Advances in Complex Systems, Vol. 14, No. 1 (2011) 31–53

DOI: 10.1142/S0219525911002883

TURNOUT INTENTION AND RANDOM SOCIAL NETWORKS

CONSTANZA FOSCO*

**Grupo Interdisciplinar de Sistemas Complejos (GISC),*
Departamento de Matemáticas, Universidad Carlos III de Madrid, Spain
cfosco@ucn.cl

ANNICK LARUELLE

Department of Economics I, University of the Basque Country UPV-EHU, Bilbao, Spain
IKERBASQUE, the Basque Science Foundation, Bilbao, Spain
a.laruelle@ikerbasque.org

ANGEL SÁNCHEZ

Grupo Interdisciplinar de Sistemas Complejos (GISC),
Departamento de Matemáticas, Universidad Carlos III de Madrid, Spain
Instituto de Ciencias Matemáticas CSIC-UAM-UC3M-UCM, Madrid, Spain
Instituto de Biocomputación y Física de Sistemas Complejos (BIFI),
Universidad de Zaragoza, Spain
anxo@math.uc3m.es

Received 24 May 2010
Revised 3 August 2010

How can networking affect the turnout in an election? We present a simple model to explain turnout as a result of a dynamic process of formation of the intention to vote within Erdös–Rényi networks. Citizens have fixed preferences for one of two parties and are embedded in a social network. They decide whether or not to vote on the basis of the attitude of their immediate contacts. They may simply follow the behavior of the majority (followers) or make an adaptive local calculus of voting (calculators). So they have the intention of voting either when the majority of their neighbors are willing to vote too, or when they perceive in their social neighborhood that elections are "close". We study the long-run average intention to vote, interpreted as the actual turnout observed in an election. Depending on the values of the average connectivity and the probability of behaving as a follower/calculator, the system exhibits monostability (zero turnout), bistability (zero and moderate/high turnout) or tristability (zero, moderate and high turnout). By obtaining realistic turnout rates for a wide range of values of both parameters, our model suggests a mechanism behind the observed relevance of social networks in recent elections.

Keywords: Turnout; random networks; voting; opinion formation; adaptive behavior.

*Corresponding author. Current position: Economics Department, Universidad Católica del Norte, Antofagasta, Chile.

1. Introduction

Two basic questions concerning the turnout in elections are: "Who votes?" and "Why do people vote?" Empirical research answers the first question by pointing out a list of individual characteristics that influence participation. The results suggest that non-participation is positively correlated with low education level, social or geographical isolation [29], being a newcomer or immigrant [21], having a low income [27], and being young [17, 22]. Moreover, those persons who voted in the previous election are more likely to vote in the next: voting is thus a habit [14, 18, 40]. The effect of external characteristics such as the electoral system or the closeness of the race between candidates has also been studied. It appears that voters are more likely to turn out under proportional electoral systems than under majority systems [15]. Closeness also matters [15, 42] although the evidence is patchy [28, 30].

Looking at the problem from a sociological viewpoint, the influence of social networks on voting behavior is well-known in political science [3, 25]. The recent empirical findings concerning what is called "behavioral contagion" [32] or "perceived pressure to vote" [4] can be summarized as follows. People whose neighbors and friends usually vote are more likely to participate [24, 32]. Interpersonal discussion influences political participation [20], although the effect of social interaction on participation is contingent on the amount and the quality of political discussion that occurs within the social network [31, 33]. Moderately informed voters tend to imitate their neighbors' voting behavior [22]. The contagion effect occurs among spouses [35], but weaker ties or even casual interactions may also determine political behavior patterns [19]. Publicizing participation increases the turnout [13]. According to some empirical studies, political disagreement within the network tends to dampen turnout [34, 16]. However, once the distinction between isolation within one's own opinion environment and the balance of exposure to two conflicting points of views is made, such political disagreement would foster participation [36].

Summing up in a stylized fashion, the empirical evidence that accounts for the social influence on turnout or voting behavior suggests that there might be two forces underlying the decision to cast a vote: on the one hand, contagion or imitation; on the other hand, the reaction to political discussions, where possibly people would tend to vote whenever they face a balanced opinion environment.

At the theoretical level, in a seminal work, Downs [7] uses rational choice to question the turnout in large elections: Why do so many people vote given that in marginal terms the cost of voting is larger than its benefits? Indeed the benefit of voting depends on the voter's probability of being decisive, which is extremely low in large electorates. Since then many explanations have been given, among them a sense of duty [41] and the objective of minimizing regret [9]. Game theoretical models have been proposed [37, 38, 26], as have group-based models of mobilization [45, 42]. (For a review of these models, see [8] or [15].) More recently, network theory has sought to explain turnout by contagion through social networks: groups of voters can convince their nearest neighbors to go and vote. It has been shown that if people imitate their neighbors' behavior, a small group of people with strong feelings about voting can bring about a massive turnout by a "domino" or "cascade" effect [1, 10]. Alternatively, when individuals base their decision to abstain or participate on what the most satisfied neighbors did in the previous period, the result is a large turnout [11].

At last, it is also worthwhile to mention the fruitful literature on dynamic opinion formation in the field of physics.[a] Most of the applications follow either the Sznajd model [44] or the majority-vote model [6]. In the Sznajd model, essentially groups of people sharing the same opinion (vote's preferences) can convince their nearest neighbors with some probability. As a result, unless some individuals are assumed to behave differently, full consensus is reached (see [43] and references therein). In the majority-vote model, agents follow the majoritarian behavior in their neighborhoods with some probability $1 - q$. This model has been studied in several types of networks and displays a critical transition at q_c (that is, if $q > q_c$ full consensus is reached). In particular, applied to Erdös–Rényi networks, such critical value increases with the average connectivity of the graph [39]. Finally, the so-called Galam model introduces the idea of group discussion in the context of physics-based models, by considering local majority rules in small groups of people that are reshuffled every timestep (see [12] for an account on the history and variants of this model).

In this paper, we bridge the gap between rational theory and network theory by combining adaptive calculus of voting and imitation within social networks. Based on the empirical evidence, we assume that individuals decide whether or not to vote on the basis of the influence of their social neighbors (e.g. family, friends, coworkers, etc.): while some individuals simply follow the observed majority behavior (imitation or contagion effect), others tend to turn out if they perceive that elections are "close" (local adaptive calculus of voting effect).

In more detail, we propose the following model. Two parties compete in an election. Each citizen has a given preference for one party or the other and that preference does not change during the relevant period. Instead, before the election takes place, the decision that evolves is the intention to participate. Citizens are embedded in a random social network and dynamically make their choice to vote or not depending on what they observe within their social neighborhoods. We study the long-run emerging average behavior which, as discussed in the next sections, can be interpreted as the actual turnout observed in the election.

We represent the social structure as a fixed random network. Nodes are citizens and links (ties) are social relationships. More specifically, we consider undirected Erdös–Rényi random networks. In this type of network, all ties have the same probability of being present and for a large number of nodes the connectivity distribution is approximately Poisson. This implies that the network can be fully characterized by the average connectivity (e.g. average number of links per

[a]For an exhaustive review, see [5, part III].

node or average degree). This feature is therefore one of the two parameters of our model. Its magnitude depends on who is really influential when one decides whether to participate or not. Do people discuss politics only with friends, with friends and family, with friends, family and co-workers, etc? Naturally, it may also depict different kinds of society, some of which are more densely connected than others.

Given the pattern of interactions, citizens form their intention to vote. Agents have limited information and are backward adaptive learners. They only have access to local information (that which they can gather in their immediate neighborhood) and use the past as a guide for making their current decision [11, 23]. That is, whenever a citizen updates his/her intention whether or not to turn out, say at time t, he/she takes into account his/her neighbors' intention to participate at $t-1$ as well as their given political preferences.[b] We consider two possible behaviors — imitation and adaptive calculus of voting — which may reflect differences in political awareness [2]. The probability of adopting one of these behaviors is the second parameter of our model. If the citizen acts as a "follower", he/she decides to vote if the majority of his/her neighbors are willing to vote too. If the citizen is a "calculator", he/she decides to vote if he/she may be "decisive" in his/her social neighborhood. In this case, if a large majority of his/her voting neighbors have preferences for his/her preferred party or his/her opposed party he/she will not vote. (Empirical evidence of not voting if one is isolated in an enemy neighborhood can be found in [16]). The "calculator" agent only votes if his/her neighborhood is divided (or, in other words, if there is opinion balance among social neighbors who are willing to vote); he/she does not vote if he/she feels that either party may win by a very large majority.[c]

We use two complementary approaches to find the long-run turnout equilibrium, i.e. the average turnout that remains stable through time. The results obtained by an analytical (mean-field) approximation are confirmed by Monte Carlo simulations. The interplay between the two key parameters, average connectivity and probability of being a follower results in a rich long-run behavior. The model often does not predict a unique stable equilibrium: the system may exhibit bistability with zero and high or moderate turnout and tristability with zero, moderate and high rates of turnout.

