were performed for each invasion process.

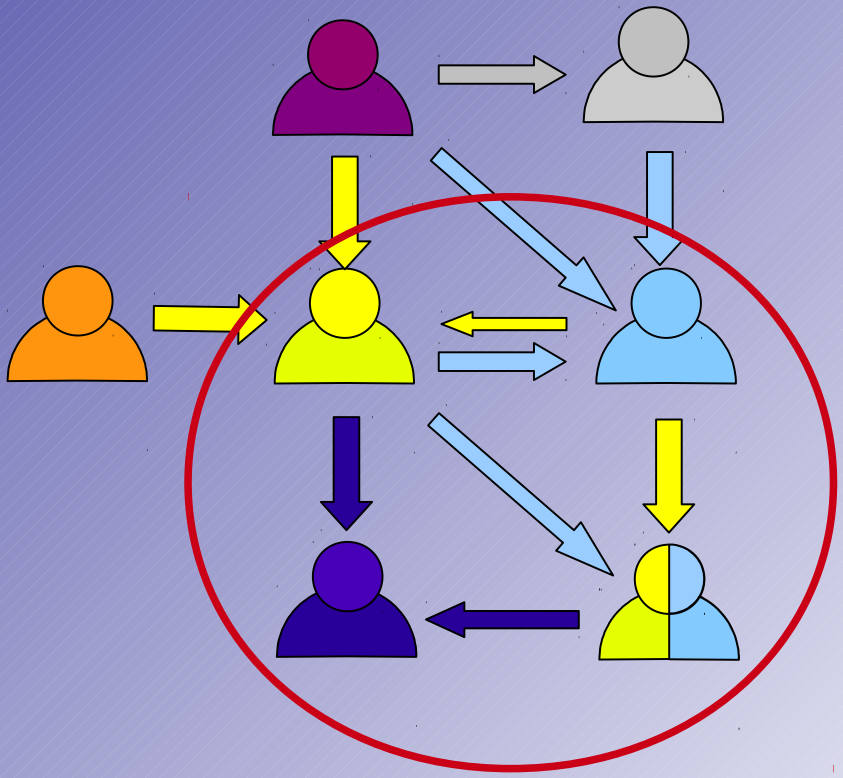
All states, pures and mixed, were invaded following this scheme.

The result of the invasion process is a weighted and directed graph whose vertices are the different equilibria attained (either pure or mixed).



Invasion process

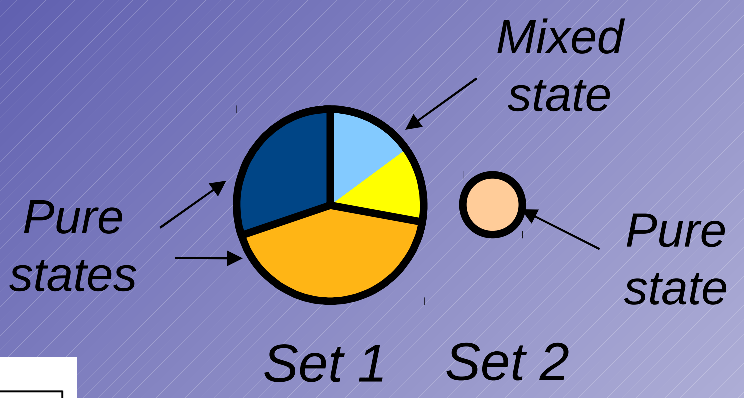
The **recurrent sets** of the graph determine which strategies dominate the process.



Each recurrent set can be formed by one (absorbing node) or a few different states with different probabilities. These states can be either pure or mixed as well.

