

## TRANSMISSION RESONANCES IN MAGNETIC STRUCTURES BASED ON NARROW-GAP SEMICONDUCTORS

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In this work we are concerned with magnetic junction structures in which a homogeneous narrow-gap semiconductor is subjected to an inhomogeneous magnetic field, in an attempt to elucidate the band-structure effect on the resonance tunneling. Careful investigation of the transmission as a function of the energy shows that the resonances in the spectrum can appear. These are remnants of the Landau levels localized near the interface boundary. Comparing the solutions obtained within two-band and single-band models we found the allowed values of the momentum to be quite different, resulting in different resonant values of the transmission coefficient of electron transport through the magnetic interface.

### 1. Introduction

Recently, experimental techniques have opened up the way to experiments in alternating magnetic fields with periods in the nanometer region.<sup>1,2</sup> This kind of field has been realized with the creation of magnetic dots, patterning of ferromagnetic materials, and deposition of superconducting materials on conventional heterostructures. Theoretically, the tunneling properties through potential-barrier structures under the influence of an inhomogeneous magnetic field have been investigated in a number of papers.<sup>3–5</sup> In contrast with tunneling through electric barriers, the tunneling probability depends not only on the electron momentum perpendicular to the tunneling barrier but also on its momentum parallel to the barrier. This renders the tunneling an inherently two-dimensional process and so the magnetic barriers possess wave-vector-filtering properties. The magnetic field localized strictly within a potential barrier was shown<sup>3</sup> to lead to resonances centered within the

barrier. This effect is related to breaking the translation invariance when going closer to the barrier. As a result, the Landau-level degeneracy is removed, and each level gives rise to bands of states, which are still localized in space and labeled by  $k_y$  (the wave vector parallel to the interface which is conserved in the tunneling). Further progress in this field was made in Refs. 6 and 7, where transport and electron properties of realistic structures with nonideal magnetic barrier forms have been studied.

Although the experimental investigations of the magnetic structure used to be performed on the basis of narrow-gap III–V semiconductors, theoretical works treated a single-band model to describe the energy spectrum. And so all band-structure effects including spin peculiarities of the electron states were neglected. Obviously, if a magnetic field is applied to the system, this can lead to inadequate transport effects. In this work, as a first step in studying the band-structure effects on the resonance tunneling, we are concerned with magnetic structures in which

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a homogeneous narrow-gap semiconductor is subjected to an inhomogeneous magnetic field. For the magnetic junction structures under consideration the magnetic field is taken to be homogeneous along the  $y$  axis and varies along the  $x$  axis. Two different magnetic structures are studied. The first one is the so-called inverted magnetic junction, where  $B_z(x) = B$  at  $x > 0$  and  $B_z(x) = -B$  at  $x < 0$ . The second structure is the normal magnetic junction, where  $B_z(x) = B_+$  at  $x > 0$  and  $B_z(x) = B_-$  at  $x < 0$ .

## 2. Theoretical Model

The simplest theoretical model for the narrow-gap semiconductors is known<sup>8</sup> to be a two-band one, which in the first approximation of the  $\mathbf{k}\hat{\mathbf{p}}$  perturbation theory reduces to the Dirac Hamiltonian:

$$\widehat{H}_{00} = \begin{pmatrix} \Delta + V & -i\hbar v \boldsymbol{\sigma} \hat{\mathbf{k}} \\ i\hbar v \boldsymbol{\sigma} \hat{\mathbf{k}} & -\Delta + V \end{pmatrix}. \quad (1)$$

Here  $\Delta$  is determined as a half band gap,  $\Delta = E_g/2$ ;  $V$  is a so-called work function, describing the shift of the gap middles;  $\hat{\mathbf{k}}$  is the momentum operator,  $\hat{\mathbf{k}} = -i\nabla$ ;  $v$  is the interband matrix element of the velocity operator;  $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$  is the vector with the components of the Pauli matrices. In the simplest form this two-band Dirac Hamiltonian describes the two nearby conduction and valence bands as two Kramers-conjugate states. So, in the very nature the Hamiltonian (1) takes into account spin properties of the wave functions, called eigenspinors in this case.

