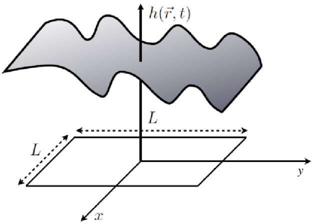
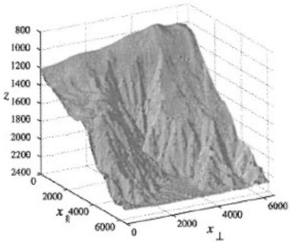
Strong anisotropy in surface kinetic roughening: theory and experiments

Edoardo Vivo
Rodolfo Cuerno and Matteo Nicoli

Growth Equations





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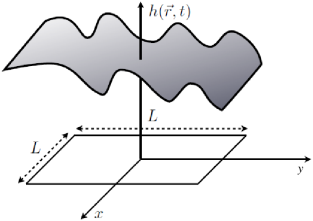
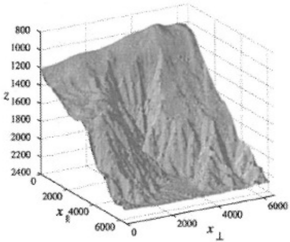
$$\partial_t h(\mathbf{r}, t) = \Xi(\mathbf{r}, t, h) + \eta(\mathbf{r}, t)$$





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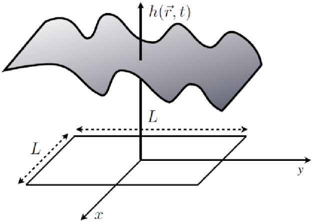
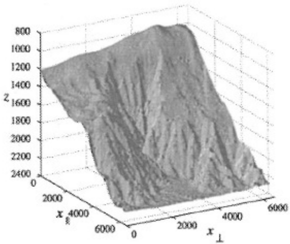


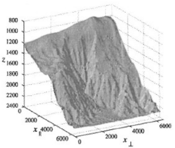


Generic Scale Invariance

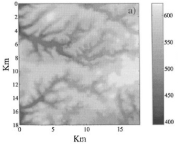
- No parameter adjustment needed
- In the context of surface growth $\longrightarrow$ Kinetic Roughening
- $\Xi(\mathbf{r}, t, h) = -\nabla \cdot \mathbf{J}$ and noise not conserved
- $\Xi(\mathbf{r}, t, h)$ depends only on derivatives of h

Grinstein, "Generic Scale Invariance and Self-Organized Criticality" (1995)

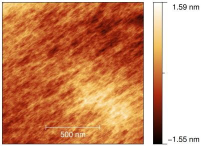
Submarine Canyon



Mountain profile



Ion Beam Sputtering



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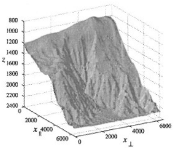

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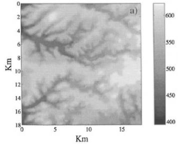

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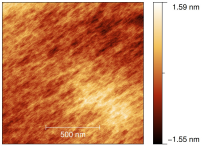

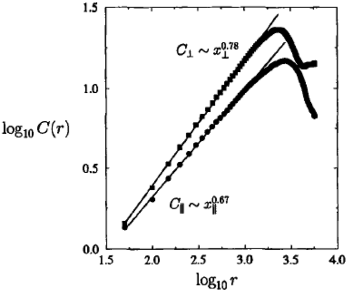

$$G_x = \left\langle \sum_{x_0} [h(x+x_0, y_0) - h(x_0, y_0)]^2 \right\rangle \sim |x|^{2\alpha_x}$$

$$G_y = \left\langle \sum_{y_0} [h(x_0, y+y_0) - h(x_0, y_0)]^2 \right\rangle \sim |y|^{2\alpha_y}$$

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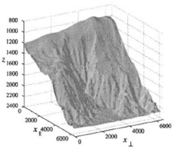

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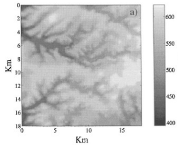

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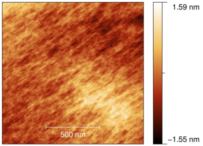