Results

Results

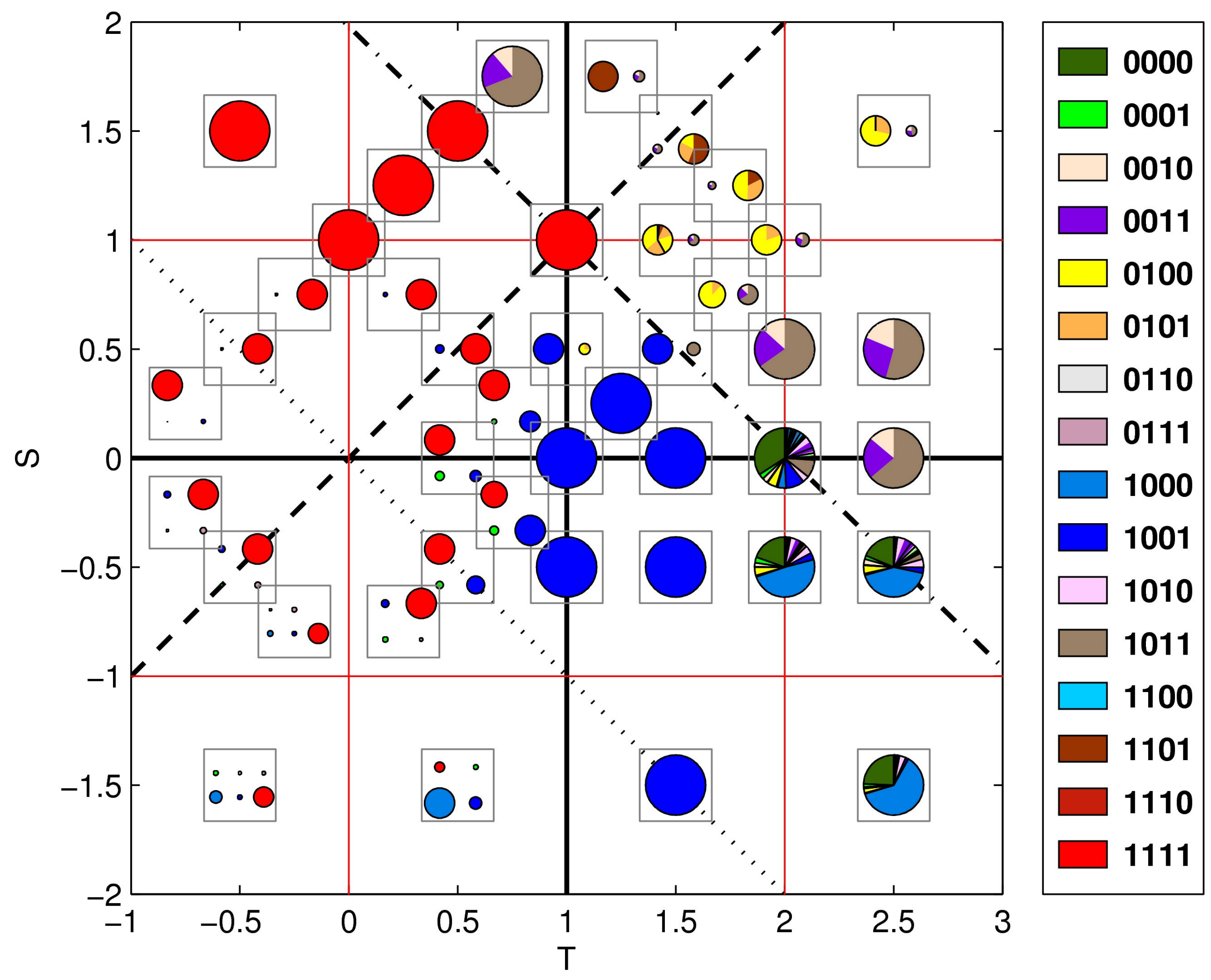

Mixed state

Pure states

Pure state

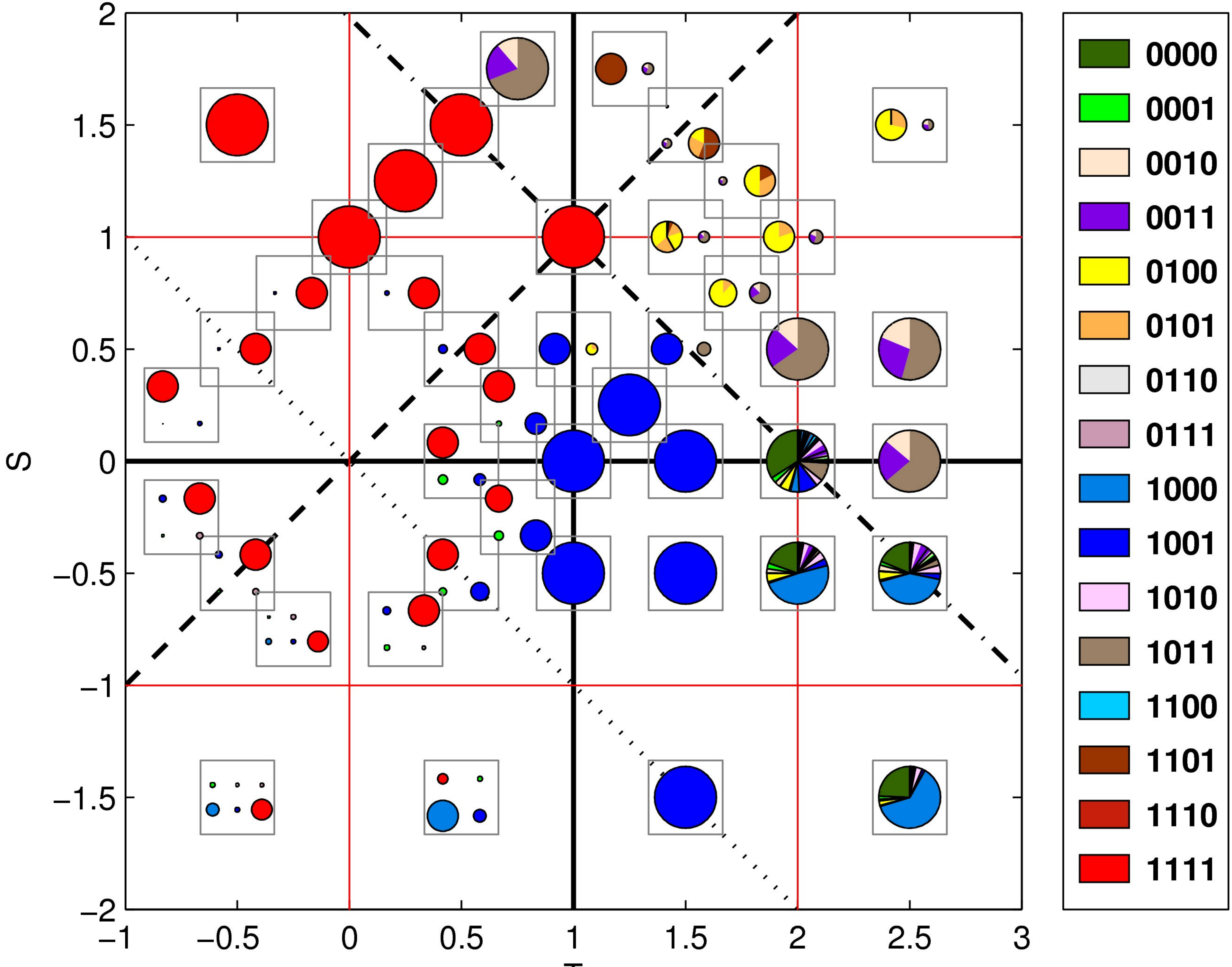
Set 1

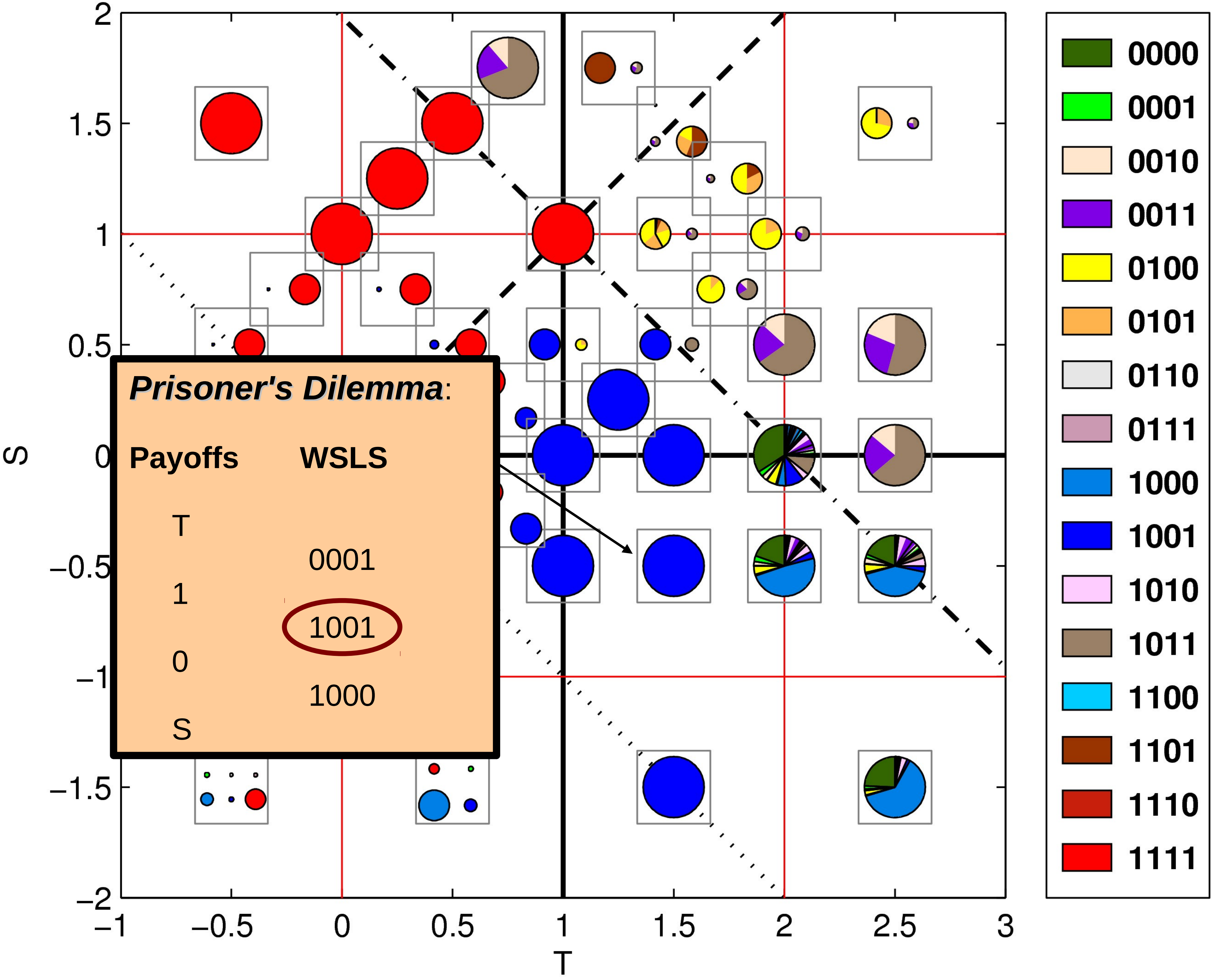
Set 2



Player 2

		C	D
Player 1	C	1	**S**
	D	**T**	0


S
T
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−1
−0.5
0
0.5
1
1.5
2
2.5
3
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111



Prisoner's Dilemma:

Payoffs **WSLS**

T

0001

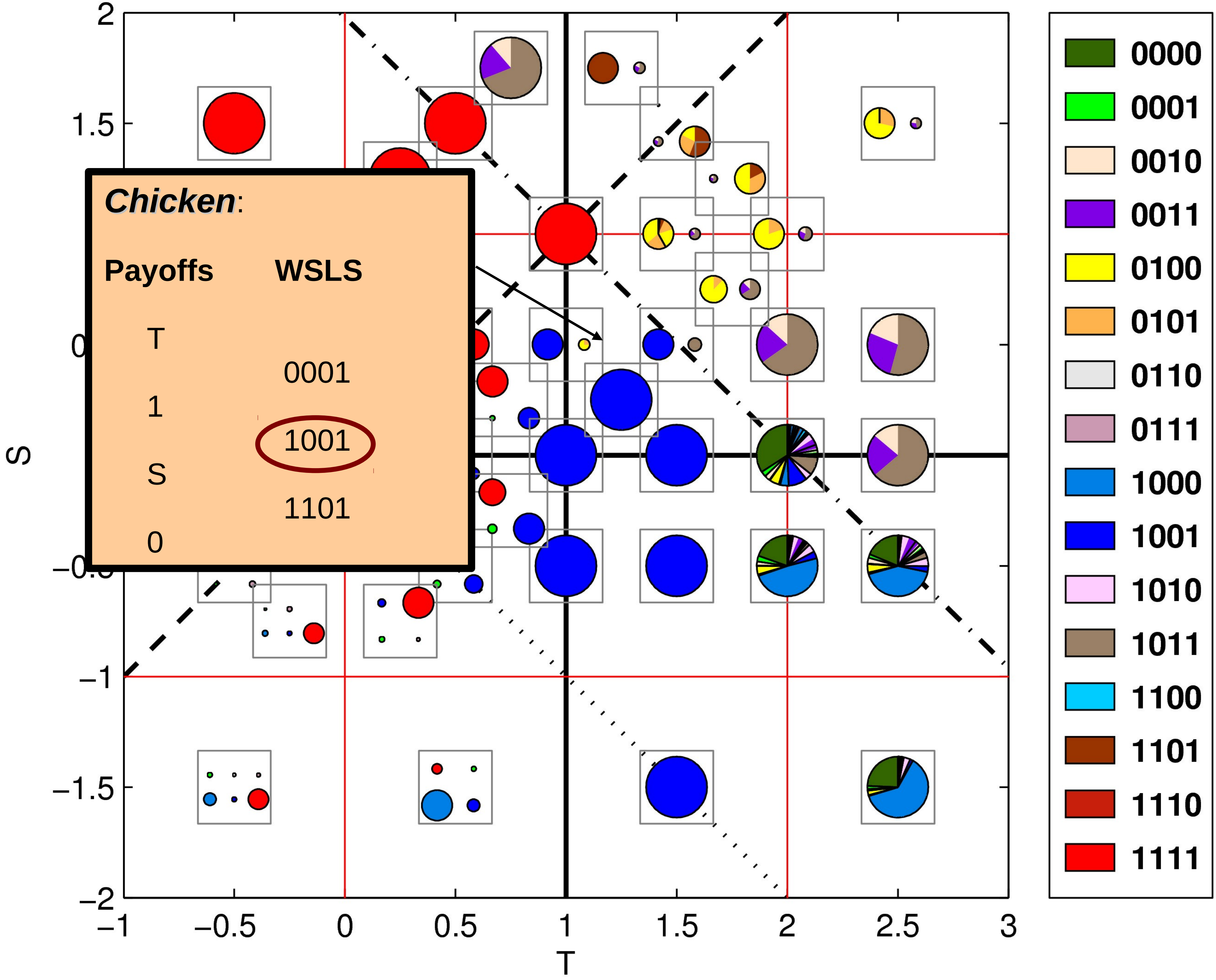
1

1001

0

1000

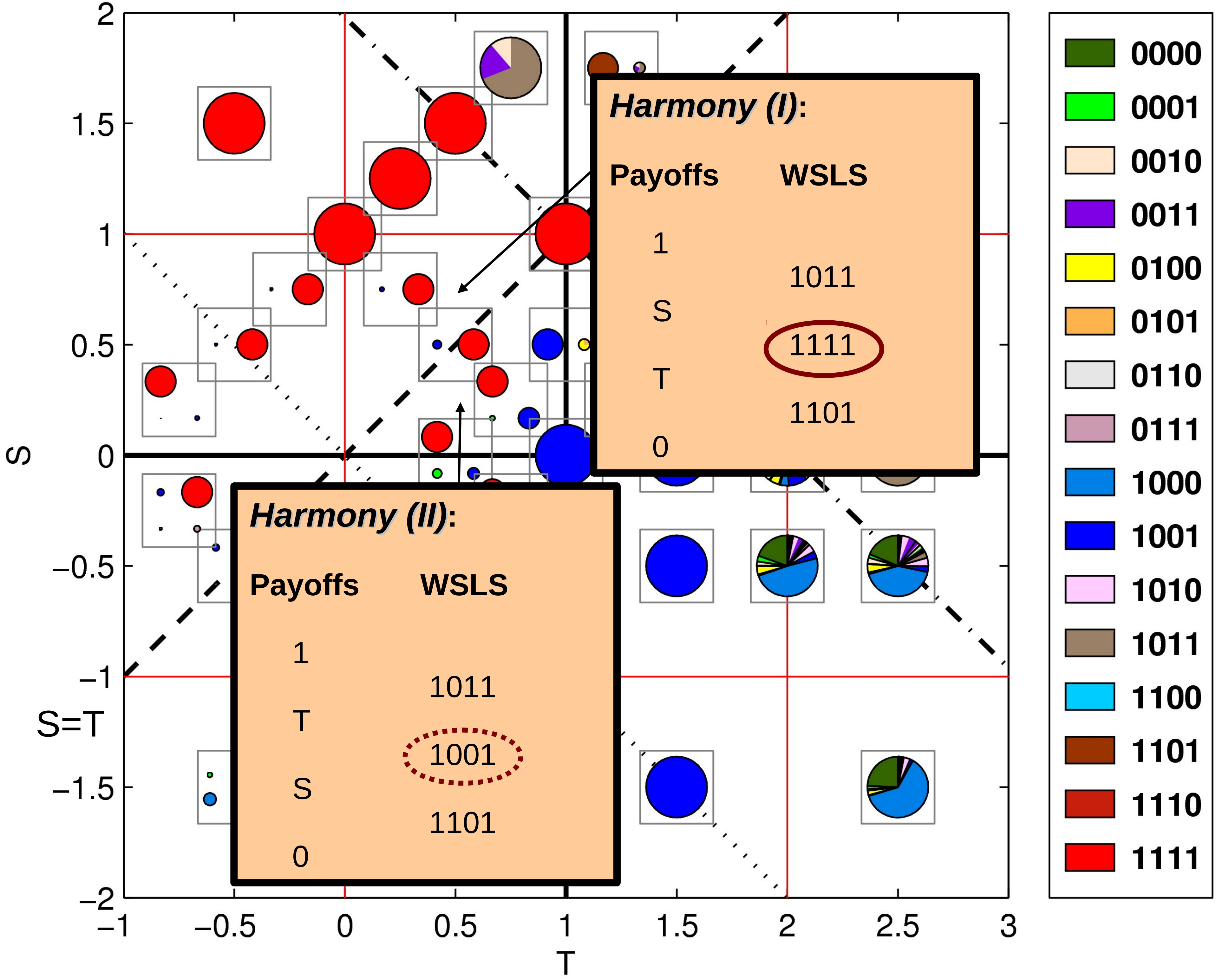
S



Chicken:

Payoffs **WSLS**

T
0001
1
1001
S
1101
0



Harmony (I):
Payoffs
WSLS
1
1011
S
1111
T
1101
0
Harmony (II):
Payoffs
WSLS
1
1011
T
1001
S
1101
0
S=T
S
T
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−1
−0.5
0
0.5
1
1.5
2
2.5
3
0000
0001
0010
0011
0100
0101
0110
0111
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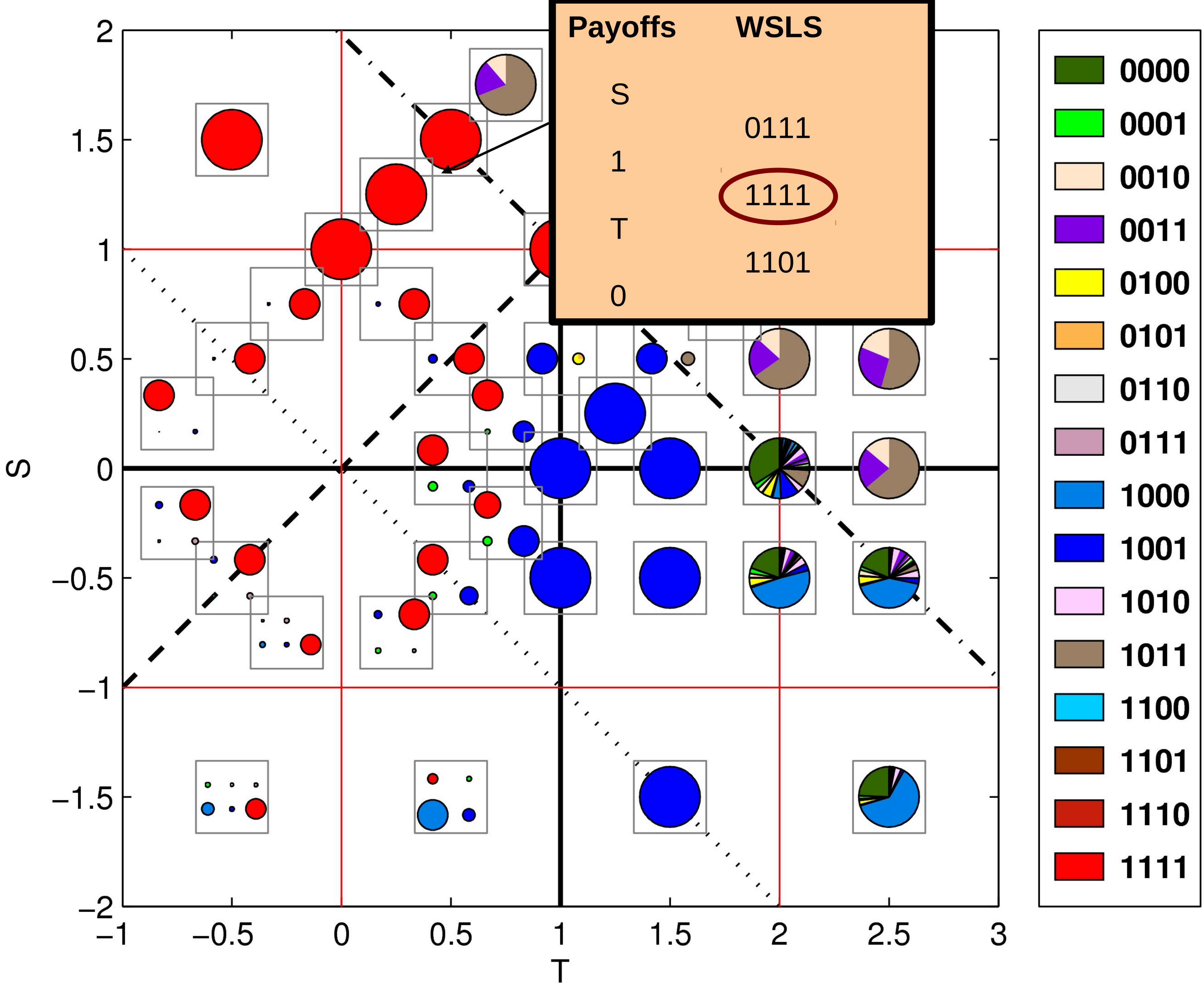
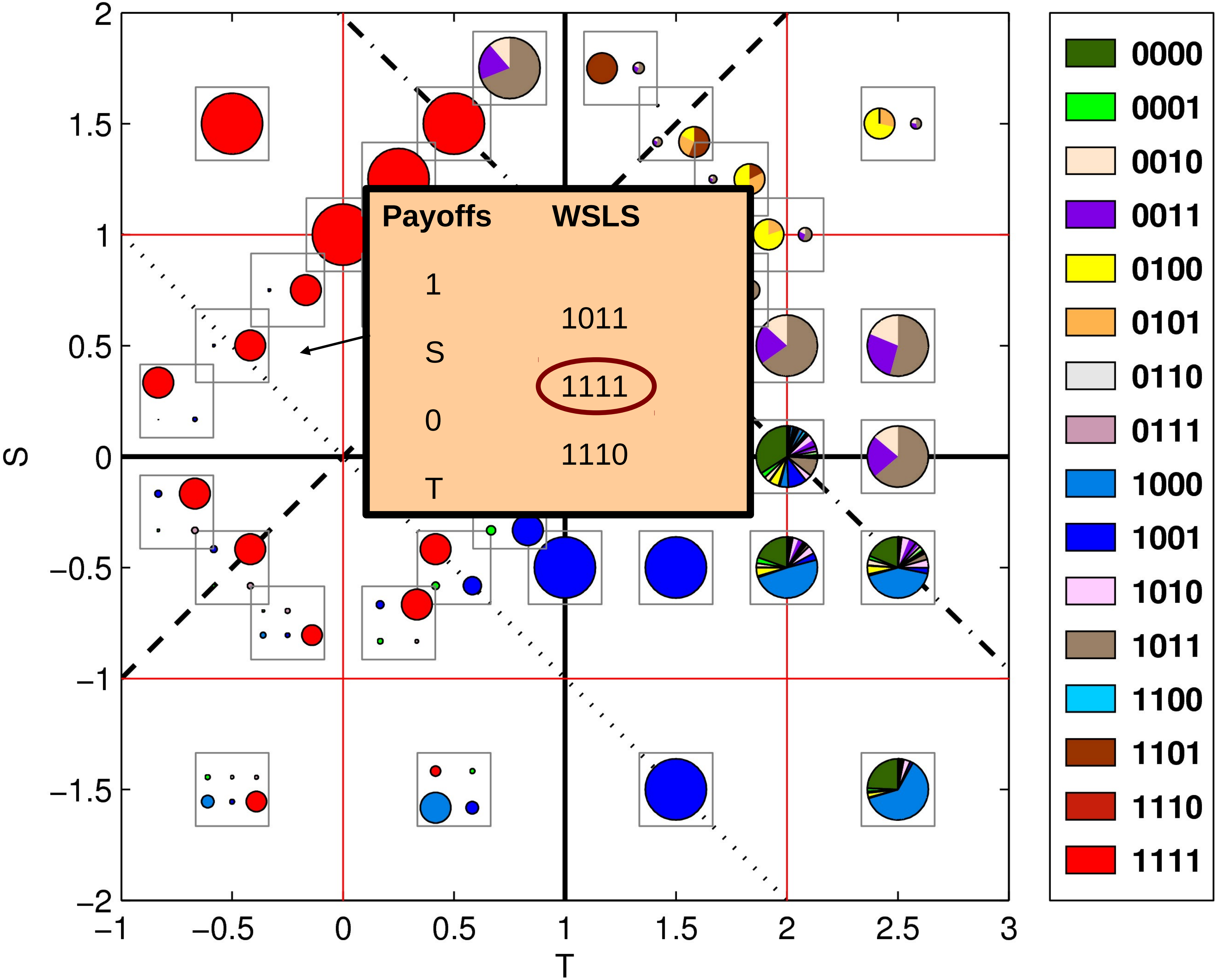
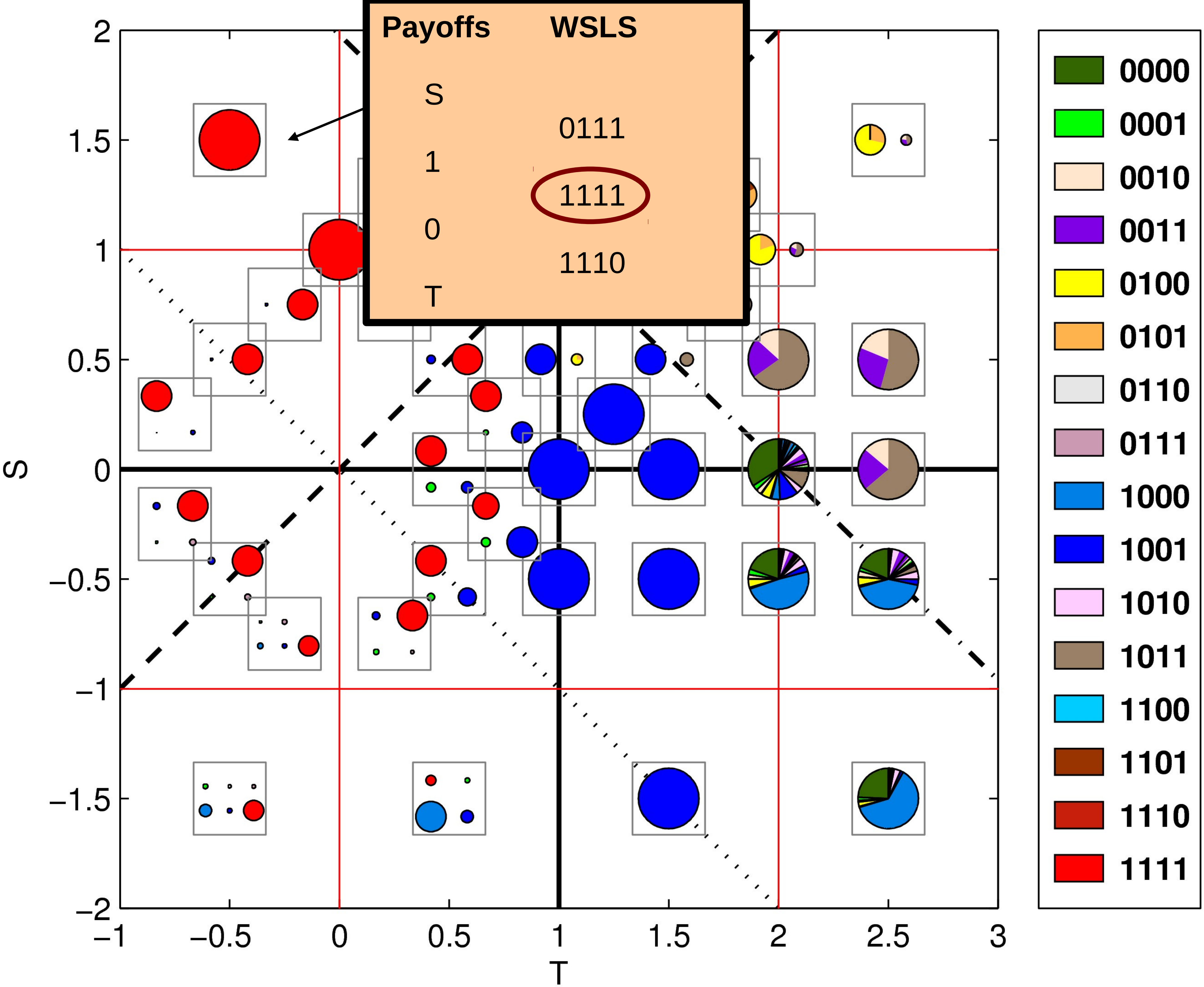
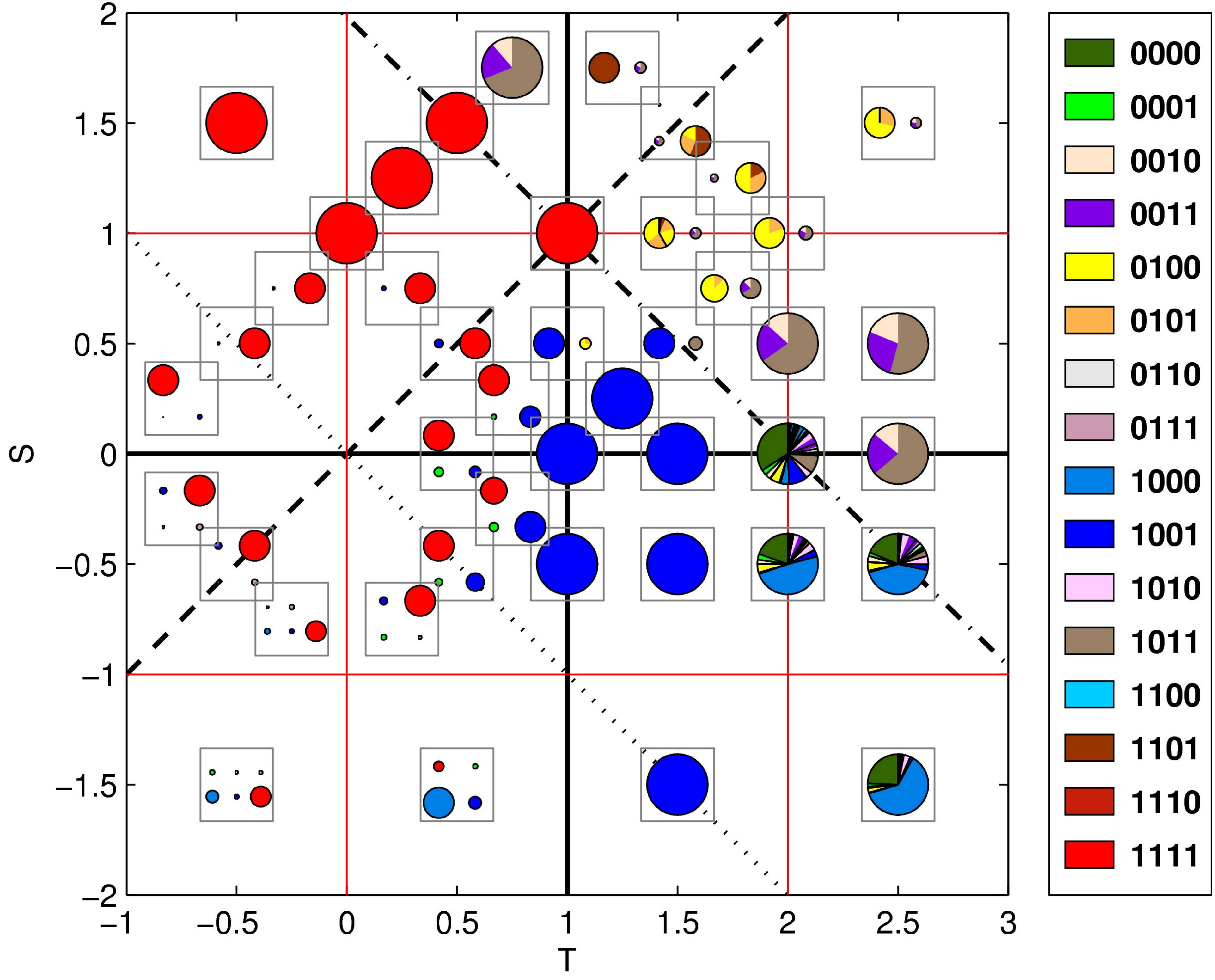
1001
1010
1011
1100
1101
1110
1111