A static magnetic field is incorporated by the standard substitution

$$\mathbf{k} \rightarrow \mathbf{k} + \frac{|e|\hbar}{c} \mathbf{A}(\mathbf{r}). \quad (2)$$

For the magnetic field  $\mathbf{B} = (0, 0, B_z)$  the electromagnetic vector potential  $\mathbf{A}$  is chosen in the Landau gauge as  $\mathbf{A}(\mathbf{r}) = (0, xB_z, 0)$ . Following the results of Ref. 9 we write the four bulk eigenfunctions for (1) in the form

$$\begin{aligned} \Psi(\mathbf{r}) &= e^{i(k_y y + \chi_\zeta \zeta)} \\ &\times \begin{pmatrix} c_1(U)\psi_{n-1}(\xi) + c_2(V)\psi_n(\xi) \\ c_3(U)\psi_{n-1}(\xi) + c_4(V)\psi_n(\xi) \end{pmatrix}, \quad (3) \end{aligned}$$

where the vector columns ( $U$ ) and ( $V$ ) are defined by

$$(U) = \frac{1}{2} \begin{pmatrix} 1 + \varphi \\ 1 - \varphi \end{pmatrix}, \quad (V) = \frac{1}{2} \begin{pmatrix} 1 - \varphi \\ 1 + \varphi \end{pmatrix}. \quad (4)$$

Here a new dimensionless variable is introduced by  $\xi = (x_0 + x)/l_H$ ,  $x_0 = \varphi k_y l_H^2$  is the center of the Landau orbits,  $\varphi = \text{sgn}(B_z) = \pm 1$  is the orientation of the magnetic field ( $\varphi = 1$  corresponds to a field pointing along the positive  $z$  axis),  $l_H = \sqrt{\hbar c/|eB_z|}$  is the magnetic length which is a measure of the lowest Landau-orbit extension,  $\chi_\zeta = l_H k_z$ , and  $\zeta = z/l_H$  are the dimensionless  $z$  momentum and coordinates. The harmonic-oscillator functions  $\psi_n(\xi)$  satisfy the canonical equation

$$\begin{aligned} \left[ -\frac{d^2}{d\xi^2} + \xi^2 \right] \psi_n(\xi) \\ = (2n + 1)\psi_n(\xi), \quad n = 0, 1, 2, \dots, \quad (5) \end{aligned}$$

and the corresponding bulk-dispersion relation is

$$\chi_\zeta^2 + 2n = \left[ \frac{l_H}{\hbar v} \right]^2 [(E - V)^2 - \Delta^2], \quad n = 0, 1, 2, \dots \quad (6)$$

As a matter of fact, the second-order differential equation (5) admits once more solution besides  $\psi_n(\xi)$ . But the geometry peculiarities of the magnetic interface structure considered demand a solution decaying at  $\xi \rightarrow \pm\infty$ , which is just allowed for by the harmonic-oscillator functions with nonnegative integer  $n$  values.

The spinor coefficients in (3) satisfy the matrix equation

$$\begin{bmatrix} -(E - V - \Delta)I & -i\hbar v \mathbf{k}_n \cdot \boldsymbol{\sigma} \\ i\hbar v \mathbf{k}_n \cdot \boldsymbol{\sigma} & -(E - V + \Delta)I \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = 0, \quad (7)$$

where  $I$  is a unit  $2 \times 2$  matrix and the vector  $\mathbf{k}_n$  is determined as follows:

$$\mathbf{k}_n = \frac{1}{l_H} (0, \sqrt{2n}, \varphi \chi_\zeta).$$

Taking into account the matrix in (7) to commute with

$$\hat{T} = \begin{bmatrix} \widehat{\sigma}_x & 0 \\ 0 & -\widehat{\sigma}_x \end{bmatrix}, \quad (8)$$