The rest of the paper is organized as follows. First, we present the framework and the assumptions (Sec. 2). Next, we analytically study the model and present some Monte Carlo simulations (Secs. 3 and 4, respectively). We highlight the main intuitions of the dynamics of the model and compare our long-run results with real

[b]It should be clear that the dynamic process of turnout formation occurs just before an election. Therefore, step t is an arbitrary time unit each time (i.e. it does not represent different election days).

[c]Note that followers' behavior resembles that of agents in a majority-vote model. The difference is that with the complementary probability our agents behave as adaptive calculators.

election data (Sec. 5). Finally, we sum up and discuss possible avenues for future research (Sec. 6).

2. The Model: Evolution of Turnout Intention

Two parties, A and B, compete in an election. Before the election takes place, citizens are involved in a dynamic process of formation of turnout intention. Let N denote the set of agents or citizens with the right to vote (indexed by $i = 1, \ldots, n$). Each agent i usually interacts with a small group, thus we model the pattern of interactions as a (exogenously given and fixed) random social network G à la Erdös–Rényi. In this network, agents are nodes and links (ij) represent social relationships or political discussions. Ties are undirected (thus if i is connected to j, so is j with respect to i) and any two agents have the same probability of being connected. We denote $N_i = \{j \neq i, j \in N : ij \in G\}$, the set of agents with whom i is connected or i's neighbors. The connectivity of i (number of neighbors) is denoted by $k_i = |N_i|$. As G belongs to the Erdös–Rényi family of networks, for large n, the connectivity is approximately Poisson-distributed and there are no correlations. In particular the clustering coefficient (i.e. the average probability of two neighbors of any agent i being connected themselves) is very small. All this implies that for a sufficiently large number of realizations and a large n, the network can be fairly described by its expected connectivity $\langle k \rangle \approx \sum_{i \in N} k_i/n$, the unique parameter of the Poisson distribution.[d]

Each agent has two basic characteristics: his/her preference and his/her turnout intention. Agent i's preference does not change during the relevant period and is denoted by $u_i \in \{A, B\}$. This means that if agent i votes, he/she chooses the candidate of party $A(B)$ whenever $u_i = A(B)$. We assume that preferences are uniformly distributed among the population and there are exactly $n/2$ agents of each type. What does change over time is the intention of agent i to vote or not. Let $v_{i,t} = 1\,(0)$ denote the intention of agent i to turnout (or not) at time t. We assume that initially a proportion of the population is willing to vote and its distribution (uniform) is independent of the network structure and the distribution of preferences. Agents are bounded rational and at each t consider the information they know from $t - 1$. More specifically, the information that they can gather is the characteristics of their neighbors; therefore at time t each agent i knows u_j and $v_{j,t-1}$ for every $j \in N_i$.

The turnout intention dynamics is as follows. At each time $t = 1, \ldots,$ one agent is random uniformly chosen to update his/her turnout intention (i.e. asynchronous updating). With probability p, the chosen agent behaves as a "follower" and is willing to vote if a majority of his/her neighbors are also willing to vote. With probability $1 - p$, the chosen agent behaves as a "calculator": he/she is willing

[d] Throughout this paper $\langle \cdots \rangle$ denotes expected value. Note that for n large, the mean connectivity $\sum_{i \in N} k_i/n$ converges in probability to the expected value or average connectivity $\langle k \rangle$.

to vote if he/she thinks that elections are more or less close. Close elections (i.e. balanced opinions among voting neighbors) are understood as a near-half division $50 \pm \beta\%$ between neighbors who are considering voting. To simplify our model, we fix such degree of "closeness" β to 10% — thus, "close" elections are a 40–60% division — and briefly discuss the implication of varying this parameter at the end of Sec. 3. Note that as citizens only have access to information within their neighborhood, this adaptive calculus of voting is done at the local level.

If agent i is chosen to update his/her turnout behavior at time t, he/she uses the information gathered at period $t-1$, namely, the number of voting neighbors, which we denote by $x_{i,t-1}$:

$$x_{i,t-1} = \sum_{j \in N_i} v_{j,t-1}, \tag{1}$$

and the number of voting neighbors with identical preferences, which we denote by $y_{i,t-1}$,

$$y_{i,t-1} = \sum_{\substack{j \in N_i: \\ u_j = u_i}} v_{j,t-1}\,. \tag{2}$$

Therefore, formally, if agent i behaves as a **follower**, his/her turnout intention at time t will be

$$v_{i,t-1} = \begin{cases} 1 & \text{if } x_{i,t-1} \geq 0.5k_i, \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

while if agent i behaves as a **calculator**,

$$v_{i,t-1} = \begin{cases} 1 & \text{if } 0.4x_{i,t-1} \leq y_{i,t-1} \leq 0.6x_{i,t-1}, \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

If agent i has no neighbors ($k_i = 0$) or no one in his/her neighborhood is willing to vote ($x_{i,t-1} = 0$), we assume that he/she simply copies his/her own past behavior, i.e. $v_{i,t} = v_{i,t-1}$.

Under the assumption that $v_{1,t}, v_{2,t}, \ldots$ are independent and identically distributed random variables, the average turnout intention at time t, $\sum_{i \in N} v_{i,t}/n$, for n large approximates the expected turnout intention at time t, i.e. $plim \sum_{i \in N} v_{i,t}/n = \langle v \rangle_t$ where *plim* denotes convergence in probability. The equilibrium turnout v is approximated by the long-run value of $\langle v \rangle_t$, that is $plim \langle v \rangle_t = v$, or in other words, when $\langle v \rangle_t$ remains stable over time, $v = \langle v \rangle_t = \langle v \rangle_{t-1}$ (see Sec. 3 for more specific details).[e]

[e]Note that assuming that $v_{1,t}, v_{2,t}, \ldots,$ are independent and identically distributed random variables allow us to invoke the weak law of large numbers. That is, for any $\delta > 0$: $plim \sum_{i \in N} v_{i,t} = \langle v \rangle_t \Leftrightarrow \lim_{n \to \infty} \Pr(|\sum_{i \in N} v_{i,t}/n - \langle v \rangle_t| > \delta) = 0$. An analogous argument lies behind $plim \langle v \rangle_t = v$, for random variables $\langle v \rangle_1, \langle v \rangle_2, \ldots$.

The long-run turnout intention obtained v represents, or can be interpreted as, the actual turnout that would be observed on the elections' day. We hence study the behavior of v as a function of the following parameters:

- $p \in [0,1]$, the probability of being a follower or the fraction of time for which any individual behaves as a follower.
- $\langle k \rangle = 5, 6, \ldots, 25$, the average connectivity of the network. We assume $\langle k \rangle \geq 5$ because for $\langle k \rangle < 5$, the fraction of isolated individuals is too great to allow the initial behavior to be updated. (This is similar to [10], where $\langle k \rangle$ is assumed to be between four and 20).

We adopt two different complementary approaches to solve the model. We approximate analytically the long-run average turnout via mean-field techniques. The approximation obtained is then confirmed and complemented by Monte Carlo simulations.

3. Mean-Field Approximation

Our aim in this section is to obtain an analytical approximation of the equilibrium turnout, i.e. the long-run state of the dynamics. As a first step, we approximate the evolution in time of the expected turnout intention $\langle v \rangle_t$. The idea is simple. We assume that the binary variables $v_{i,t}$ are independent and identically distributed Bernoulli random variables with a probability of success of $E[v_{i,t}] = \langle v \rangle_t$, therefore for n large the average turnout at time t converges in probability to the expected value $\langle v \rangle_t$, i.e. $plim \sum_{i \in N} v_{i,t}/n = \langle v \rangle_t$. In this context, $\langle v \rangle_t$ is the approximated probability that any agent i is willing to vote at time t.