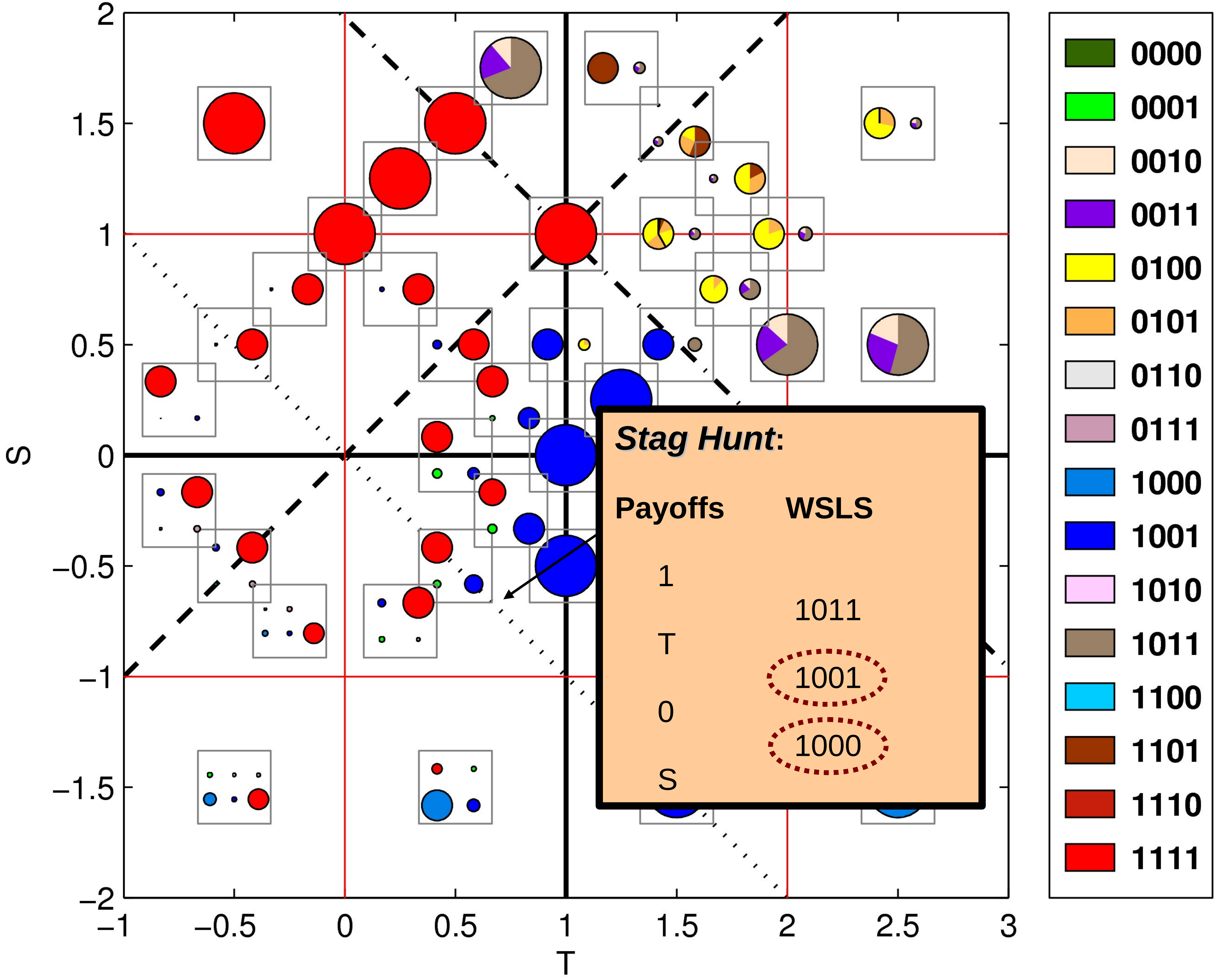

Payoffs
WSLS
S
1
T
0
0111
1111
1101
S
T
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−1
−0.5
0
0.5
1
1.5
2
2.5
3
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111

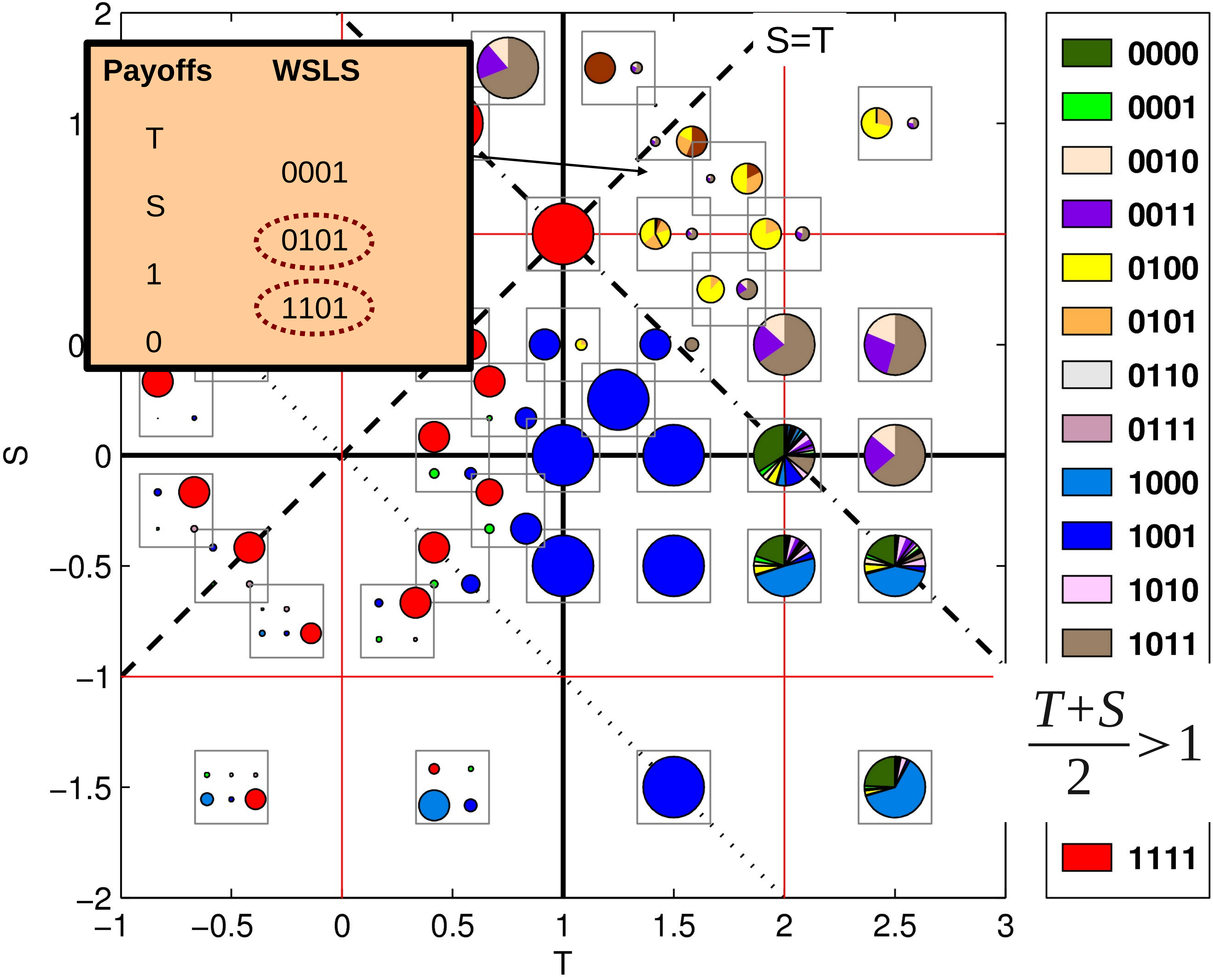

2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
S
−1
−0.5
0
0.5
1
1.5
2
2.5
3
T
Payoffs
WSLS
1
1011
S
1111
0
1110
T
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111

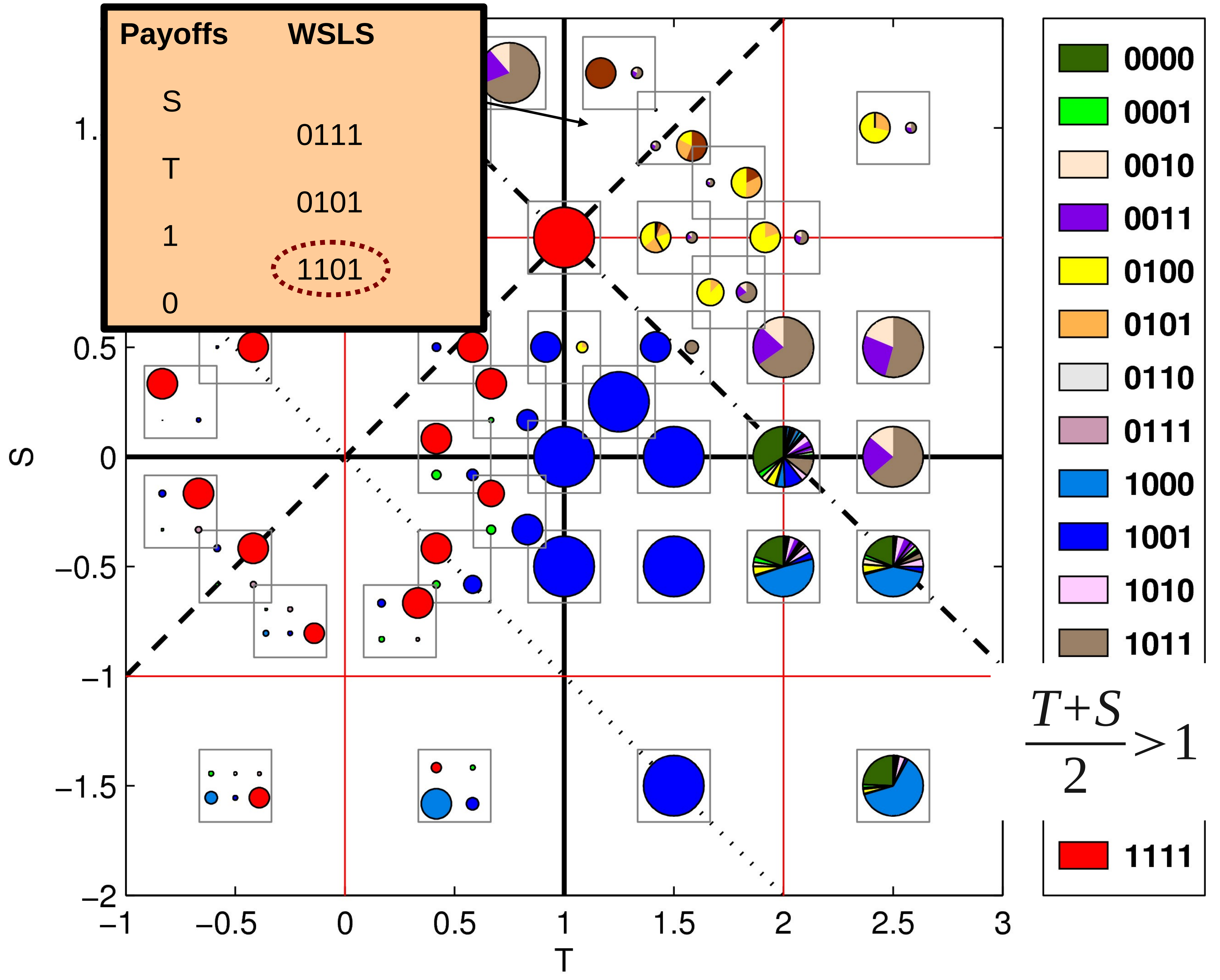

Payoffs
WSLS
S
1
0
T
0111
1111
1110
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111
S
T
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−1
−0.5
0
0.5
1
1.5
2
2.5
3