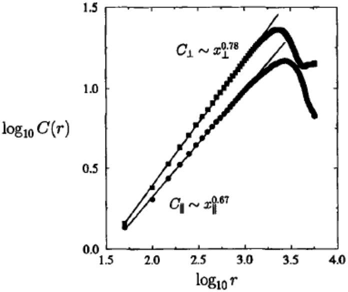

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Anisotropy Exponent

$$\zeta = \frac{\alpha_x}{\alpha_y} \begin{cases} = 1 & \text{Weak Anisotropy (WA)} \\ \neq 1 & \text{Strong Anisotropy (SA)} \end{cases}$$

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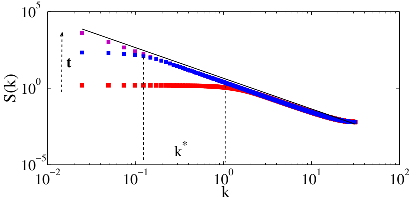
Scaling of the Observables



A.-L. Barabási, H. E. Stanley, "Fractal concepts in surface growth" (1995)

$$S(\mathbf{k},t) = \langle \tilde{h}_{\mathbf{k}}(t)\tilde{h}_{-\mathbf{k}}(t)\rangle$$

$$S(\mathbf{k},t) \sim |\mathbf{k}|^{-(2\alpha+d)} \qquad |\mathbf{k}| \ll 1$$

$$|\mathbf{k}^*| \propto \frac{1}{\xi} \sim t^{-1/z}$$

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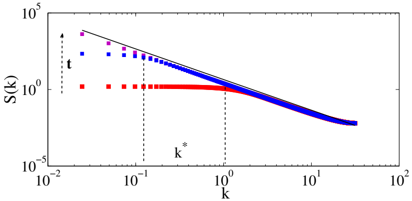

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Other Important Observables

$$G(\mathbf{r}, t) = \left\langle \sum_{\mathbf{x}} [h(\mathbf{x} + \mathbf{r}, t) - h(\mathbf{x}, t)]^2 \right\rangle = \frac{1}{L^d} \sum_{\mathbf{k}} [1 - \cos(\mathbf{k} \cdot \mathbf{r})] \, S(\mathbf{k}, t)$$

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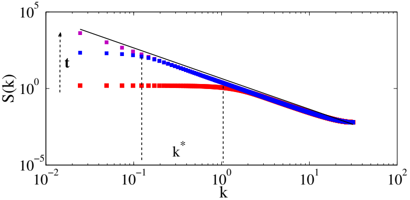

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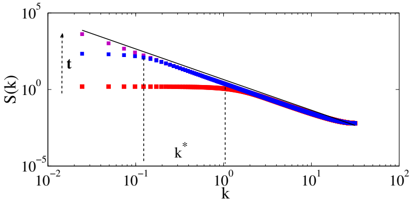

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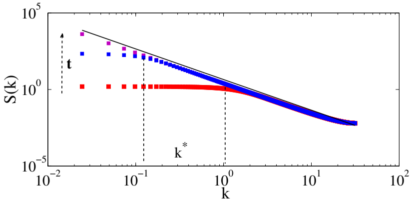
$$W(L,t) = \left\langle \left[\frac{1}{L^d}\sum_{\mathbf{r}}[h(\mathbf{r},t)-\bar{h}(t)]^2\right]^{1/2}\right\rangle = \left[\frac{1}{L^d}\sum_{\mathbf{k}\neq 0} S(\mathbf{k},t)\right]^{1/2}$$

Scaling of the Observables



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$$W(L,t) = \left\langle \left[\frac{1}{L^d} \sum_{\mathbf{r}} [h(\mathbf{r},t) - \bar{h}(t)]^2 \right]^{1/2} \right\rangle \sim \begin{cases} t^{\beta} & t \ll L^z, \\ L^{\alpha} & t \gg L^z \end{cases}$$

Problems addressed

- No (or incomplete) anisotropic generalization(s) of the FV Ansatz available. → Formulation of the Ansatz and verification through numerical simulations.