The probability that an agent i is willing to vote at time t depends on his/her behavior at time t (either calculator or follower) and his/her local information, which in turn depends on what his/her neighbors did at time $t-1$. Recalling the mean-field basic hypothesis, we assume that any neighbor intended to vote at time $t-1$ with probability $\langle v \rangle_{t-1}$ and that i has $k_i \approx \langle k \rangle$ neighbors, where $\langle k \rangle$ is the average connectivity of the network.[f]

If i behaves as a follower, what matters is the fraction of neighbors who were willing to vote at time $t-1$, $(x_{i,t-1}/k_i)$. If i behaves as a calculator, he/she cares about the fraction of voting neighbors with the same preferences as himself/herself $(y_{i,t-1}/x_{i,t-1})$. Our assumptions imply that $x_{i,t-1} \approx x_{t-1}$ and $y_{i,t-1} \approx y_{t-1}$ for all i. They also allow us to interpret x_{t-1} as a random variable with Binomial distribution $(\langle v \rangle_{t-1}, \langle k \rangle)$; and y_{t-1}, as a random variable with Binomial distribution $(1/2, x_{t-1})$.

The probability that any agent i is willing to vote at time t is hence approximated by

$$\langle v \rangle_t \approx p \Pr(x_{t-1} \geq 0.5\langle k \rangle) + (1-p)\Pr(0.4x_{t-1} \leq y_{t-1} \leq 0.6x_{t-1}); \tag{5}$$

[f] Here we assume that the average connectivity is an integer number, while it may be a real number.

where the probability of voting *conditional* on behavior as a follower (in the first term) can be computed as

$$\begin{aligned}\Pr(x_{t-1} \geq 0.5\langle k\rangle) &= \sum_{l=\lceil 0.5\langle k\rangle\rceil}^{\langle k\rangle} \Pr(x_{t-1} = l)\\ &= \sum_{l=\lceil 0.5\langle k\rangle\rceil}^{\langle k\rangle} \binom{\langle k\rangle}{l} \langle v\rangle_{t-1}^{l} \,(1 - \langle v\rangle_{t-1})^{\langle k\rangle - l},\end{aligned} \tag{6}$$

where $\lceil z\rceil$ denotes the ceiling integer value of z, i.e. the smallest integer larger than z and the probability of turnout *conditional* on calculating behavior (second term of (5)) as

$$\begin{aligned}&\Pr(0.4x_{t-1} \leq y_{t-1} \leq 0.6x_{t-1})\\ &\quad= \sum_{l=1}^{\langle k\rangle} \Pr(x_{t-1} = l) \Pr(\lceil 0.4x_{t-1}\rceil \leq y_{t-1} \leq \lfloor 0.6x_{t-1}\rfloor | x_{t-1} = l)\\ &\quad= \sum_{l=1}^{\langle k\rangle} \Pr(x_{t-1} = l) \sum_{m=\lceil 0.4\,l\rceil}^{\lfloor 0.6\,l\rfloor} \Pr(y_{t-1} = m)\\ &\quad= \sum_{l=1}^{\langle k\rangle} \binom{\langle k\rangle}{l} \langle v\rangle_{t-1}^{l} \,(1 - \langle v\rangle_{t-1})^{\langle k\rangle - l} \sum_{m=\lceil 0.4\,l\rceil}^{\lfloor 0.6\,l\rfloor} \binom{l}{m} \frac{1}{2^l},\end{aligned} \tag{7}$$

where $\lfloor z\rfloor$ denotes the floor integer value of z, i.e. the largest integer smaller than z. Substituting the probabilities into (5) we obtain $\langle v\rangle_t$ as a function of $\langle v\rangle_{t-1}$. As discussed above, we are interested in the long-run emergent behavior, which is therefore approximated by the asymptotically stable solutions of (5).

3.1. *"Long-run" solutions: existence and stability*

In the long run, $\langle v\rangle_t = \langle v\rangle_{t-1} = v$, thus given $\langle k\rangle$ and p, the turnout intention v verifies:

$$\begin{aligned}v = p &\sum_{l=\lceil 0.5\langle k\rangle\rceil}^{\langle k\rangle} \binom{\langle k\rangle}{l} v^{l} \,(1 - v)^{\langle k\rangle - l}\\ &+ (1 - p) \sum_{l=1}^{\langle k\rangle} \binom{\langle k\rangle}{l} v^{l} \,(1 - v)^{\langle k\rangle - l} \sum_{m=\lceil 0.4\,l\rceil}^{\lfloor 0.6\,l\rfloor} \binom{l}{m} \frac{1}{2^l}.\end{aligned} \tag{8}$$

The right-hand side of (8) is a function of v, $f(v)$, thus by definition any fixed point v^* satisfies condition (8). Fixed points reflect long-run behavior as long as they are (locally) asymptotically stable. Therefore, we require v^* to meet the additional condition $|f'(v^*|\langle k\rangle, p)| < 1$ (that is, the absolute value of the slope of f evaluated at the fixed point should be smaller than 1).

First, we address two results that can be easily shown analytically.

Proposition 1. *$v^* = 0$ is always a local asymptotically stable solution for any choice of parameters.*

Proof. As the right-hand side of (8) as a function of v is a polynomial of degree $\langle k \rangle$ with an independent coefficient of zero, $f(v) - v = 0$ has always at least one solution, which is $v^* = 0$, whatever the values of $\langle k \rangle$ and p. To check its stability, we consider $f'(v)$, which after rearranging is given by

$$f'(v) = p \sum_{l=\lceil 0.5\langle k \rangle \rceil}^{\langle k \rangle} \binom{\langle k \rangle}{l} \big((1-v)^{\langle k \rangle - l - 1} v^{l-1} (l - v\langle k \rangle)\big)$$
$$+ (1-p) \sum_{l=1}^{\langle k \rangle} \binom{\langle k \rangle}{l} \big((1-v)^{\langle k \rangle - l - 1} v^{l-1} (l - v\langle k \rangle)\big) \sum_{m=\lceil 0.4\, l \rceil}^{\lfloor 0.6\, l \rfloor} \binom{l}{m} \frac{1}{2^l}.$$

In all the terms of $f'(v)$, v or a power of v appears, except in the term for $l = 1$ preceded by $(1-p)$. But as $l = 1$, the factor in that term $\sum_{m=\lceil 0.4\, l \rceil}^{\lfloor 0.6\, l \rfloor} \binom{l}{m} \frac{1}{2^l} = 0$ for, if i has only one voting neighbor, he/she never votes. Thus, $|f'(0|\langle k \rangle, p)| = 0 < 1$ and $v^* = 0$ is an asymptotically super-stable fixed point. □

The intuition is simple. If at some point in time no one has the intention to vote, followers follow this behavior and calculators are not able to update, so they keep their past behavior. The equilibrium turnout is then equal to zero.

One might wonder whether the other extreme, that is, a 100% turnout, may also be a stable equilibrium. The answer is yes, but only if all voters are followers. Formally:

Proposition 2. *If $v^* = 1$ is a local asymptotically stable solution, then $p = 1$.*

Proof. We first show that if $v^* = 1$ is a fixed point of (8), then $p = 1$. Suppose that $v^* = 1$ and $p < 1$. If $v \to 1$ in condition (8) all the terms except those for $l = \langle k \rangle$ are zero, thus (after rearranging) (8) becomes

$$(1-p)\left(1 - \sum_{m=\lceil 0.4\, \langle k \rangle \rceil}^{\lfloor 0.6\, \langle k \rangle \rfloor} \binom{\langle k \rangle}{m} \frac{1}{2^{\langle k \rangle}}\right) = 0.$$

Then, as the second factor is *strictly positive*, it must be the case that $p = 1$, otherwise $v^* = 1$ would not be a fixed point, a contradiction. Next, we check the stability of $v^* = 1$ *under the assumption of* $p = 1$. Consider $f'(v)$ for $p = 1$,

$$f'(v) = \sum_{l=\lceil 0.5\langle k \rangle \rceil}^{\langle k \rangle} \binom{\langle k \rangle}{l} \big((1-v)^{\langle k \rangle - l - 1} v^{l-1} (l - v\langle k \rangle)\big).$$

In all the terms of $f'(v)$, $(1-v)$ or a power of $(1-v)$ appears, except those for $l = \langle k \rangle$ and $l = \langle k \rangle - 1$. These two terms (symplified) are equal to

$\left(v^{\langle k\rangle-2} - v^{\langle k\rangle-1}\right)\langle k\rangle\left(\langle k\rangle - 1\right)$ and for $v = 1$ also vanish. Thus, $|f'(1|\langle k\rangle, 1)| = 0 < 1$ and $v^* = 1$ is an asymptotically super-stable fixed point. □

These two propositions together imply that for $p = 1$ there is bistability, that is, we have two different (although extreme and unrealistic) equilibrium turnouts. This means that depending on the initial conditions either or both of these equilibria may emerge.[g]

For other values of p, the solutions of $f(v) - v = 0$ have to be found numerically, as the degree of the polynomial ($\langle k\rangle \geq 5$) is too high for analytical solutions to be obtained. We fix $\langle k\rangle$ and show the typical bifurcation diagrams taking p as the bifurcation parameter. The function $v^*(p)$ describes branches of fixed points and the bifurcation diagram presents all those branches in the (p, v^*) space. When for a value of p, say p_0, several branches come together, the point (p_0, v^*) is said to be a bifurcation point and p_0, its bifurcation value. As Fig. 1 depicts, in the diagrams of our model there can be one or two bifurcation points, at which saddle-node or fold bifurcations emerge. In a saddle-node or fold bifurcation, two branches of fixed points emerge. One of them is stable (points are attracting), the other unstable (points are repelling).