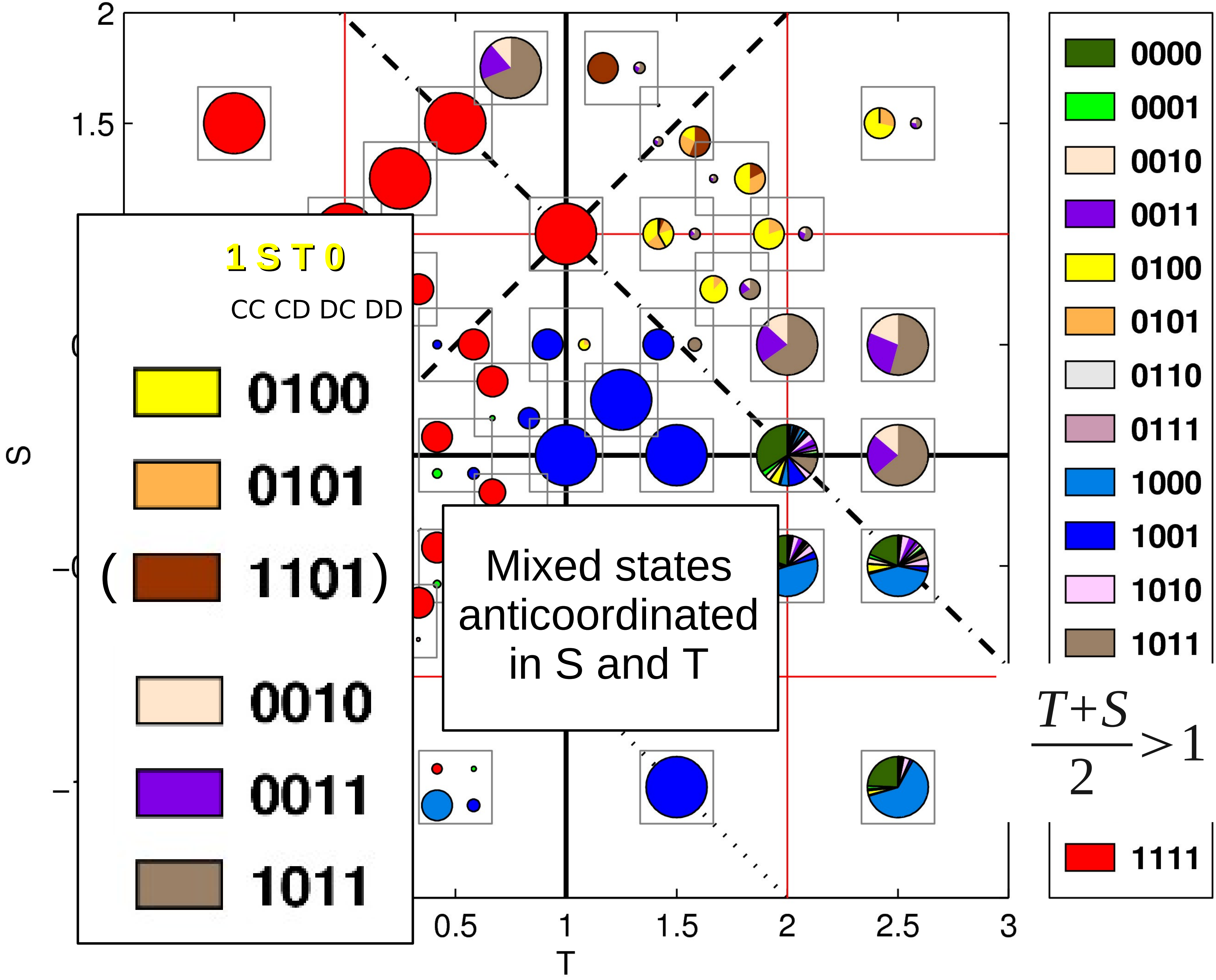


2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
S
−1
−0.5
0
0.5
1
1.5
2
2.5
3
T
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111


S
T
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−1
−0.5
0
0.5
1
1.5
2
2.5
3
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111
Stag Hunt:
Payoffs
WSLS
1
1011
T
1001
0
1000
S

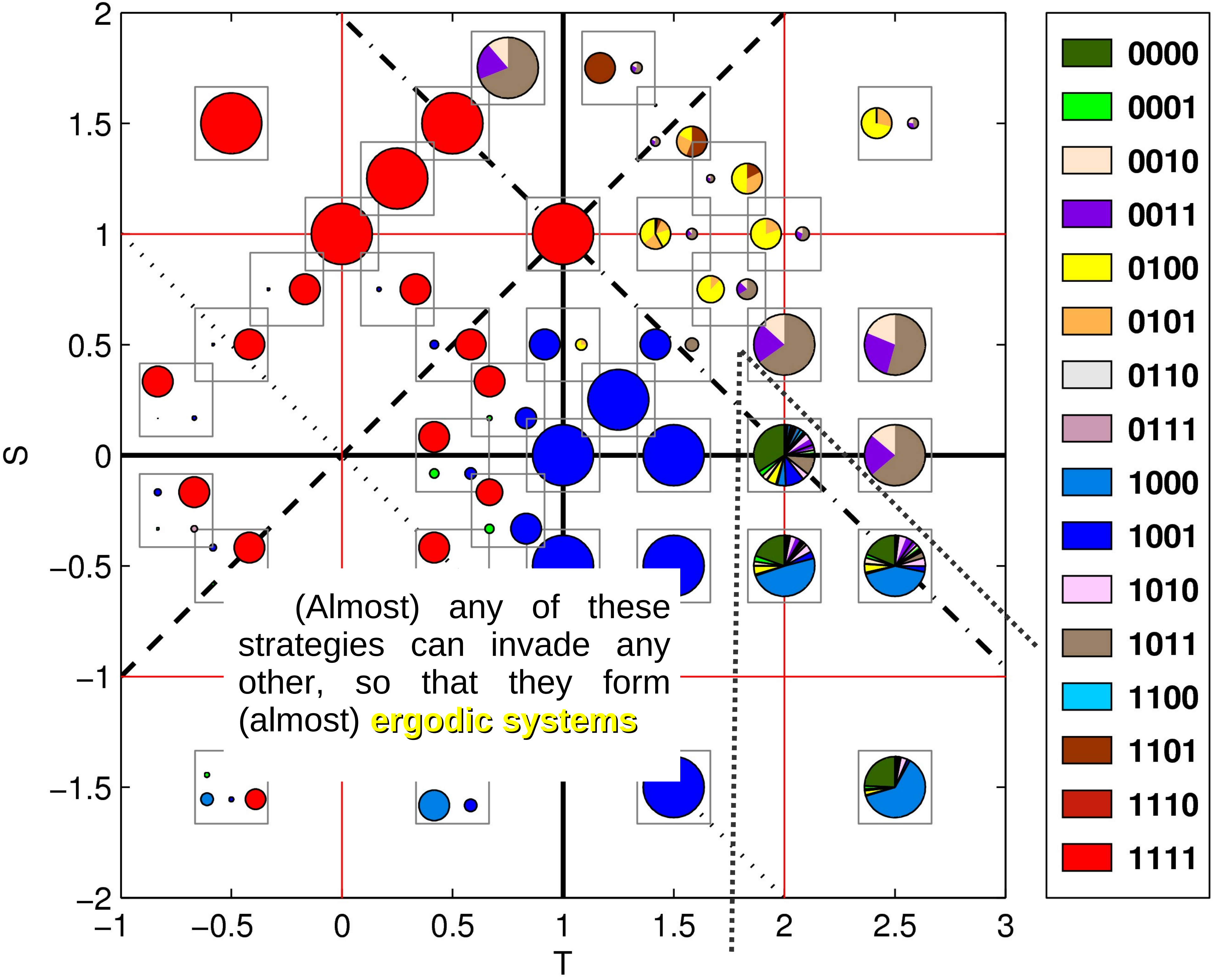

Payoffs
WSLS
T
0001
S
0101
1
1101
0
S=T
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1111
$\frac{T+S}{2}>1$
S
T
2
1
0.5
0
−0.5
−1
−1.5
−2
−1
−0.5
0
0.5
1
1.5
2
2.5
3