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Anisotropic Scaling Ansatz for the PSD in $d = 2$

We suppose that, at large times and long length-scales,

$$S(k_x, k_y) \sim \frac{1}{|k_x|^{2\tilde{\alpha}_x} + |k_y|^{2\tilde{\alpha}_y}} \longrightarrow \begin{cases} S(k_x, 0) \sim |k_x|^{-2\tilde{\alpha}_x} \\ S(0, k_y) \sim |k_y|^{-2\tilde{\alpha}_y} \end{cases}$$

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One-dimensional Power Spectral Densities

$$h(x, y_0) \longrightarrow S_x(k_x) = \langle \tilde{h}_{y_0}(k_x) \tilde{h}_{y_0}(-k_x) \rangle$$
$$h(x_0, y) \longrightarrow S_y(k_y) = \langle \tilde{h}_{x_0}(k_y) \tilde{h}_{x_0}(-k_y) \rangle$$

Widely used in analysis of experimental data

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- Relations between $S_{x,y}$ and $S(k_x, k_y)$.

$$S_x(k_x) \sim k_x^{-(2\tilde{\alpha}_x - \zeta)},$$
$$S_y(k_y) \sim k_y^{-(2\tilde{\alpha}_y - 1/\zeta)}$$

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- Relations between $S_{x,y}$ and $S(k_x, k_y)$.
- Relations between $S_{x,y}$ and $G_{x,y}$.

$$2\alpha_x = 2\tilde{\alpha}_x - \zeta - 1$$
$$2\alpha_y = 2\tilde{\alpha}_y - 1/\zeta - 1$$
$$\zeta = \frac{\alpha_x}{\alpha_y} = \frac{\tilde{\alpha}_x}{\tilde{\alpha}_y}$$

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- It is possible do define $z_{x,y}$, and it turns out that $\zeta = \frac{z_y}{z_x}$.
- The growth exponent β is the same in every direction.
- Only three independent exponents, α_x, z_x, and ζ.

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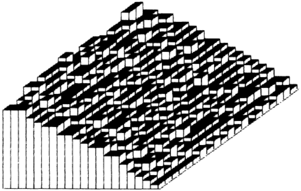
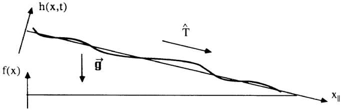
The Hwa-Kardar equation

$$\partial_t h = \nu_x \partial_x^2 h + \nu_y \partial_y^2 h - \frac{\lambda}{2}\partial_x(h^2) + \eta$$





T. Hwa and M. Kardar, PRA **45** (1992)

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Properties

- $\partial_t h + \nabla \cdot \mathbf{J} = \eta(\mathbf{r}, t) \rightarrow$ Generic scale invariance

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Properties

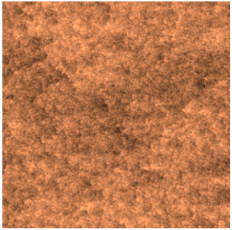
- $\partial_t h + \nabla \cdot \mathbf{J} = \eta(\mathbf{r}, t) \to$ Generic scale invariance
- Scaling exponents are believed to be exactly known for all dimensions. In 2D

$$\alpha_x = -\frac{1}{5}, \quad z_x = \frac{6}{5}, \quad \zeta = \frac{3}{5}$$

T. Hwa and M. Kardar, PRA **45** (1992)

Numerical Simulations for the HK equations

- Pseudo-Spectral Method.
- Wait for the sytem to reach a steady state.
- Compute the 2D and 1D PSDs and see if they scale as expected.