An important conclusion that can be derived is that, depending on the combination of p and $\langle k\rangle$, the system may exhibit monostability, bistability or tristability. That is, for some pairs $(p, \langle k\rangle)$, the model predicts multiple equilibria. This does not mean, however, that all the equilibria are equally likely to emerge: the equilibrium observed depends on the initial conditions, i.e. the initial fraction of population willing to vote (v_0). The arrows in Fig. 1 describe the basins of attraction of each asymptotically stable equilibrium.[h]

Consider for example the diagram of Fig. 1(c), for p between p_1 and p_2, where the model predicts tristability (zero, moderate and high turnout). If the initial condition lies strictly above the upper-dashed line, we will observe only the high turnout equilibrium. If the initial condition lies approximately on the upper-dashed line, both the high and moderate long-run turnouts are likely to be observed (in this situation we can say that there exists "true" multistability). Similarly, if v_0 lies strictly below the upper-dashed and strictly above the lower-dashed line, only the moderate turnout will emerge; if it lies on the bottom-dashed line, again two equilibria are possible (moderate and zero turnout); and, finally, if it lies strictly below the lower-dashed line we will observe only the zero turnout equilibrium.

The stability zones in the $(p, \langle k\rangle)$ space are shown in Fig. 2. The critical probabilities are the aforementioned bifurcation values. We observe that

- If the connectivity is low ($5 \leq \langle k\rangle \leq 9$), there is a critical probability p_1 such that for $p < p_1$ there is a unique zero turnout equilibrium; while for $p > p_1$ the

[g]It also implies that there must be at least one more fixed point between 0 and 1 which is unstable, i.e. a repelling fixed point. See Fig. 1.
[h]See Sec. 4 for a numerical example.

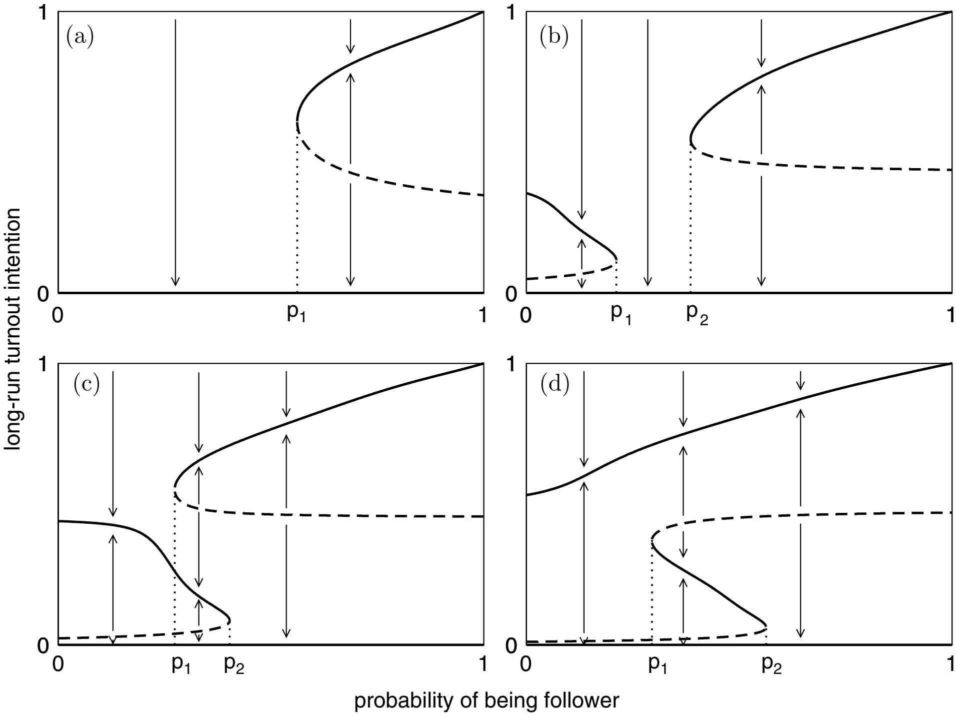

Fig. 1. Bifurcation Diagrams, $v^*(p)$. Solid lines depict stable fixed points; dashed, unstable fixed points. p_1, p_2 are bifurcation values. Panels: (a), $\langle k \rangle = 6$; (b), $\langle k \rangle = 12$; (c), $\langle k \rangle = 16$; (d), $\langle k \rangle = 22$.

system exhibits bistability, with either zero or high turnout. An example is given in Fig. 1(a) ($\langle k \rangle = 6$).

- If the connectivity is intermediate ($10 \leq \langle k \rangle \leq 14$), there are two critical values, p_1 and p_2, ($0 < p_1 < p_2 < 1$). For $p < p_1$ the system exhibits bistability, with either zero or moderate turnout. For $p_1 < p < p_2$, there is a unique zero turnout equilibrium, and for $p > p_2$ the system exhibits bistability, either zero or high turnout. The typical diagram is shown in Fig. 1(b) ($\langle k \rangle = 12$).
- Finally, if the connectivity is high ($15 \leq \langle k \rangle \leq 25$), there are two critical values, p_1 and p_2, ($0 < p_1 < p_2 < 1$). For $p < p_1$, we have either zero or moderate turnout (bistability). For $p_1 < p < p_2$, turnout can be zero, moderate or high (tristability), and for $p > p_2$ there can be either zero or high turnout (bistability). In this case, there are two kinds of bifurcation diagram (cf. Fig. 1(c) for $\langle k \rangle = 16$ and Fig. 1(d) for $\langle k \rangle = 22$).

Remark. Although there is no difference between the two bottom panels of Figs. 1(c) and 1(d) in terms of stability zones, an important difference emerges if we study the effect of gradual variations of p. Starting from one extreme of the range of p (either 0 or 1), we let the system stabilize at the corresponding fixed point

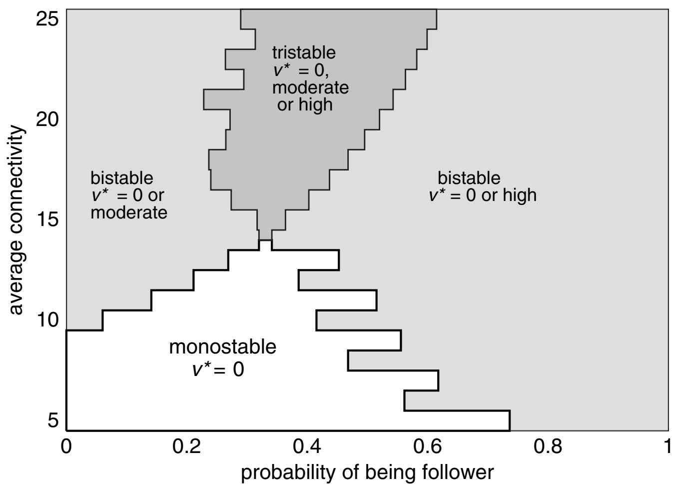

Fig. 2. Stability zones. Interplay between average connectivity $\langle k \rangle$ and follower's probability p. The non-monotonic shape of the borders is due to the fact that $\langle k \rangle$ is assumed integer and the subsequent parity effect on the location of the bifurcation points.

and gradually vary the value of p (either up or down, respectively) until another steady state is reached, then change p again and so forth. Thus for example, when $\langle k \rangle$ is very high (between 18 and 25, as in Fig. 1(d)), if we start from $p = 1$ and vary p downwards, we move along the upper stable branch (i.e. without "jumps"). However when $\langle k \rangle$ is medium-high (between 15 and 17, as in Fig. 1(c)), we observe a discontinuity or discrete jump at p_1.

We conclude by discussing what would happen if we varied some parameters that we have kept constant so far, namely the size of the majority and how "closeness" is defined. Imagine that a follower is willing to vote if $x_{i,t-1} \geq r\, k_i$ where $r \in [1/2, 1]$; while a calculator is willing to vote if $(1/2 - \beta)x_{i,t-1} \leq y_{i,t-1} \leq (1/2 + \beta)x_{i,t-1}$, with $\beta \in [0, 1/2]$. Roughly, increasing (decreasing) r (β) reduces the average turnout and, depending on $\langle k \rangle$, changes the bifurcation diagram. In particular, when $\langle k \rangle$ is large, instead of a diagram like Fig. 1(d), we would have one similar to Figs. 1(c) or 1(b) because when either r is close to 1 or β is close to 0, we are introducing a bias towards non-voting behavior, the effect of which is similar to reducing connectivity $\langle k \rangle$. The opposite occurs when r (β) is reduced (increased), i.e. r and β are close to 1/2.

