

# Design of an efficient spin filter device

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Received 2 June 2006, in final form 16 October 2006

Published 10 November 2006

Online at [stacks.iop.org/SST/21/1764](http://stacks.iop.org/SST/21/1764)

## Abstract

In a quantum dot in which the spin degeneracy of a carrier is lifted, the Fano effect may be used as an effective means to generate spin polarization of transmitted carriers. In this work, we propose a new and more effective design of a spin-dependent polarizer. The proposed device consists of a quantum wire with two side-coupled quantum dots. A detailed analysis of the spin-dependent polarized current is carried out, and we find some improvements as compared to more conventional designs.

(Some figures in this article are in colour only in the electronic version)

## 1. Introduction

Recently, there has been much interest in understanding the manner in which the unique properties of nanostructures may be exploited in spintronic devices, which utilize the spin degree of freedom of the electron as the basis of their operation [1–6]. A natural feature of these devices is the direct connection between their conductance and their quantum-mechanical transmission properties, which may allow their use as an all-electrical means for generating and detecting spin-polarized distributions of carriers.

Quantum dots are man-made nanostructures in which electrons are confined in all space dimensions [7]. Energy and charge quantization results from this confinement. As both features are present in real atomic systems, useful analogies between *real* and *artificial* atomic systems have been exploited recently. Enforcing this analogy, Fano [8] and Dicke effects [9] were also found to be present in quantum dot configurations. On the other hand, Song *et al* [2] described how a spin filter may be achieved in open quantum dot systems by exploiting the Fano resonances that occur in their transmission characteristic. In a quantum dot in which the spin degeneracy of the carrier is lifted, they showed that the Fano effect may be used as an effective means to generate spin polarization of transmitted carriers and that electrical detection of the resulting polarization should be possible. Moreover, nonlinear effects arising in electronic systems due to interactions could help to finely tune Fano resonances in quantum devices [10].

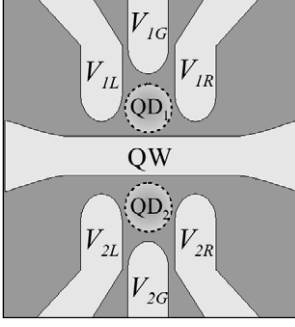
In a previous work [11], we showed that in a side-coupled double quantum dot system the transmission shows a large peak-to-valley ratio. Moreover, the difference of energy

between the resonances and antiresonances can be controlled adjusting the difference between the energy levels of the two quantum dots by gate voltages. In this work, we show that the above properties of the side-coupled double quantum dot system can be extended to design an efficient spin filter. We compare the spin-dependent polarization of this design and the polarization obtained with two other devices, namely one side-coupled quantum well and a T-shape double quantum dot. As a main result, we find better spin-polarization capabilities as compared to those more conventional designs.

## 2. Model

The system under consideration is formed by two quantum dots connected by tunnel coupling to a long quantum wire waveguide, as shown schematically in figure 1. We consider that the magnetic field is localized in the quantum dots. This confinement can be done by placing nano-magnets on top of each quantum dot [5, 6]. The full system is modelled by the Anderson Hamiltonian with two laterally connected impurities, namely  $H = H_W + H_D + H_{int}$  with

$$\begin{aligned}
 H_W &= -v \sum_{(i \neq j)} \sum_{\sigma = \uparrow \downarrow} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}), \\
 H_D &= \sum_{\alpha = u, l} \sum_{\sigma = \uparrow \downarrow} \varepsilon_\alpha d_{\alpha\sigma}^\dagger d_{\alpha\sigma} + \sum_{\alpha = u, d} U n_{\alpha\uparrow} n_{\alpha\downarrow} \\
 &\quad + \sum_{\alpha = u, d} \sum_{\sigma, \bar{\sigma} = \uparrow \downarrow} g \mu B \sigma_\sigma^z d_{\alpha\sigma}^\dagger d_{\alpha\bar{\sigma}}, \\
 H_{int} &= -V_0 \sum_{\alpha = u, l} \sum_{\sigma = \uparrow \downarrow} (d_{\alpha\sigma}^\dagger c_{0\sigma} + c_{0\sigma}^\dagger d_{\alpha\sigma}), \quad (1)
 \end{aligned}$$