we can write  $\mathbf{c}$  as eigenfunctions of  $\hat{T}$ . So, one obtains

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 1 \\ \tau \end{bmatrix} \\ \frac{i\hbar v}{E - V + \Delta} (\mathbf{k}_n \cdot \boldsymbol{\sigma}) \begin{bmatrix} 1 \\ \tau \end{bmatrix} \end{bmatrix}, \quad (9)$$

where  $\tau = \pm 1$ ,  $n = 1, 2, 3, \dots$ . Note that (9) implies the eigenspinors with  $n \neq 0$  to be doubly  $\tau$ -degenerate, while the eigenspinor with  $n = 0$  is not  $\tau$ -degenerate.

### 3. Magnetic-Junction Structure

Starting from the Schrödinger equation determined by the Hamiltonian (1) with the replacement (2), we are looking for a solution as a linear combination of the eigenfunctions (3). For the steplike magnetic interface with the sharp change of the magnetic field at  $x = 0$ , the boundary conditions need to be applied to the eigenvalue problem. Assuming the wave function to be continuous at the interface and integrating the Schrödinger equation across the interface boundary, we obtain the boundary conditions to be reduced just to the demand for continuity of the wave functions in this case.

#### 3.1. Inverted magnetic junction

Since for the inverted magnetic junction the sign  $\varphi$  in the vectors ( $U$ ) and ( $V$ ) is changed at the interface boundary, the dispersion relation for each Landau level reads

$$(\chi_\zeta^2 + 2n)[\psi_n^2(\xi_0) - \psi_{n-1}^2(\xi_0)] = 0, \quad (10)$$

where  $\xi_0 = k_y l_H$  is a dimensionless  $x$  coordinate of the Landau-orbit center. This equation defines the allowed values for the momentum  $k_y$ , which in its turn is related to the location of the Landau-orbit centers.

The wave function (3) for  $n = 0$  is not  $\tau$ -degenerate. Hence, for the opposite directed magnetic fields the corresponding eigenspinors with  $n = 0$  are characterized by the opposite directions of the spins, with an average spin vector being

$$\langle \Psi^+ | \hat{\Sigma} | \Psi \rangle \sim \pm(0, 0, 1),$$

where the signs  $\pm$  are related to the states at  $x < 0$  and  $x > 0$ , respectively. That is why the wave function with  $n = 0$  admits a trivial solution only, being completely forbidden.

For  $n = 1$ , Eq. (10) is satisfied at  $k_y = \pm \frac{1}{\sqrt{2}l_H}$ . The centers of the Landau orbits for  $n = 1$  eigenspinors are allowed to be  $\xi_0 = \pm \frac{1}{\sqrt{2}}$  distant from the interface boundary. Thus, the wave function of the inverted magnetic contact is strongly localized near the interface boundary, the decay length being determined by the magnetic length  $l_H$ . In the bulk each Landau level is well known to be degenerate because it does not depend on  $k_y$ . Obviously, only a few allowed values for  $k_y$  result in (particular or complete) removal of the Landau-level degeneracy in the magnetic structure.

Going on this analysis we find that for  $n = 2$  Eq. (10) is satisfied at  $k_y = \frac{\pm 1 \pm \sqrt{3}}{2l_H}$ , i.e.  $\xi_0 = \frac{\pm 1 \pm \sqrt{3}}{2}$ . The Landau levels are localized again near the interface boundary, but this localization is not so strong as for  $n = 1$ .