4. Simulations

In this section, we run simulations to confirm our theoretical analysis and insights about the model. The aim is to check whether the long-run solution of the mean-field approximation describes, at least qualitatively, the long-run state of the model.[i]

[i]Software used: Python 2.5.2 (Copyright 2001–2008 Python Software Foundation. All rights reserved). The codes are available at https://sites.google.com/a/ucn.cl/constanza-fosco/research.

In one realization (or run), we generate an Erdös–Rényi network of 5×10^3 nodes and given average connectivity $\langle k \rangle$. Starting with an initial condition v_0, the behavior evolves for $T = 5 \times 10^5$ timesteps (one individual's update each timestep).[j] The turnout of each realization is the average over the last 2×10^3 timesteps. This is repeated 100 times (with the same values of p and $\langle k \rangle$), and the fixed points v^* are the average over all runs. Whenever the dynamics ended in different fixed points, we calculated the different averages. For technical reasons, we have introduced the possibility of an agent with a low probability (ε) randomly choosing whether to participate or not. This is done to prevent the possibility of being stuck in the extreme turnout of 0%, although this is very unlikely given that v_0 is not too small. (In other words, we introduce ε to restore ergodicity.)

The observed long-run solutions depend on the initial condition. For an initial condition of 50% of voting citizens (i.e. $v_0 = 0.5$), the typical behavior of the simulated diagrams, from low to high average connectivity, is exemplified in Fig. 3. Panels (a), (b), (c) and (d) depict the typical behavior for low, medium-low, medium-high, and high average connectivity, respectively. Then, we modify the initial condition in order to see whether other equilibria (stable fixed points) can be observed. (See Fig. 4 for some examples.)

For the extreme case when there are only followers ($p = 1$), the initial condition completely determines the equilibrium turnout: if the initial turnout is smaller than 50%, by contagion we end up with a null turnout. By contrast, the initial condition has almost no effect in the case of purely calculating behavior ($p = 0$). The reason is simple. On the one hand, when the average connectivity is small ($5 \leq \langle k \rangle \leq 9$), the only possible equilibrium for $p = 0$ is zero turnout for all initial conditions. On the other hand, even if there are two possible equilibria (zero and moderate) for $\langle k \rangle \geq 10$, the initial condition would have to be extremely low for the zero turnout equilibrium to emerge. As explained in the previous section in the analysis of Fig. 1, for the zero turnout equilibrium to emerge, initial conditions would have to lie below the lower dashed line (unstable or repelling fixed points). These unstable branches are, in turn, very close to zero.

Two aspects of the simulations can be stressed:

- First, the mean-field approximation describes the actual long-run turnout intention quite accurately. Almost all the simulated turnout points (symbols)

[j] The maximum number of timesteps is quite safe, as can be observed in Fig. 6. We previously ran several realizations starting from $T = 10^5$ and observed whether the dynamics tended to stabilize around any particular value. Then, we have increased T until the stabilization was evident. (Depending on $\langle k \rangle$, this occurred around $T = 3 \times 10^5$.)

We also ran several simulations assuming a synchronous mechanism, i.e. all agents updating simultaneously at each timestep (code available at https://sites.google.com/a/ucn.cl/constanza-fosco/research). We did not find significant differences. The intuition is simple. There would be more important differences if, under the asynchronous mechanism, agents who are network neighbors were chosen to update in subsequent periods t and $t + 1$. Since the mechanism consists of time independent draws and n is large, this event is very unlikely.

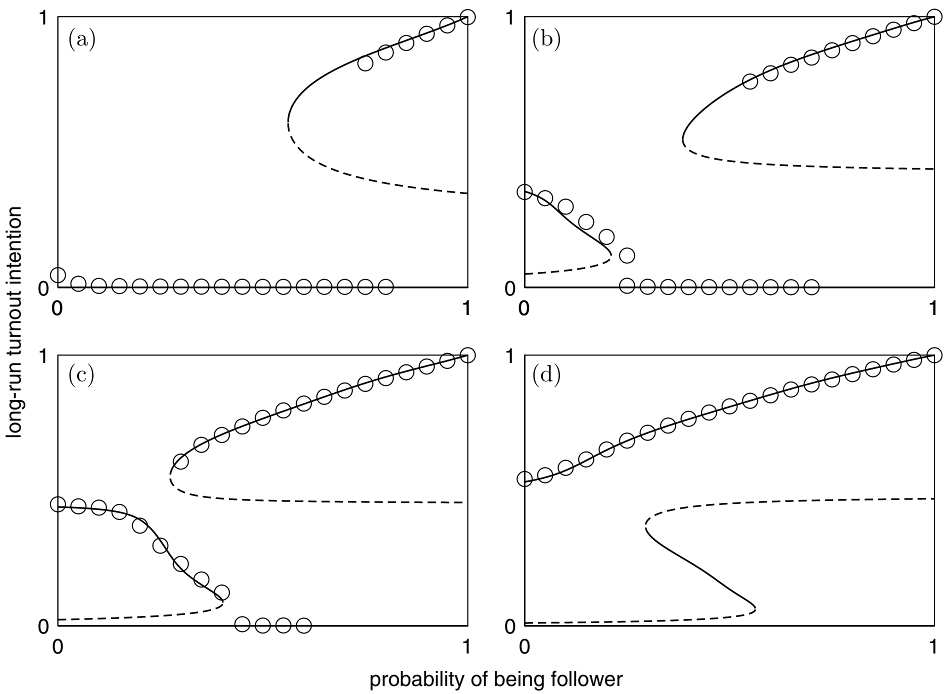

Fig. 3. Turnout intention v against the probability of being follower p. Solid lines are theoretical predictions; circles, MC simulations ($v_0 = 0.5$, $n = 5 \times 10^3$, $\varepsilon = 0.001$, $T = 5 \times 10^5$). Each circle is the average of 100 runs; each run is the average of the last 2×10^3 timesteps. Panels: (a) $\langle k \rangle = 6$; (b) $\langle k \rangle = 12$; (c) $\langle k \rangle = 16$; (d) $\langle k \rangle = 22$.

correspond to stable fixed points of condition (8) (solid lines). When there are multiple equilibria, the initial condition v_0 matters. Therefore, in general, to reach low equilibria, an initial condition of less than 50% is needed.

- Second, with the initial condition of 50%, for some combinations of parameters the system exhibits what we call "true" multistability. This is observed for medium-low connectivity and a relatively large p, and for medium-high connectivity and intermediate values of p (cf. Figs. 3(b) and 3(c), respectively).

Given that multiple equilibria are possible, what is the probability that a particular equilibrium emerges? We approach to this question in two different ways.

First, we consider the fixed initial condition $v_0 = 0.5$. Since repelling fixed points for medium or high p lie around 0.5, the small noise ε perturbs the dynamics so that some realizations reach the high turnout equilibrium (above the repelling point), and other realizations reach the low turnout equilibrium (below the repelling point).[k] Simulations suggest that the dynamics yields the high turnout equilibrium

[k]It should be clear, though, that given that ε also restores the ergodicity of the system, once a particular realization reaches an asymptotically stable equilibrium, it *could* eventually reach another asymptotically stable equilibrium, but *only* if $t \to \infty$.

