Casimir Effect between Chern Insulators.

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LPTMS

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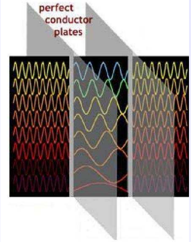
Outline

Casimir Effect

Vacuum energy $E = \frac{\hbar}{c}\sum_n \omega_n$.
Perfect metal plates



$$\phi_n(0) = \phi_n(L) = 0 \quad \Rightarrow \quad k_n = \frac{2\pi}{L} n \quad \forall\, n \in \mathbb{N}$$

$$\begin{array}{lll} \text{Inside:} & k_n = \frac{2\pi}{L} n & \forall n \in \mathbb{N} \\ \text{Outside:} & k_n & \text{continuous.} \end{array}$$

$$\langle E \rangle_{in} = \frac{\hbar c}{2} \sum_{n=1}^{\infty} k_n \to \infty$$

$$\langle E \rangle_{out} = \frac{\hbar c}{2} \int_0^{\infty} k(n)\, dn \to \infty$$

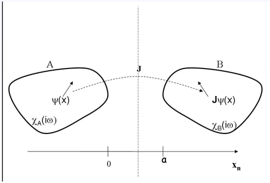
L	$\langle p \rangle$(atm)
1 m	10^{-33}
1 mm	10^{-21}
1 μm	10^{-9}
1 nm	10^{3}

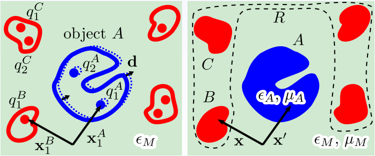
$$\boxed{\langle E \rangle_{in} - \langle E \rangle_{out} = -\frac{\hbar c \pi^2}{720 L^3} A} \quad \Rightarrow \quad \langle F \rangle = -\partial_L \langle E \rangle = -\frac{\hbar c \pi^2}{240 L^4} A$$

Looking for Casimir repulsion

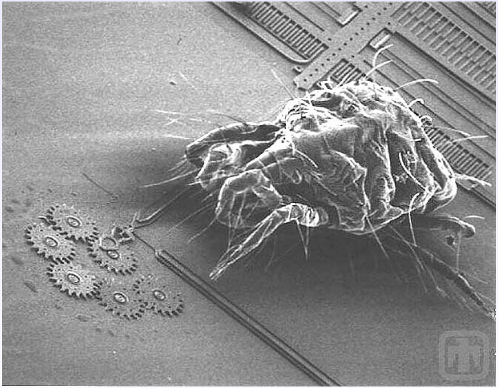
- Why?
  - Casimir interaction between usual dielectrics is **attractive**.
  - A proposal to avoid Stiction phenomena in vacuum
  - New designs for NEM and MEMs
  - Fundamental Physics: Is it possible to obtain repulsion with the Casimir effect?
- Stability theorems
  - Symmetric configurations attract [Kenneth & Klich. PRL **97**, 160401 (2006)]
  - Stable equilibria not accessible for dielectrics in vacuum.

    [Rahi, Kardar & Emig. PRL **105**, 070404 (2010).]



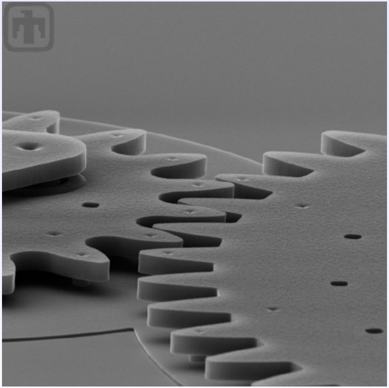


Human scale vs. Nanoscale



$$\left.\begin{array}{l}\text{Same mechanism}\\ \text{Different scales}\end{array}\right\} \Rightarrow \text{Different behavior!}$$

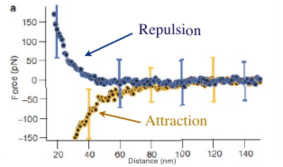
Stiction

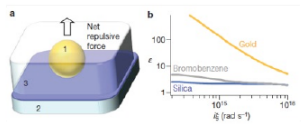


- At nanoscale, the gear sticks.
- The reason: The strong attractive Casimir force between gearwheels.
- Everyone is looking for ways to avoid it.

