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LOCALIZATION–DELOCALIZATION TRANSITION IN LOW-DIMENSIONAL SYSTEMS DRIVEN BY NONRANDOM LONG-RANGE HOPPING

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We analyze numerically the spatial extent of eigenfunctions of a one-dimensional Anderson model with *nonrandom* long-range hopping, which is assumed to fall proportionally to a power μ of the distance between sites m and n ($J_{mn} \sim |m-n|^{-\mu}$). We show that at $\mu < 3/2$ the eigenstates at the top of the band undergo the Anderson localization–delocalization transition, while at $\mu \geq 3/2$ all the states are localized.

Keywords: Anderson localization; Disordered solids

1 INTRODUCTION

After the advent of the one-parameter scaling theory [1], it was conjectured that localization effects [2] in low-dimensional systems are more dramatic than in bulk materials, in the sense that all eigenstates are exponentially localized and that the localization–delocalization transition no longer exists in the thermodynamic limit. This conjecture has been rigorously proven in one-dimensional (1D) tight-binding models with both *uncorrelated*

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diagonal disorder and nonrandom *short-range* hopping (see Ref. [3] for a comprehensive review). However, correlations in the disorder or long-range hopping are often found in different physical systems (*e.g.* Frenkel excitons).

As it has been suggested recently [4], extended states may appear at the top of the band in 1D disordered systems due to *nonrandom* long-range hopping. In this contribution, we report further progress along these lines. In particular, we deal with multifractal properties of the eigenfunctions close to the localization–delocalization transition and estimate the critical value of disorder for the transition to occur.

2 ANDERSON MODEL WITH NONRANDOM LONG-RANGE HOPPING

We consider the 1D Anderson Hamiltonian with diagonal disorder and nonrandom long-range hopping on a regular 1D lattice with N sites [4]

$$\mathcal{H} = \sum_{n=1}^{N} \varepsilon_n |n\rangle\langle n| + \sum_{m,n=1}^{N} J_{mn} |m\rangle\langle n|, \tag{1}$$

where $|n\rangle$ is the ket vector of a state with energy ε_n which is assumed to be a stochastic variable, uncorrelated for different sites and uniformly distributed within an interval of width Δ. The hopping is chosen to be of the form $J_{mn} = J|m-n|^{-\mu}$ with $1 < \mu < 3/2, J > 0$ and $J_{nn} \equiv 0$.

The level spacing at the top of the band of the unperturbed system (without disorder), $\delta E \sim JN^{1-\mu}$ [4], decreases upon increasing the system size slower than the strength of *effective* disorder (reduced by the quasi-particle motion), $\Delta_{\text{eff}} \sim \Delta/\sqrt{N}$. Therefore, if the disorder is a *perturbative* magnitude for a given lattice size, it will remain *perturbative* even on increasing the size. Consequently, the Hamiltonian supports extended states at this particular band edge, despite its 1D nature [4]. On the contrary, if the degree of disorder Δ is large compared to the width of the unperturbed band, it will localize all the states. Thus, a critical value of Δ should exist, at which the system undergoes the localization–delocalization transition. At $\mu > 3/2$, the level spacing δE decreases faster than the effective disorder σ_{eff}, and therefore all the states become localized when $N \to \infty$.

3 SCALING OF THE PARTICIPATION NUMBER

Here we provide further evidences of the above statement from a detailed numerical study of the participation number of the normalized eigenstate $|\nu\rangle = \sum_n \psi_{\nu n}|n\rangle$, given by $P_\nu = [\sum_{n=1}^{N} |\psi_{\nu n}|^4]^{-1}$.

We take advantage of the Lanczos method [5], enabling one to calculate a few eigenstates of the Hamiltonian (1) for rather large system sizes. We have found that the participation number of the states close to the top of the band scales proportional to N for a moderate disorder (of the order of or smaller than the width of the unperturbed band, being $\simeq 10J$ for the sizes studied in the present work), thus revealing the extended nature of these eigenstates. However, these eigenstates are localized for stronger disorder, as it is indeed the case for $\Delta = 40J$, where the participation number does not show any size scaling (see Fig. 1).