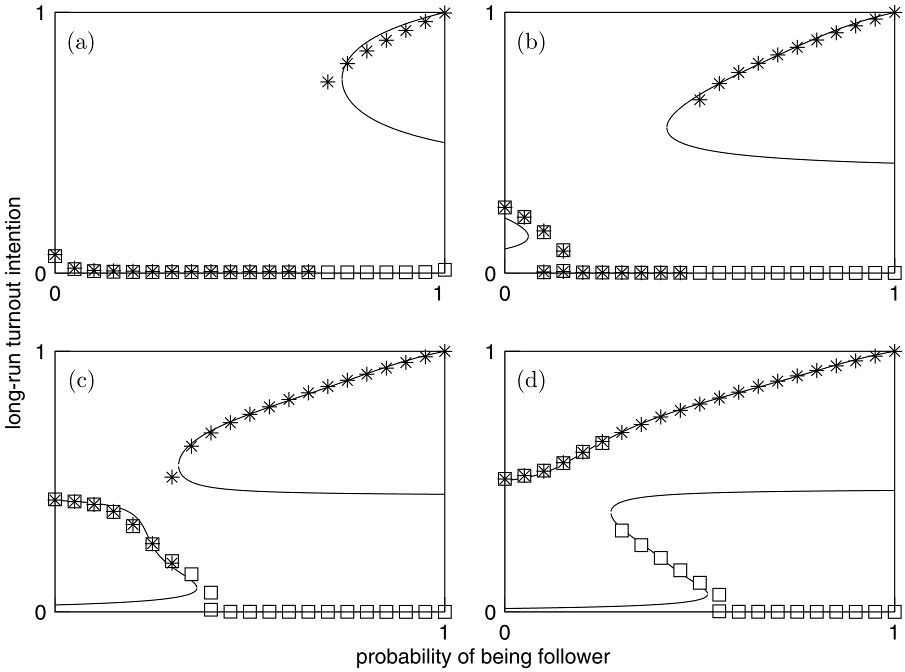

Fig. 4. Turnout intention v against the probability of being follower p. Solid lines are theoretical predictions; squares ($v_0 = 0.3$) and stars ($v_0 = 0.7$) are MC simulations ($n = 5 \times 10^3$, $\varepsilon = 0.001$, $T = 5 \times 10^5$). Each symbol is the average of 100 runs; each run, of the last 2×10^3 timesteps. Panels: (a) $\langle k \rangle = 5$; (b) $\langle k \rangle = 10$; (c) $\langle k \rangle = 15$; (d) $\langle k \rangle = 20$.

with a probability increasing in p. (See in Table 1 the relative frequencies for $\langle k \rangle = 16$.)

Second, we assume that *any* initial condition v_0 is equally probable, i.e. v_0 is uniformly distributed between 0 and 1. Then, for each pair $(p, \langle k \rangle)$, the probability that some equilibrium emerges can be theoretically computed as the probability

Table 1. Relative frequency of different equilibria for $\langle k \rangle = 16$, given an initial condition of $v_0 = 0.5$. The possible equilibria are denoted by v_1^*, v_2^*, and v_3^*, with $v_1^* = 0 < v_2^* < v_3^*$. Cells contain the relative frequency (over 200 realizations) of each equilibrium, conditional on the probability of being follower (p). Theoretically, for $0.27012 < p < 0.40287$ there are three equilibria; otherwise, there are two.

p	<0.27	0.27	0.28	0.3	0.32	0.34	0.36	0.38	0.4	0.42
v_1^*	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
v_2^*	1.0	0.99	0.97	0.89	0.78	0.72	0.64	0.52	0.46	0.30
v_3^*		0.01	0.03	0.11	0.22	0.28	0.36	0.48	0.54	0.70
p	0.44	0.46	0.48	0.5	0.52	0.54	0.56	0.58	0.6	>0.6
v_1^*	0.23	0.17	0.13	0.08	0.06	0.05	0.03	0.02	0.01	0.0
v_2^*	0.77	0.83	0.87	0.92	0.94	0.95	0.97	0.98	0.99	1.0

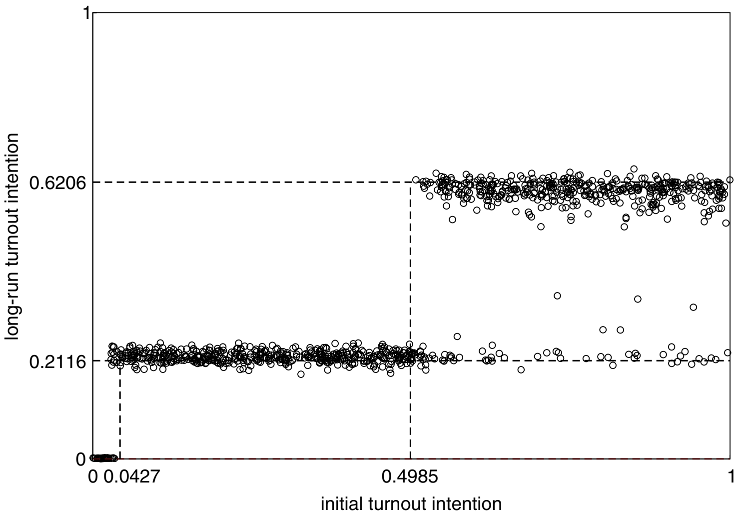

Fig. 5. Multiple equilibria and initial conditions v_0. These are 1000 MC realizations (runs) for $\langle k\rangle = 16$ and $p = 0.3$, with random uniformly distributed initial conditions ($n = 5\times10^3$, $\varepsilon = 0.001$, $T = 5\times10^5$). Each circle is the average of the last 2×10^3 timesteps. The three theoretically possible equilibria are $v^* = 0$, $v^* = 0.2116$, and $v^* = 0.6206$. The repelling fixed points (0.0427 and 0.4985) delimit approximately the correspondent basins of attraction.

that v_0 belongs to its basin of attraction. For example, consider the case $\langle k\rangle = 16$ (Fig. 1(c)), and assume that the probability of being follower is $p = 0.3$ (between p_1 and p_2). For $p = 0.3$, the fixed points $\{0, 0.2116, 0.6206\}$ are asymptotically stable (attracting), while the fixed points $\{0.0427, 0.4985\}$ are unstable (repelling). Then the probability of observing $v^* = 0$ can be approximated by $0.0427 - 0 = 0.0427$, the probability of observing $v^* = 0.2116$, by $0.4985 - 0.0427 = 0.4558$, and the probability of observing $v^* = 0.6206$, by $1 - 0.4985 = 0.5015$. In Fig. 5, we show 1000 realizations for random uniformly chosen initial conditions that approximately confirm these calculations.

5. Discussion

In this section, we outline the intuition of the dynamics underlying our main results and compare the theoretical levels of long-run turnout intention with real election data.

Let us start with the extreme case where only follower or imitation behavior prevails in the population ($p = 1$). Any agent is willing to vote if a majority of his/her neighbors intends to turnout. This does not depend much on the connectivity but rather on the average turnout in the previous period. For an initial turnout slightly larger than 50%, agents are likely to start out as willing to vote, and so on in the following periods, which should increase the turnout. Contagion thus spreads participation through the whole population and a very high turnout can be expected in the long run. Indeed we show that in this case the equilibrium turnout is 100%. Similarly if the initial turnout is smaller than 50%, non-participation should spread through the population and the equilibrium turnout should be 0%. More generally, the qualitative effect of follower behavior on turnout is to reinforce the prevailing conditions.

Now let us focus on the other extreme case, i.e. when there are only calculators ($p = 0$). The connectivity matters for the calculus of voting. The probability of close division at local level depends on the number of voting neighbors and also on the preferences of those neighbors. If we assume that supporters of A and B are uniformly distributed, the more voting neighbors there are, the larger the probability of close division is (though the parity effect plays a role). Qualitatively, if average connectivity increases, the number of voting neighbors should also increase, and thus the equilibrium of turnout should be higher. Note, however, that an equilibrium turnout equal to 100% cannot arise in the presence of pure calculating behavior. The intuition is simple. Assume that initially all agents are willing to vote. As preferences are uniformly distributed within the population an agent is more likely to find that elections are "close" in his/her neighborhood. However, this is *not certain* for *all* agents, and some will change their intention towards non-participation. By contrast a null turnout can be an equilibrium: if no one is willing to vote, no one can make his/her calculus of voting and thus all agents maintain their prevalent behavior. In sum, the effect of calculator behavior is less obvious than the effect of follower behavior, but there seems to be a positive relation between connectivity and turnout.

Now let us consider the interplay between the two types of behavior, depending on the initial turnout and the connectivity. We observe two kinds of self-reinforcing dynamics, one yielding low turnout and the other high turnout:

- **Low Turnout, the "Vicious Cycle":** Consider a situation with an initial turnout below 50% and/or low connectivity so that calculators are likely not to vote. As a consequence followers will not vote either, which decreases the number of voting neighbors. If the calculators face smaller subsets of voting neighbors they tend to vote less, followers continue to reinforce this behavior, and so on. In the long run, followers are likely not to vote at all, while the turnout of calculators may be moderate or null. The process thus stabilizes around zero turnout or, under some conditions, at a positive but moderate turnout (below 50%).
- **High Turnout, the "Virtuous Cycle":** Next, consider a situation with an initial turnout above 50% and/or high connectivity so that calculators are likely to vote, and push the average voting above 50%. In this case, followers are likely to vote, calculators have many voting neighbors, which increases the probability of a close division, and thus vote, followers reinforce this outcome, etc. The long-run outcome will be a high turnout.