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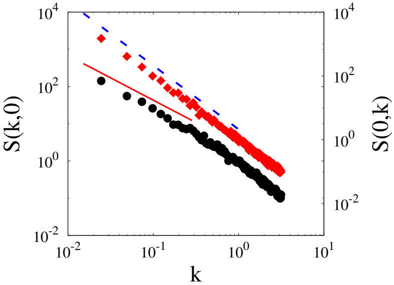

— $2\tilde{\alpha}_x = 6/5$

- - $2\tilde{\alpha}_y = 2$

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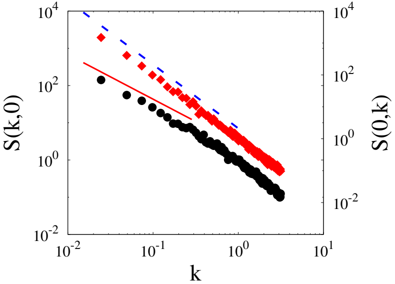

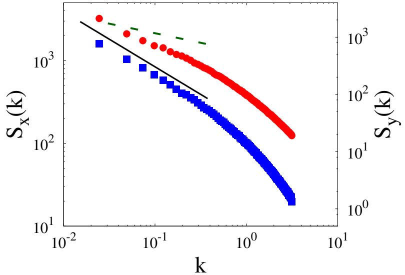

- $2\tilde{\alpha}_x = 6/5$
- $2\tilde{\alpha}_y = 2$
- $2\tilde{\alpha}_x - \zeta = 3/5$
- $2\tilde{\alpha}_y - 1/\zeta = 1/3$

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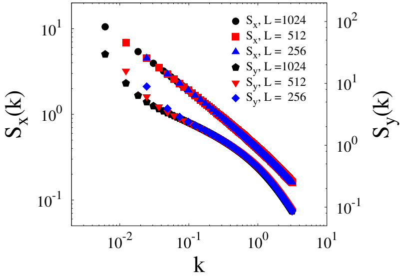

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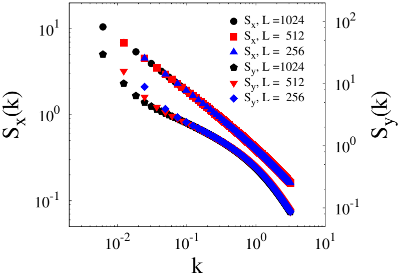
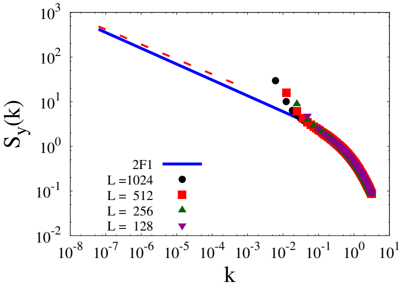
- $$S_y(k_y) = \frac{1}{\pi} \int_0^{\pi/a} \frac{dk_x}{k_x^{2\tilde{\alpha}_x} + k_y^{2\tilde{\alpha}_y}} = \frac{1}{a} k_y^{-2\tilde{\alpha}_y} \, {}_2F_1\left(\frac{1}{2\tilde{\alpha}_x}, 1; 1 + \frac{1}{2\tilde{\alpha}_x}, -\left(\frac{\pi}{a}\right)^{2\tilde{\alpha}_x} k_y^{-2\tilde{\alpha}_y}\right)$$





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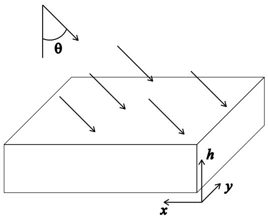
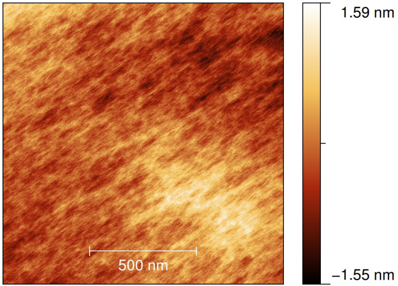
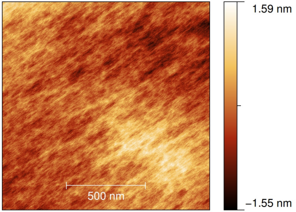
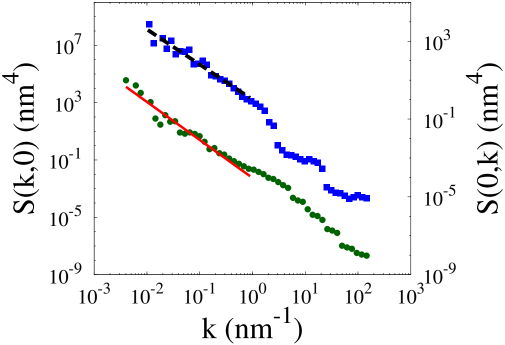




Stable case

- 2 keV Kr^+ ions on a Si(100) target.
- $\theta = 81^\circ$.
- Samples lateral size $\sim 1 - 2\,\mu$m.
- Average over 8 samples.