4 MULTFRACTALITY OF THE EIGENSTATES

The discussion in the preceeding paragraph makes it possible to conclude that a localization–delocalization crossover should be observed upon

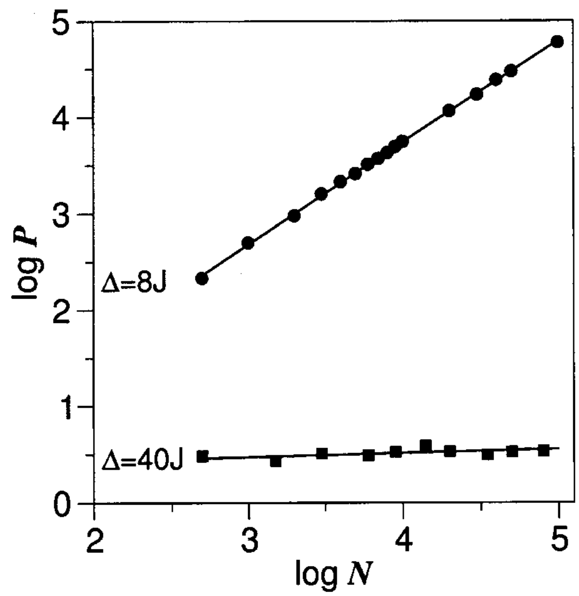

FIGURE 1 Size scaling of the participation number of the uppermost eigenstate for $\mu = 5/4$ and different degrees of disorder. The participation number scales as N for $\Delta = 8$ J (extended state) while remains constant for $\Delta = 40$ J (localized state).

increasing the degree of disorder. To monitor this transition, we have assumed that the multifractal character of the eigenfunctions at the transition is one of the most salient features in the Anderson localization problem (see Refs. [6, 7] and references therein: they feature fluctuations on all length scales). Consequently, strong fluctuations in the participation number close to the transition might be one of its clearest fingerprints. We have undertaken the task of computing numerically the width of the distribution function of the participation number as a function of the degree of disorder. Figure 2 shows the results for a particular value of the exponent $\mu = 5/4$. As it can be seen, the typical fluctuation of the participation number, σ_P, displays a strong enhancement around $\Delta = 11J$. From the previous discussion, we argue that this value corresponds to the critical degree of disorder for the transition to occur.

5 CONCLUDING REMARKS

The above value of the critical disorder has been obtained for a system of finite length ($N = 1000$) with a particular value of the exponent ($\mu = 5/4$).

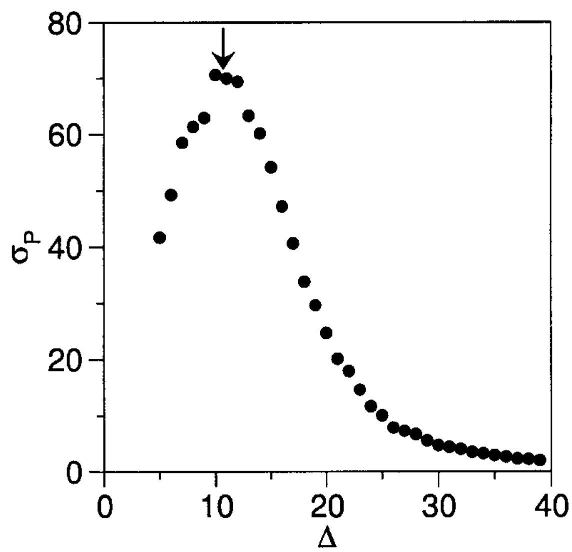

FIGURE 2 Typical fluctuation of the participation number σ_P as a function of the degree of disorder Δ for $N = 1000$. The curve comprises the average over 1000 realizations of the disorder. The vertical arrow indicates the critical value at which the localization–delocalization transition takes place.

Due to the relatively slow decay of the hopping as a function of the distance between sites, finite size effects are much more important than those found in the standard Anderson problem with nearest-neighbor interactions. For instance, the width of the unperturbed band is the natural energy scale determining the value of the critical value of disorder. Within the standard Anderson model this band width rapidly approaches the value $4dJ$ (d being the dimensionality of the lattice) upon increasing the system size. On the contrary, it slowly converges towards its limiting value ($\simeq 10.6J$ for $N \to \infty$) in the case of long-range hopping. Consequently, the critical value of disorder displays the same trend, slowly going up to the magnitude corresponding to the infinite system.

Finally, we would like to mention that the above conclusions can easily be generalized to two-dimensional disordered systems with long-range hopping. In particular, the localization–delocalization transition occurs whenever $2 < \mu < 3$, but the critical value of the disorder strength depends on the system size even stronger than in 1D geometries. Work along these lines is now in progress.

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