Of course, both cycles are mediated by the probability of an agent being a follower, and a cycle may start because followers reinforce an increasing or decreasing

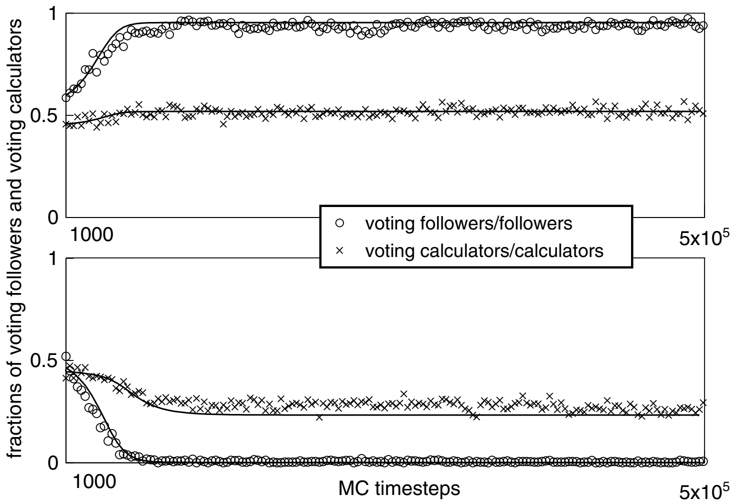

Fig. 6. Evolution of turnout intention (v) conditional on behavior. These are two realizations for $n = 5 \times 10^3$; $\langle k \rangle = 16$, $p = 0.35$, initial condition $v_0 = 0.5$ and $T = 5 \times 10^5$. One realization (upper panel) converges to a high turnout equilibrium, the other (bottom panel) to a low turnout equilibrium. Each 1000 steps, we calculate the fraction of times a calculator (follower) agent votes (symbols). Full lines are theoretical predictions cf. Eqs. (6) and (7) in Sec. 3, respectively.

turnout or because calculators increase or decrease overall participation. Moreover both cycles can be observed on networks with identical numbers of agents, parameters, and initial turnout. In Fig. 6, we show an example of the evolution over time of the probability of voting conditional on each type of behavior. These are two realizations for a network of $n = 5 \times 10^3$ agents and average connectivity $\langle k \rangle = 16$; a probability of being a follower of $p = 0.35$, and an initial turnout of 50%. Every 1000 steps we plot the fraction of times that a calculator votes and a follower votes. As the average connectivity is medium-high, calculators tend to vote, but the key is whether or not they are able to maintain an average turnout above 50%. If so, a virtuous cycle starts and if not, a vicious cycle starts. For the realization that leads to the low turnout equilibrium, the calculator turnout falls below 50%, the followers have no incentive to vote, and thus the vicious cycle operates. In the high turnout equilibrium, initially driven by calculators, the followers tend to vote *en masse*, and the virtuous cycle starts. As we shown in the previous section, this is not an exception. There are many combinations of parameters that lead to non-uniqueness of the turnout equilibrium.

Finally, let us focus on the levels of the equilibrium turnout and compare them with the turnout that we observe in real elections. The international Institute for Democracy and Electoral Assistance (International IDEA) has a voter turnout website on which statistics are available on political participation.[1] From these data, it

[1]See http://www.idea.int/vt/index.cfm.

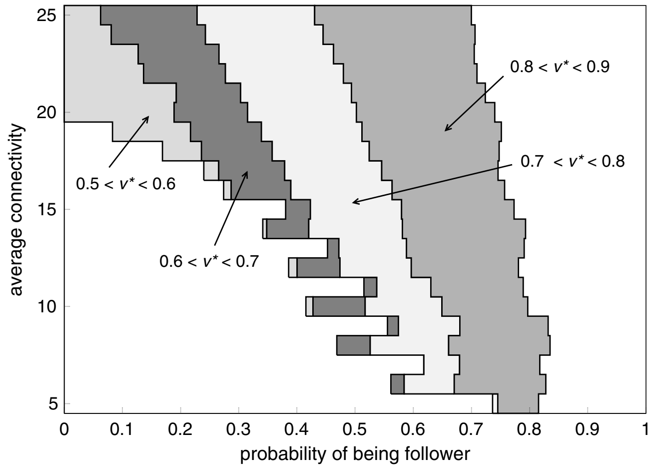

Fig. 7. Regions where the predicted turnout rates are realistic. The four regions depict the combinations of average connectivity $\langle k \rangle$ and probability of being follower p that yield realistic long-run turnout rates. The non-monotonic shape of the borders is due to the fact that $\langle k \rangle$ is assumed integer and the subsequent parity effect.

can be said than more than half of the countries listed have turnouts of between 60% and 80%, and four-fifths have turnouts of between 50% and 90%. The countries with turnouts of more than 90% are usually countries where voting is compulsory (such as Australia and Belgium).

The question is whether our model can lead to equilibria compatible with real data, that is, with turnout rates between 50% and 90%. The answer is yes, but not for all values of our parameters. In Fig. 7, we plot the regions of pairs $(p, \langle k \rangle)$ with which we obtain realistic turnout rates. These rates are one of the two or three equilibria that the model predicts, specifically the highest one. Thus, it should be clear that these realistic turnout rates could arise if the dynamic process of turnout formation stabilizes around the highest equilibrium, given the adequate initial condition.

- A turnout rate of between 80% and 90% can be obtained for all connectivity levels $\langle k \rangle$. Roughly speaking, the more densely connected the network is, the lower the probability of being a follower p needs to be, but at the same time, the range of possible values of p increases. For instance, if $\langle k \rangle = 5$, p should be between 0.75 and 0.82; while if $\langle k \rangle = 25$, it should be between 0.43 and 0.7.
- Something similar occurs with a turnout of between 70% and 80%. The difference is that all the possible values of p are smaller than in the previous case.
- A turnout of between 60% and 70% can be obtained for low levels of probability of being a follower and connectivity levels greater than five.
- A turnout of between 50% and 60% can be obtained for very low levels of probability of being a follower and connectivity levels greater than 9. Note that due to the parity effect, for $\langle k \rangle = 11, 13, 15$, the theoretical turnout is always greater

than 60%, and that for $\langle k \rangle = 10, 12, 14, 16, 17$, the turnout in this region is greater than 55%, meaning that only for $\langle k \rangle \geq 18$, would turnouts of close to 50% be observed.

For realistic turnout rates to be obtained, the probability cannot take extreme values: for $p = 1$, we have either 0% or 100% participation, while for $p = 0$, the turnout is below 60%, even for high connectivity. Realistic levels of turnout are obtained either for a large proportion of follower behavior and not very densely connected networks, or for more connected networks and a larger fraction of adaptive calculating behavior.

6. Conclusion

We present a simple model to explain turnout rates under the assumption that before any election takes place, i.e. during the campaign, citizens dynamically form their intention to vote as a consequence of their social interactions. The pattern of interactions is fixed and modeled as an Erdös–Rényi random network and hence it can be characterized by its average connectivity. Individuals may simply follow the majority behavior (and vote or not) or they may behave as local adaptive calculators. In the latter case, they tend to vote if they perceive that the election is "close" in their neighborhood. These two behaviors intend to capture what empirically has been found to be the main determinants of voting behavior with respect to the influence of the social neighborhoods: contagion or imitation and the effect of balanced/unbalanced opinion environments in political discussions.

We study the long-run average turnout intention, which in this model represents the turnout observed in an election. When all agents behave as pure followers, long-run turnout rates are very unrealistic (either all vote or no-one does) and connectivity plays no role. Indeed, this unrealistic result holds for dynamics driven by some contagion or imitation effect whenever the possibility of noisily (or reversed) decisions is small enough. The introduction of calculating behavior then has two interesting effects. On the one hand, the resulting turnout rates are in general more realistic. On the other hand those outcomes depend on the average connectivity of the network. Depending on the combination of values of the two key parameters (average connectivity and the probability of being a follower/calculator), the system exhibits monostability (zero turnout), bistability (zero turnout and either moderate or high turnout) or tristability (zero, moderate and high turnout). When there is more than one possible equilibrium, different initial conditions converge to different stable stationary states. In some cases, the same initial condition yields different equilibria, so turnout eventually becomes unpredictable.

For a wide range of the parameters values, this model predicts realistic turnout rates, i.e. comparable to the average turnout observed in the real-world elections. This yields interesting normative questions about the possibility of improving the turnout rates by somehow impinging on the key parameters of the model. The type of behavior (mediated by p) seems difficult to assess and therefore to "control".