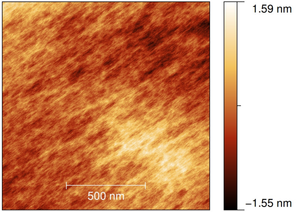


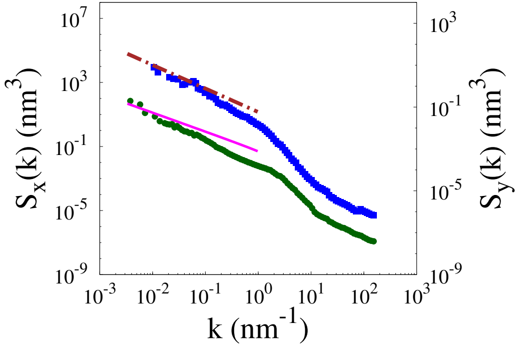


$$2\tilde{\alpha}_x = 2.66 \pm 0.02,$$
$$2\tilde{\alpha}_y = 1.80 \pm 0.02,$$
$$\zeta = 1.48 \pm 0.02,$$

Stable case

- 2 keV Kr^+ ions on a Si(100) target.
- $\theta = 81°$.
- Samples lateral size $\sim 1 - 2\,\mu m$.
- Average over 8 samples.





$$2\tilde{\alpha}_x = 2.66 \pm 0.02,$$

$$2\tilde{\alpha}_y = 1.80 \pm 0.02,$$

$$\zeta = 1.48 \pm 0.02,$$

$$S_x(k) \sim k^{-1.18},$$

$$S_y(k) \sim k^{-1.12}$$

Problems addressed

- No (or incomplete) anisotropic generalization(s) of the FV Ansatz available. → Formulation of the Ansatz and verification through numerical simulations.
- Lack of experimental verification. → Analysis of experimental data using our Ansatz.
- Paradigmatic anisotropic equations (AKPZ) do not display SA. → Are there conditions for SPDEs to display SA? DRG for several equations.

Linear equations

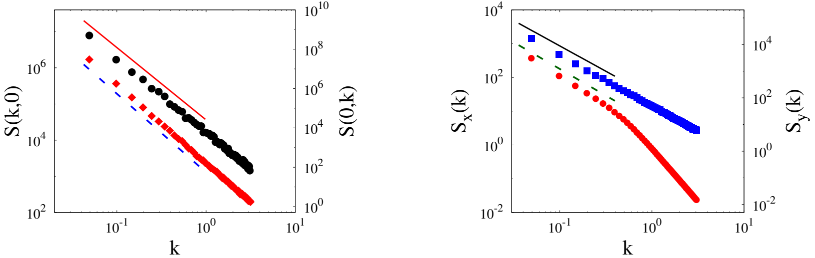
aEW equation: $\partial_t h = \nu_x \partial_x^2 h + \nu_y \partial_y^2 h + \eta$

Isotropic, in the Edwards-Wilkinson universality class

Linear equations

2-4 equation: $\partial_t h = \nu_x \partial_x^2 h - \nu_y \partial_y^4 h + \eta$

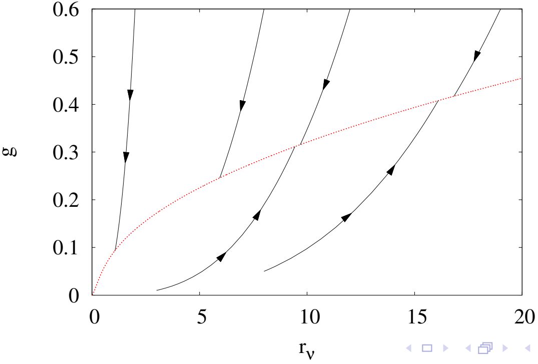
Strongly Anisotropic



E. Vivo, M. Nicoli, and R. Cuerno, PRE **86** (2012)

Non-Linear equations: conserved dynamics

HK equation: $\partial_t h = \nu_x \partial_x^2 h + \nu_y \partial_y^2 h - \frac{\lambda_x}{2}(\partial_x h^2) \qquad + \eta$