#### 3.2. Normal magnetic junction

For the normal magnetic junction the magnetic field just alternates its absolute value at the interface boundary. Hence the eigenspinors have a similar form on both sides of the interface without changing the sign  $\varphi$ . By solving the boundary-value problem one obtains the dispersion relation

$$2n(\chi_\zeta^{-2} + 2n) \left\{ \frac{1}{l_H^+ l_H^-} [\psi_n(\xi_0^-) \psi_{n-1}(\xi_0^+) - \psi_n(\xi_0^+) \times \psi_{n-1}(\xi_0^-)]^2 - \left( \frac{1}{l_H^+} - \frac{1}{l_H^-} \right)^2 \psi_n(\xi_0^-) \times \psi_n(\xi_0^+) \psi_{n-1}(\xi_0^-) \psi_{n-1}(\xi_0^+) \right\} = 0. \quad (11)$$

Here  $l_H^\pm$  are the values of the magnetic length for  $x > 0$  and  $x < 0$ , respectively, and  $\xi_0^\pm = k_y l_H^\pm$  are again the dimensionless  $x$  coordinates of the Landau-orbit centers on the different sides of the interface boundary.

We note that for  $n = 0$  the dispersion relation (11) is reduced to an identity. In contrast with the inverted junction, a nontrivial solution for  $n = 0$  is admitted for all  $k_y$  values. This is an apparent consequence of the fact that for the normal junction the

average spin vector has the same direction on both sides of the interface boundary. Analysis of Eq. (11) shows that for  $n = 1$  and  $n = 2$  only the eigen-spinors with  $k_y = 0$  are allowed, i.e. the centers of the Landau orbits are strongly located at the interface boundary. Now the average spin vector is directed along the  $z$  axis just at the point  $x = 0$ . Going away from the interface boundary, the spin vector is beginning to rotate around the  $z$  axis.

To compare these results with the single-band model, we shall study the well-known one-band Schrödinger equation<sup>3</sup>

$$\frac{1}{2m} \left( \hbar \hat{\mathbf{k}} + \frac{e}{\hbar c} \mathbf{A} \right)^2 \Psi = e \Psi, \quad (12)$$

which is satisfied by the harmonic-oscillator functions again. The boundary conditions are now reduced to the demand for continuity of both the wave functions and their derivatives at the interface boundary.

Spin peculiarities of the one-band wave functions are completely neglected within this approximation. This leads to different results obtained in the framework of the one-band and two-band models. For example, for the inverted magnetic junction treated by the one-band model the state with  $n = 1$  is allowed for  $k_y = 0$  and  $k_y = \pm \frac{1}{l_H}$ . However, for the two-band model the allowed  $k_y$  values were shown to be  $\pm \frac{1}{\sqrt{2}l_H}$ . We note that within the two-band model the value  $k_y = 0$  is not allowed, in contrast with the one-band model. Moreover, the fact of forbidding the value  $k_y = 0$  remains valid for all Landau levels. When considering transport properties of the magnetic-barrier structures, this effect is revealed by changing the transmission-resonance values. For the normal junction the spin properties appear to be not so important. That is why the results obtained for the lowest-lying Landau levels are quite similar for the two models. A difference appears for  $n > 2$  only.

#### 4. Conclusions

Since for the magnetic structures considered the magnetic field extends everywhere, the single-parti-

cle stationary solutions to the Schrödinger equation vanish asymptotically on both sides of the magnetic junction. These correspond to Landau-level wave functions centered at some point on the  $x$  axis which is fixed by  $k_y$  in the Landau gauge. Deep in the bulk each eigenvalue is degenerated because it does not depend on  $k_y$ . Getting closer to the interface, the breaking of the translational invariance removes the degeneracy and each level gives rise to bands of states, which are still localized in space and labeled by  $k_y$ . Transport in the presence of an external bias needs scattering against impurities which allows for the hopping between these states. Careful investigation of the transmission as a function of the energy is in progress now, showing the appearance of the resonances in the spectrum. These are remnants of the Landau levels localized near the interface boundary, and show up as peaks with strong dependence on  $k_y$ . Comparison between the solutions obtained for the Dirac two-band and the single-band models shows that for the inverted magnetic junction the allowed  $k_y$  values are quite different, resulting in different resonant values of the transmission coefficient of the electron transport through the magnetic interface. As for the normal magnetic junction, the results obtained within the single-band model appear to describe the electron transport through the magnetic interface structure quite adequately.

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