Instead, given our interpretation, increasing the average connectivity would mean increasing the average size of the individual's group of influence. This would be achieved, for example, through the use of communication channels which take explicitly advantage of the structure of social networks (internet social networks or mobile phone). This is consistent with the observation in recent elections that the use of social networks and other Web 2.0 resources is a key factor in increasing turnout (at least for the candidates that use these media in their campaign).

In our model, citizens have fixed preferences for two parties and their decision is whether to vote or not. There is no correlation among neighbors' preferences. This is not completely consistent with voting literature, where it is found that citizens tend to segregate in groups of identical preferences: citizens with identical political preference are more likely to be connected. In our setup, this segregation would only decrease turnout. To see why, consider an agent with calculating behavior who shares the same preference with all his/her neighbors. Then he/she will never vote. This would induce a trend of non-participation that would spread through the whole population by contagion. The fact that segregation depresses turnout was found previously in [11]. Further research could include the co-evolution of preferences and turnout intention, perhaps assuming that a fraction of the population does not have clear preferences for one particular party or may change its preference. This would reflect what was found in [46]: half of the electorate switch their decision at least one over three ballots. Differences between supporters of the two parties could also be introduced, that is, the initial fraction of supporters may differ for one party or the other.

Acknowledgments

The authors are grateful to an anonymous referee for helpful comments and suggestions. Fosco and Sánchez acknowledge financial support from MICINN, Spain (grant MOSAICO) and Comunidad de Madrid, Spain (grant SIMUMAT-CM). Fosco and Laruelle acknowledge financial support from the Spanish Ministerio de Ciencia y Tecnología under project ECO2009-11213, co-funded by ERDF. Laruelle acknowledges financial support from the Spanish Ministerio de Ciencia y Tecnología under project SEJ2006-05455, co-funded by ERDF.

References

[1] Amaro de Matos, J. and Barros, P. P., Social norms and the paradox of elections' turnout, *Public Choice* **121** (2004) 239–255.

[2] Beck, P. A., Dalton, R. J., Greene, S. and Huckfeldt, R., The social calculus of voting: Interpersonal, media, and organizational influences on presidential choices, *American Political Science Review* **96** (2002) 57–73.

[3] Berelson, B., Lazarsfeld, P. F. and McPhee, W. N., *Voting* (University of Chicago Press, Chicago, 1954).

[4] Blais, A. and Young, R., Why do people vote? An experiment in rationality, *Public Choice* **99** (1999) 39–55.

[5] Castellano, C., Fortunato, S. and Loreto, V., Statistical physics of social dynamics, *Review of Modern Physics* **81** (2009) 591–646.

[6] de Oliveira, M. J., Isotropic majority-vote model on a square lattice, *Journal of Statistical Physics* **66** (1992) 273–281.

[7] Downs, A., *An Economic Theory of Democracy* (Harper and Row, New York, 1957).

[8] Feddersen, T. J., Rational choice theory and the paradox of not voting, *Journal of Economic Perspectives* **18** (2004) 99–112.

[9] Ferejohn, J. and Fiona, M., The paradox of not voting: A decision theoretic analysis, *American Political Science Review* **68** (1974) 525–536.

[10] Fowler, J. H., Turnout in a small world, in *The Social Logic of Politics: Personal Networks as Contexts for Political Behavior*, ed. Zuckerman, A. (Temple University Press, Philadelphia, 2005) 269–287.

[11] Fowler, J. H. and Smirnov, O., Dynamic parties and social turnout: An agent-based model, *American Journal of Sociology* **110** (2005) 1070–1094.

[12] Galam, S., Sociophysics: A review of Galam models, *International Journal of Modern Physics C* **19** (2008) 409–440.

[13] Gerber, A. S., Green, D. P. and Larimer, C. W., Social pressure and voter turnout: Evidence from a large-scale field experiment, *American Political Science Review* **102** (2008) 33–48.

[14] Gerber, A. S., Green, D. P. and Shachar, R., Voting may be habit-forming: Evidence from a randomized field experiment, *American Journal of Political Science* **47** (2003) 540–550.

[15] Geys, B., 'Rational' theories of voter turnout: A review, *Political Studies Review* **4** (2006) 16–32.

[16] Gimpel, J. G., Dyck, J. J. and Shaw, D. R., Registrants, voters and turnout variability across neighborhoods, *Political Behavior* **26** (2004) 343–375.

[17] Gimpel, J. G., Morris, I. L. and Armstrong, D. R., Turnout and the local age distribution: Examining political participation across space and time, *Political Geography* **23** (2004) 71–95.

[18] Green, D. and Shachar, R., Habit formation and political behaviour: Evidence of consuetude in voter turnout, *British Journal of Political Science* **30** (2000) 561–573.

[19] Huckfeldt, R., Beck, P. A., Dalton, R. J. and Levine, J., Political environments, cohesive social groups, and the communication of public opinion, *American Journal of Political Science* **39** (1995) 1025–1054.

[20] Huckfeldt, R. and Sprague, J., *Citizens, Politics, and Social Communication: Information and Influence in an Election Campaign* (Cambridge University Press, Cambridge, 1995).

[21] Jackson, R. A., Differential influences on latino electoral participation, *Political Behavior* **25** (2003) 339–366.

[22] Johnson, M., Revisiting the ties that bind: A rational-psychological perspective on social connectedness (2002), revised to 2002.

[23] Kanazawa, S., A possible solution to the paradox of voter turnout, *The Journal of Politics* **60** (1998) 974–995.

[24] Kenny, C. B., Political participation and effects from the social environment, *American Journal of Political Science* **36** (1992) 259–267.

[25] Lazarsfeld, P. F., Berelson, B. and Gaudet, H., *The People's Choice* (Columbia University Press, New York, 1948).

[26] Ledyard, J., The pure theory of two candidates elections, *Public Choice* **44** (1984) 7–41.

[27] Lijphart, A., Unequal participation: Democracy's unresolved dilemma, *American Political Science Review* **91** (1997) 1–14.

[28] Matsusaka, J. G., Election closeness and voter turnout: Evidence from California ballot propositions, *Public Choice* **76** (1993) 313–334.

[29] Matsusaka, J. G., Explaining voter turnout patterns: An information theory, *Public Choice* **84** (1995) 91–117.

[30] Matsusaka, J. G. and Palda, F., The Downsian voter meets the ecological fallacy, *Public Choice* **77** (1993) 855–878.

[31] McClurg, S. D., Social networks and political participation: The role of social interaction in explaining political participation, *Political Research Quarterly* **56** (2003) 449–464.

[32] McClurg, S. D., Indirect mobilization: The social consequences of party contacts, *American Politics Research* **32** (2004) 406–443.

[33] McClurg, S. D., The electoral relevance of political talk: Examining disagreement and expertise effects in social networks on political participation, *American Journal of Political Science* **50** (2006) 737–754.

[34] Mutz, D. C., The consequences of cross-cutting networks for political participation, *American Journal of Political Science* **46** (2002) 838–855.

[35] Nickerson, D. W., Is voting contagious? Evidence from two field experiments, *American Journal of Political Science* **102** (2008) 49–57.

[36] Nir, L., Ambivalent social networks and their consequences for participation, *International Journal of Public Opinion Research* **17** (2005) 422–442.

[37] Palfrey, T. and Rosenthal, H., A strategic calculus of voting, *Public Choice* **41** (1983) 7–53.

[38] Palfrey, T. and Rosenthal, H., Voter participation and strategic uncertainty, *American Political Science Review* **79** (1985) 62–78.

[39] Pereira, Luiz F. C. and Brady Moreira, F. G., Majority-vote model on random graphs, *Physical Review E* **71** (2005) 016123.

[40] Plutzer, E., Becoming a habitual voter: Inertia, resources, and growth in young adulthood, *American Political Science Review* **96** (2002) 41–56.

[41] Riker, W. H. and Ordeshook, P. C., A theory of the calculus of voting, *American Political Science Review* **62** (1968) 25–43.

[42] Shachar, R. and Nalebuff, B., Follow the leader: Theory and evidence on political participation, *American Economic Review* **89** (1999) 525–547.

[43] Stauffer, D., Monte Carlo simulations of Sznajd models, *Journal of Artificial Societies and Social Simulation* **5** (2001).

[44] Sznajd-Weron, K. and Sznajd, J., Opinion evolution in closed community, *International Journal of Modern Physics C* **11** (2000) 1157–1165.

[45] Uhlaner, C. J., Rational turnout: The neglected role of groups, *American Journal of Political Science* **33** (1989) 390–422.

[46] Zuckerman, A. S., Valentino, N. A. and Zuckerman, E. W., A structural theory of vote choice: Social and political networks and electoral flows in Britain and the United States, *Journal of Politics* **56** (1994) 1008–1033.