Payoffs
WSLS
S
0111
T
0101
1
1101
0
1.
0.5
0
−0.5
−1
−1.5
−2
S
−1
−0.5
0
0.5
1
1.5
2
2.5
3
T
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1111
$\frac{T+S}{2}>1$

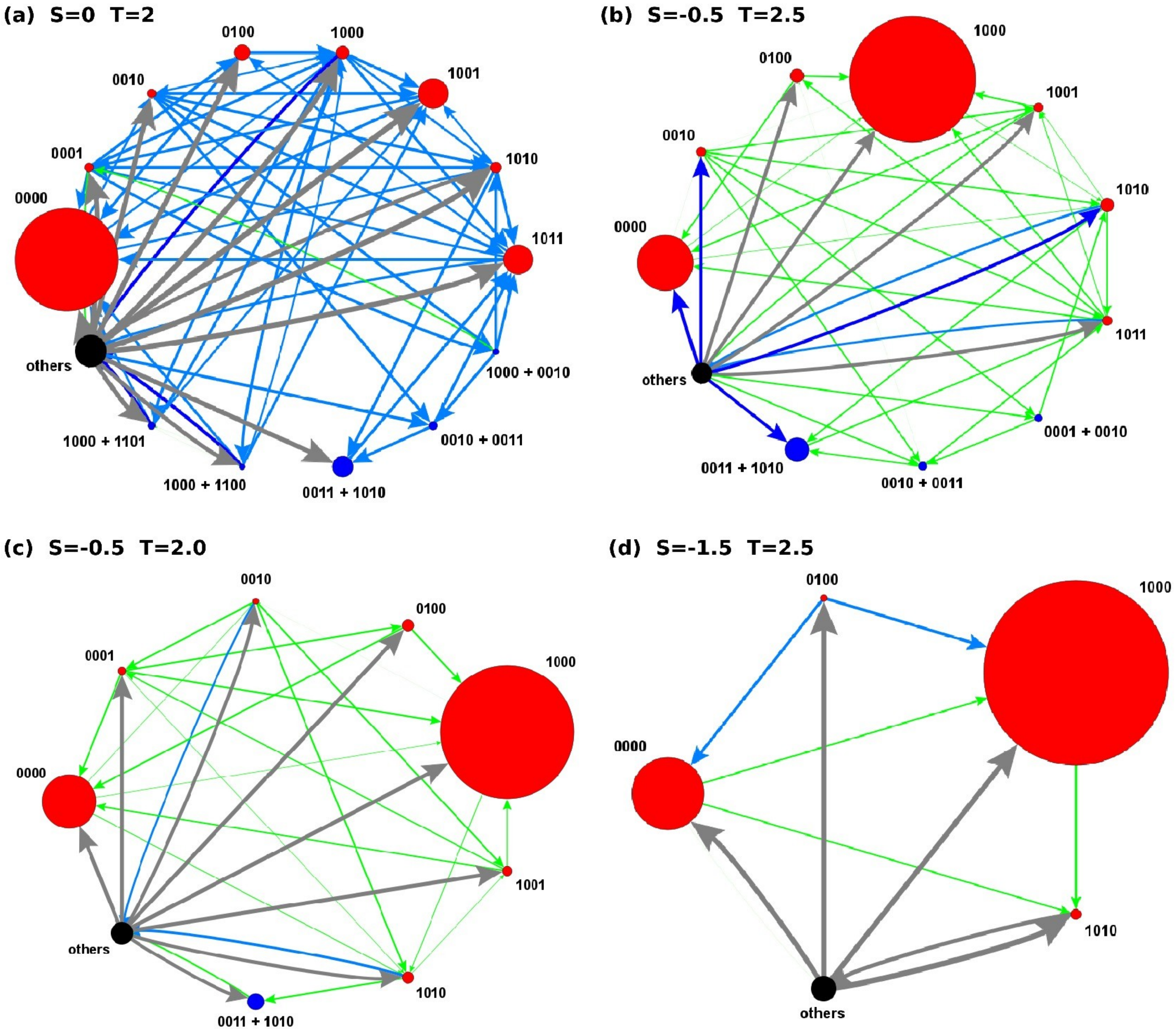


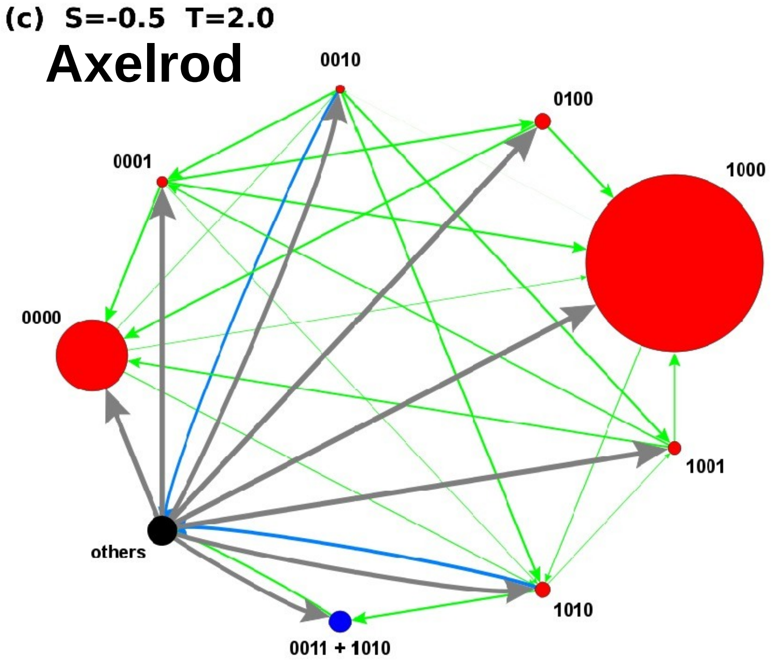
Mixed states anticoordinated in S and T

$$\frac{T+S}{2} > 1$$



(Almost) any of these strategies can invade any other, so that they form (almost) ergodic systems


(a) S=0 T=2
0100
1000
1001
0010
0001
1010
0000
1011
others
1000 + 0010
1000 + 1101
0010 + 0011
1000 + 1100
0011 + 1010
(b) S=-0.5 T=2.5
0100
1000
1001
0010
1010
0000
1011
others
0001 + 0010
0011 + 1010
0010 + 0011
(c) S=-0.5 T=2.0
0010
0100
0001
1000
0000
1001
others
1010
0011 + 1010
(d) S=-1.5 T=2.5
0100
1000
0000
1010
others



Axelrod

Grim (1000) and ***All-D*** (0000) strategies have the highest probabilities.

But **other strategies are necessary to mediate** between the main ones.

Summary

- Study of the role of the different strategies and their interrelations in a wide spectrum of games.

- Characterization of emergence and domination of the strategies via an evolutionary invasion process

- Incorporation of all the mixed states born in all the systematic analysis

Conclusions

- The best strategies **depend on the game: WSLS**

- **No ambitious WSLS** strategies are present

- Influence of **mixed states**. Mixed states anticoordinated in S and T dominate T+S>2 region

- In **ergodic systems** different strategies are important to **mediate** between the main ones

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