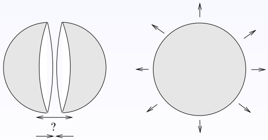
Several proposals to obtain repulsion

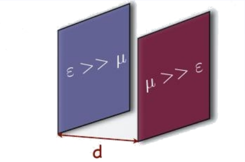
- Lifshitz formula, changes the sign of Casimir effect in dielectric media (proved) [Munday et al. Nature 457, 170-173 (2009)]





- Boyer: Sphere shell formula
- Boyer: dielectric ($\epsilon \to \infty$) vs. perfect magnetic ($\mu \to \infty$) [Boyer, PRA 9, 2078 (1974)]

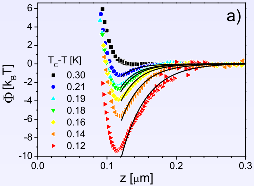


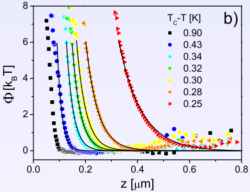


$$E_B(d) = -\frac{7}{8}E_c(d)$$

Several proposals to obtain repulsion

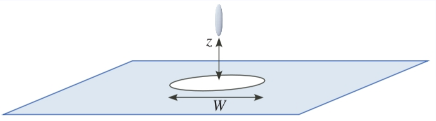
- Repulsion in Critical Casimir effect (proved) [Hertlein et al., Nature **451**, 172 (2008)]

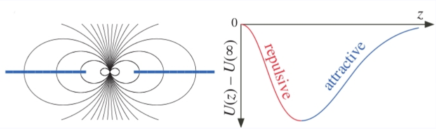




- In some geometric configurations, unstable repulsion between perfect metals

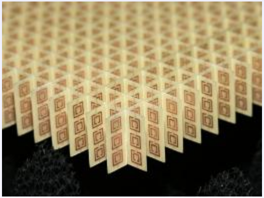
[Levin et al. PRL **105**, 090403 (2010)]

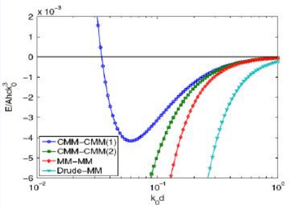




Several proposals to obtain repulsion

- Several proposals with metamaterials seem to give repulsion in vacuum, but several problems remains. [Zhao et al. PRL **103**, 103602 (2009)] [ULeonhardt and Philbin, New J. Phys. **9**, 254 (2007)] [Rosa et al. PRL. **100**, 183602 (2008), PRA **78**, 032117 (2008)] . . .





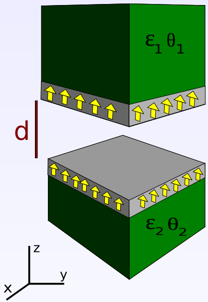
- In this case, the materials are Veselago lensing material for all frequencies.

$$\begin{aligned} \mathbf{D} &= \epsilon \mathbf{E} + i\kappa \mathbf{H} \\ \mathbf{B} &= \mu^{-1}\mathbf{H} - i\kappa \mathbf{E} \end{aligned}$$

Several proposals to obtain repulsion

- Topological Insulators. [Grushin and Cortijo, PRL **106**, 020403 (2011)] [Grushin et. al. PRB **84**, 045119 (2011)] [Rodriguez-Lopez PRB. **84**, 165409 (2011)]

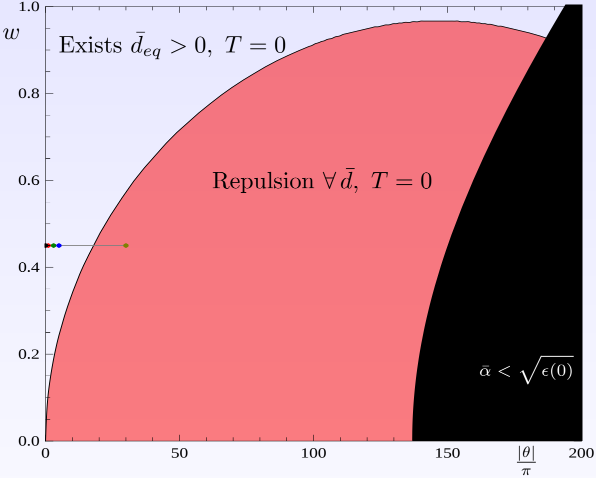
$$\begin{cases} \mathbf{D} = \epsilon \mathbf{E} + \alpha\theta/\pi \mathbf{B} \\ \mathbf{B} = \mu^{-1}\mathbf{H} - \alpha\theta/\pi \mathbf{E} \end{cases}$$