Non-Linear equations: conserved dynamics

gHK equation: $\partial_t h = \nu_x \partial_x^2 h + \nu_y \partial_y^2 h - \frac{\lambda_x}{2}(\partial_x h^2) - \frac{\lambda_y}{2}(\partial_y h^2) + \eta$

Complete case

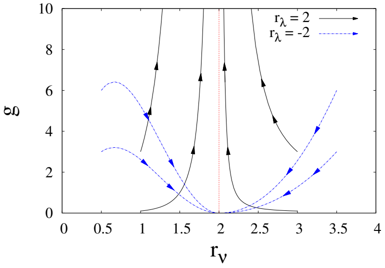
Isotropic, with exponents given by

$$\alpha_x = -1/3, \quad z_x = 4/3, \quad \zeta = 1$$

Non-Linear equations: non-conserved dynamics

aKPZ equation: $\partial_t h = \nu_x \partial_x^2 h + \nu_y \partial_y^2 h + \frac{\lambda_x}{2}(\partial_x h)^2 + \frac{\lambda_y}{2}(\partial_y h)^2 + \eta$

Isotropic, but universality class depends on the relative signs of λ_x, λ_y

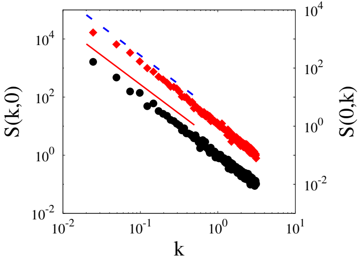


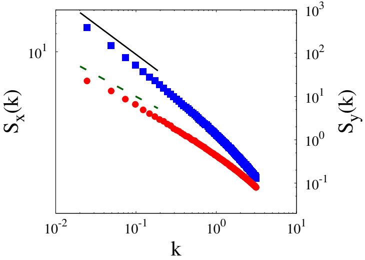
D. Wolf, PRL **67** (1991)

Non-Linear equations: non-conserved dynamics

aKPZ equation: $\partial_t h = \nu_x \partial_x^2 h + \nu_y \partial_y^2 h + \frac{\lambda_x}{2} (\partial_x h)^2 \qquad + \eta$

Numerical simulations show that it is still isotropic, in the EW universality class





Non-Linear equations: summary

	Conserved		Not Conserved
	ACKPZ	HK	AKPZ
Complete	WA	WA	WA
Incomplete ($\lambda_y = 0$)	SA	SA	WA

Non-Linear equations: summary

	Conserved		Not Conserved
	ACKPZ	HK	AKPZ
Complete	WA	WA	WA
Incomplete ($\lambda_y = 0$)	SA	SA	WA

We suggest that, in order to display SA, the equation must be conserved (with non-conserved noise), and must have a strongly anisotropic form.

Conclusions

- We put forward an Ansatz for anisotropic scaling, that relates in a consistent way scaling exponents measured from different observables.
- We verified the Ansatz against linear and non-linear strongly anisotropic SPDEs, and found experimental evidence for it to occurr.
- Using DRG techniques, together with numerical simulations, we suggest that SA can only be displayed by a restricted class of non-linear SPDEs.