- $\mu = \mu_0$
- $\alpha = \frac{e^2}{\hbar c}$
- $\theta = (2n+1)\pi$ for $n \in \mathbb{Z}$
- Oscillator model (Pseudo–Drude) for $\epsilon(k)$:

$$\epsilon(i\kappa) = \epsilon_0 + \sum_i \frac{\omega_{e,i}^2}{\omega_{R,i}^2 + \gamma_{R,i} c\kappa + c^2\kappa^2}.$$

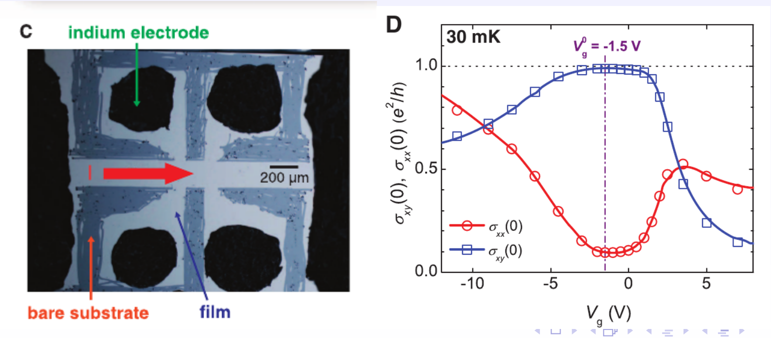
- Naive model for the magnetoelectric coupling $\theta(\omega) = cte$



- If $\text{sign}(\theta_1) = \text{sign}(\theta_2)$, Kenneth & Klich theorem imply attraction.
- If $\text{sign}(\theta_1) \neq \text{sign}(\theta_2)$, any theorem constrain the dynamics, thus we can expect any behavior.
- The attraction due to the bulk opposes to the repulsion due to θ.

Chern Insulators

- A new 2D topological state of matter.
- Characterized by their Chern number C.
- C is the number of Dirac cones weighted by the sign of their masses.
- Em properties: $\sigma_{xx}(\omega = 0) = 0$ and $\sigma_{xy}(\omega = 0) = C\frac{e^2}{h}$, with $C \in \mathbb{Z}$.
- Quantum Hall surface states without magnetic field.
- Experientally observed. [Cui-Zu Chang et al. Science 340, 167 (2013)]



Casimir effect between CIs plates: exact calculations

- Casimir energy for two parallel plates [Dzyaloshinskii, Lifshitz & Pitaevskii, Adv. in Phys. **10**, 38, (1961)] [Rahi et al. PRD **80**, 085021 (2009)]

$$E = \frac{\hbar c}{2\pi}\int_0^\infty d\xi \int \frac{d\mathbf{k}_\perp^2}{(2\pi)^2}\log\left|\mathbb{I} - \mathbb{R}_1\mathbb{R}_2 e^{-2d\sqrt{\xi^2+\mathbf{k}_\perp^2}}\right|$$

- $\mathbb{R}$ matrix for zero thickness (pure 2D) dielectrics with surface currents

$$\mathbb{R} = \frac{2\pi}{c\Delta}\begin{pmatrix} -\frac{\sigma_{xx}}{\lambda} - \frac{2\pi}{c}\left(\sigma_{xx}^2+\sigma_{xy}^2\right) & \sigma_{xy} \\ \sigma_{xy} & \lambda\sigma_{xx} + \frac{2\pi}{c}\left(\sigma_{xx}^2+\sigma_{xy}^2\right) \end{pmatrix}$$

  - $\Delta = 1 + \frac{2\pi}{c}\sigma_{xx}\left(\frac{1}{\lambda}+\lambda\right) + \frac{4\pi^2}{c^2}\left(\sigma_{xx}^2+\sigma_{xy}^2\right)$
  - $\lambda = \frac{ck_z}{\omega}$
  - $k_z^2 = \left(\frac{\omega}{c}\right)^2 + \mathbf{k}_\perp^2$
  - $\xi = i\frac{\omega}{c}$