Anisotropic Scaling Ansatz for the PSD in $d = 2$

$$S(k_x, k_y) \sim \frac{1}{|k_x|^{2\tilde{\alpha}_x} + |k_y|^{2\tilde{\alpha}_y}} \longrightarrow \begin{cases} S(k_x, 0) \sim |k_x|^{-2\tilde{\alpha}_x} \\ S(0, k_y) \sim |k_y|^{-2\tilde{\alpha}_y} \end{cases}$$

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$$h(x, y_0) \longrightarrow S_x(k_x) = \langle \tilde{h}_{y_0}(k_x) \tilde{h}_{y_0}(-k_x) \rangle$$
$$h(x_0, y) \longrightarrow S_y(k_y) = \langle \tilde{h}_{x_0}(k_y) \tilde{h}_{x_0}(-k_y) \rangle$$

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Questions

- What is the relation between $S_{x,y}(k_{x,y})$ and $S(k_x, k_y)$?
- What is the scaling behavior of the one-dimensional PSDs?
- What is the relation between $\tilde{\alpha}_{x,y}$ and $\alpha_{x,y}$?

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1D vs 2D PSD

$$S_x(k_x) = \frac{1}{L} S(k_x, 0) + \frac{2}{L} \sum_{n_y=1}^{\infty} S(k_x, k_y)$$

$$S_y(k_y) = \frac{1}{L} S(0, k_y) + \frac{2}{L} \sum_{n_x=1}^{\infty} S(k_x, k_y)$$

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1D vs 2D PSD - Thermodynamic limit

$$S_x(k_x) = \frac{1}{\pi} \int_0^\infty dk_y S(k_x, k_y) \sim k_x^{-(2\tilde{\alpha}_x - \zeta)}$$

$$S_y(k_y) = \frac{1}{\pi} \int_0^\infty dk_x S(k_x, k_y) \sim k_y^{-(2\tilde{\alpha}_y - 1/\zeta)}$$

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Anisotropic Scaling Ansatz for the PSD in $d = 2$

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Relations among exponents

$$G_x(x) = \frac{2}{\pi} \int_0^\infty dk_x \, [1 - \cos(k_x \, x)] \, S_x(k_x) \sim |x|^{2\tilde{\alpha}_x - \zeta - 1} = |x|^{2\alpha_x}$$

$$G_y(y) = \frac{2}{\pi} \int_0^\infty dk_y \, [1 - \cos(k_y \, y)] \, S_y(k_y) \sim |y|^{2\tilde{\alpha}_y - 1/\zeta - 1} = |y|^{2\alpha_y}$$

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Relations among exponents

$$2\alpha_x = 2\tilde{\alpha}_x - \zeta - 1$$
$$2\alpha_y = 2\tilde{\alpha}_y - 1/\zeta - 1$$
$$\zeta = \frac{\alpha_x}{\alpha_y} = \frac{\tilde{\alpha}_x}{\tilde{\alpha}_y}$$

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- It is possible do define $z_{x,y}$ such that $\zeta = \frac{z_y}{z_x}$.
- The growth exponent β is the same in every direction.

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Gaussian Approximation for the HK equation

$$\partial_t \tilde{h}(\mathbf{k},t) = -\left(\nu_x |k_x|^{6/5} + \nu_y |k_y|^2\right) \tilde{h}(\mathbf{k},t) + \tilde{\eta}(\mathbf{k},t),$$

- Linear, and yet shares the values of the exponents with the HK equation.
- The Ansatz is the exact solution.
- It's a Gaussian approximation: $\mathcal{P}\{h\} \propto \exp[-(1/2D)\int \left(\nu_x|k_x|^{6/5} + \nu_y|k_y|^2\right)|\tilde{h}(\mathbf{k})|^2 \mathrm{d}\mathbf{k}]$

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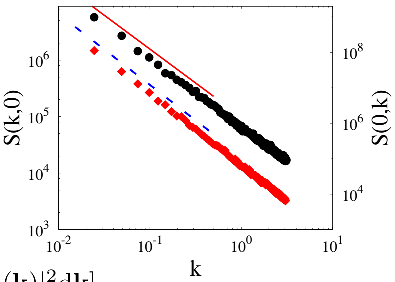

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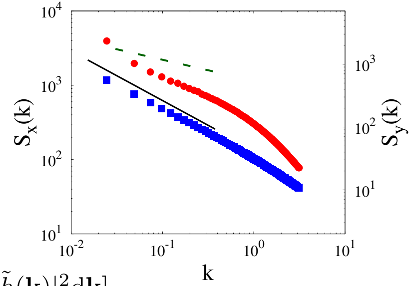

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