Casimir effect between CIs plates

- Generic one band model for Chern Insulators

$$H = \sum_{\mathbf{k}\in BZ} c_{\mathbf{k}}^{\dagger} d_{\mathbf{k}} \cdot \sigma c_{\mathbf{k}}$$

$$d_1 + i\, d_2 = t\left(\sin(k_1) + i\, \sin(k_2)\right)$$

$$d_3 = h_1 \cos(k_1) + h_2 \cos(k_1) + h_3 + h_4 \left(\cos(k_1 + k_2) + \cos(k_1 - k_2)\right)$$

- Masses in the Dirac points

$$m^{(ij)} = (-1)^i h_1 + (-1)^j h_2 + h_3 + (-1)^{i+j} 2h_4$$

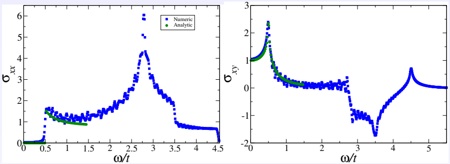
- Chern Number

$$C = \frac{1}{2} \sum_{i,j} (-1)^{i+j} \mathrm{sign}(m^{(ij)}) \in \{-2, -1, 0, 1, 2\}$$

Casimir effect between CIs plates

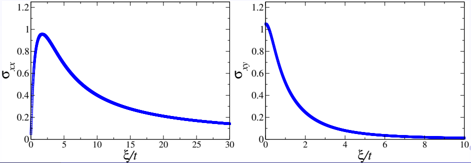
- Kubo formula for $\sigma_{xx}(\omega)$ and $\sigma_{xy}(\omega)$

$$\sigma_{ij}(\omega) = \frac{i}{\omega}\frac{1}{\Omega N}\sum_{s,t=\{+,-\}}\sum_{\mathbf{k}\in BZ}\frac{\mathrm{Tr}\left(j^i_{\mathbf{k}}P_{s,\mathbf{k}}j^j_{\mathbf{k}}P^i_{t,\mathbf{k}}\right)}{\omega - E_{s,\mathbf{k}} + E_{t,\mathbf{k}}}\left(n_{t,\mathbf{k}} - n_{s,\mathbf{k}}\right)$$



- Kramers-Krönig relations $(\sigma(\omega) = \sigma_R(\omega) + i\,\sigma_I(\omega))$

$$\sigma_{ij}(i\xi) = \frac{2}{\pi}\int_0^{\infty} d\omega\,\frac{\xi\sigma_{ij,R}(\omega)}{\omega^2+\xi^2}$$

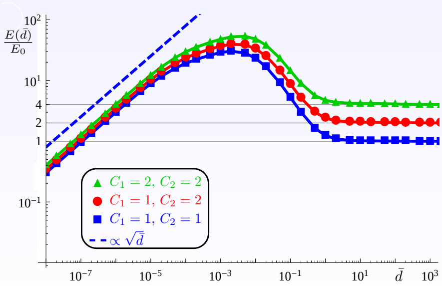


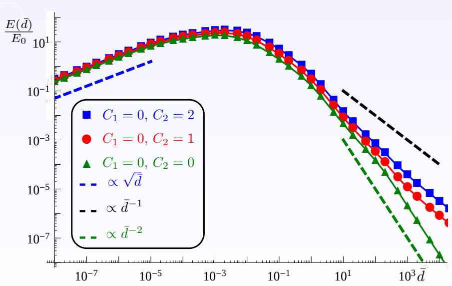
Casimir effect between CIs plates

- Large distance limit

$$\lim_{\xi \ll 1} \sigma_{xx}(i\xi) = b\alpha\xi, \qquad \lim_{\xi \ll 1} \sigma_{xy}(i\xi) = C\frac{\alpha}{2\pi}.$$

$$\lim_{d\to\infty} E = -\frac{\hbar c\alpha^2}{8\pi^2 d^3} C_1 C_2 - \frac{\hbar c\alpha^3}{4\pi^2 d^4}\left[C_1^2 b_2 + C_2^2 b_1\right] - \frac{9\hbar c\alpha^2}{10 d^5} b_1 b_2 + \cdots$$

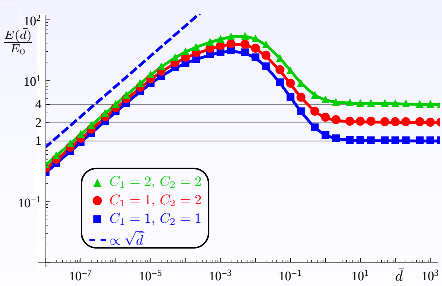


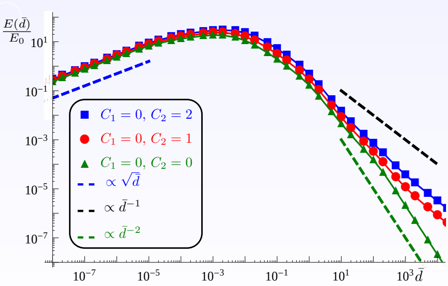


- Short distance limit

$$\lim_{\xi \gg 1} \sigma_{xx}(i\xi) = \alpha \frac{s_{xx}}{\xi}, \qquad \lim_{\xi \gg 1} \sigma_{xy}(i\xi) = C\alpha \frac{s_{xy}}{\xi^2}.$$

$$\lim_{d \to 0} E = -\frac{3\hbar c}{128} \sqrt{\frac{\alpha}{d^5}} \frac{\sqrt{s_{xx,1} s_{xx,2}}}{\sqrt{s_{xx,1}} + \sqrt{s_{xx,2}}}.$$



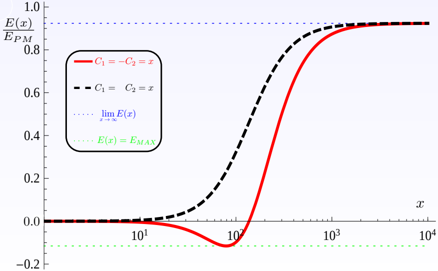


Casimir effect between CIs plates

- Large distance limit and large Chern number

$$\lim_{\xi \ll 1} \sigma_{xx}(i\xi) = 0, \qquad \lim_{\xi \ll 1} \sigma_{xy}(i\xi) = C\frac{\alpha}{2\pi}.$$

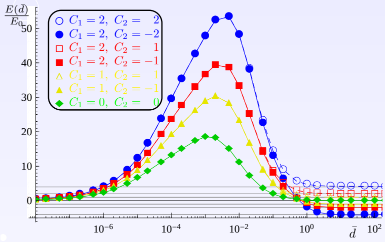
$$\lim_{d\to\infty} E = -\frac{\hbar c \alpha^2}{8\pi^2 d^3} C_1 C_2 \frac{1 + C_1 C_2 \alpha^2}{(1 + C_1^2\alpha^2)(1 + C_2^2\alpha^2)} \approx -\frac{\hbar c \alpha^2}{8\pi^2 d^3} C_1 C_2$$

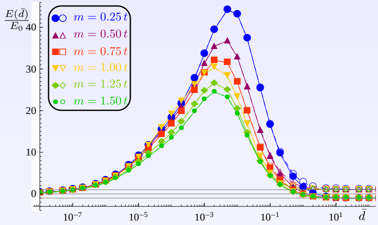


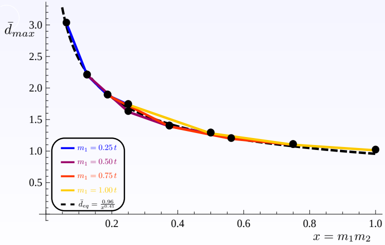
$$\lim_{C \gg 1} E = -\frac{\hbar c}{8\pi^2 d^3} \approx 0.92 E_{PM}$$

$$\lim_{C = 1/\sqrt{3}\alpha} E = \frac{\hbar c}{64\pi^2 d^3} \approx -0.12 E_{PM}$$

Numerical computation for intermediate distances.







$$\bar{d}_{eq} \sim \frac{1}{\sqrt{C_1 C_2 m_1 m_2}}$$

Conclussions

1. Casimir effect between Chern Insulators can be repulsive at large distances if $C_1 C_2 < 0$...
2. ... but it is always attractive at short distances.
3. The effect is small because $E \propto \hbar c \alpha^2$ ($F_{CIs} \approx 10^{-3} F_{PM}$)
4. Proposal: Avoid (or at least reduce) Stiction with CIs?

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