**Figure 1.** Schematic view of the two quantum dots attached to a quantum wire. Current passing from the source to the drain is controlled by the gate voltages  $V_{1G}$  and  $V_{2G}$ .

where  $c_{i\sigma}^\dagger$  is the creation operator of an electron at site  $i$  of the wire in the  $\sigma$  spin state ( $\sigma = \uparrow, \downarrow$ ), and  $d_{\alpha\sigma}^\dagger$  is the corresponding operator of an electron with spin  $\sigma$  of the upper ( $\alpha = u$ ) or lower ( $\alpha = l$ ) quantum dot. Moreover  $n_{\alpha\sigma} = d_{\alpha\sigma}^\dagger d_{\alpha\sigma}$ . Here,  $\varepsilon_\alpha$  is the energy level of the  $\alpha$  dot and  $V_0$  is the coupling between the quantum wire and one of the quantum dots. The magnetic field  $B$  is applied perpendicular to the electron gas;  $\sigma^z$  is the  $z$  Pauli matrix,  $g$  is the Landé  $g$ -factor of electrons,  $\mu$  is the Bohr magneton and  $U$  is the Coulomb coupling. The potential of the wire is taken to be zero and the hopping in the wire is  $-v$ . Furthermore, we set the energy levels of the quantum dots as  $\varepsilon_u = \varepsilon_0 + \Delta V$  and  $\varepsilon_l = \varepsilon_0 - \Delta V$ . The potential drop across the two quantum dots,  $2\Delta V$ , will be a major parameter controlling the width of the spin-polarized transmission band, thus allowing for a good control of the device performance.

### 3. Conductance and spin polarization

The linear conductance in the coherent transport regime can be obtained from the well-known Landauer formula at zero temperature  $G_\sigma = (e^2/h)T_\sigma(\omega = \varepsilon_F)$ , where  $T_\sigma(\omega)$  is the transmission probability for the  $\sigma$  spin state given by

$$T_\sigma(\omega) = \frac{2\Gamma_L(\omega)\Gamma_R(\omega)}{\Gamma_L(\omega) + \Gamma_R(\omega)} \text{Im}[G_{0,\sigma}^W]. \quad (2)$$

Here,  $G_{0,\sigma}^W$  is the Green function at site 0 of the wire for the  $\sigma$  spin state and  $\Gamma_{L(R)}$  is the coupling of the site 0 to the left (right) side of the wire. By using a Dyson equation  $G = G^0 + G^0 H_{\text{int}} G$ , we calculate the Green function of the site 0 of the quantum wire coupled to the dots (with  $G^0$  unperturbed the Green function).

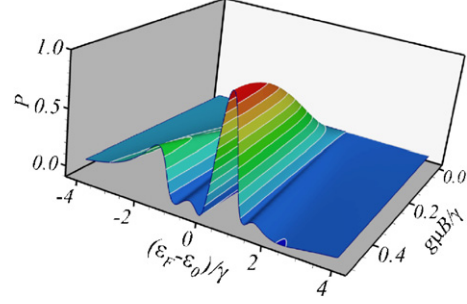
We first consider the transmission probability in the case of noninteracting electrons, namely  $U = 0$  in equation (1). The calculation is straightforward and for brevity we quote only the obtained transmission probability

$$T_\sigma(\omega) = \frac{\mathcal{F}(\Delta V)\mathcal{F}(-\Delta V)}{\mathcal{F}(\Delta V)\mathcal{F}(-\Delta V) + 4\gamma^2\mathcal{F}(0)}, \quad (3)$$

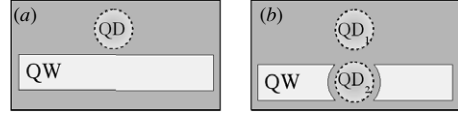
where  $\gamma = V_0^2/2v$ . For simplicity, we defined  $\mathcal{F}(z) \equiv [\omega - z - \varepsilon_0 - \sigma g\mu B]^2$ .

Following [2], we introduce the weighted spin polarization as

$$P_\sigma = \frac{|T_\uparrow - T_\downarrow|}{|T_\uparrow + T_\downarrow|} T_\sigma, \quad \sigma = \uparrow\downarrow. \quad (4)$$



**Figure 2.** Weighted spin polarization ( $\sigma = \uparrow$ ) for the side-coupled double quantum dot, as a function of the Fermi energy and magnetic field.



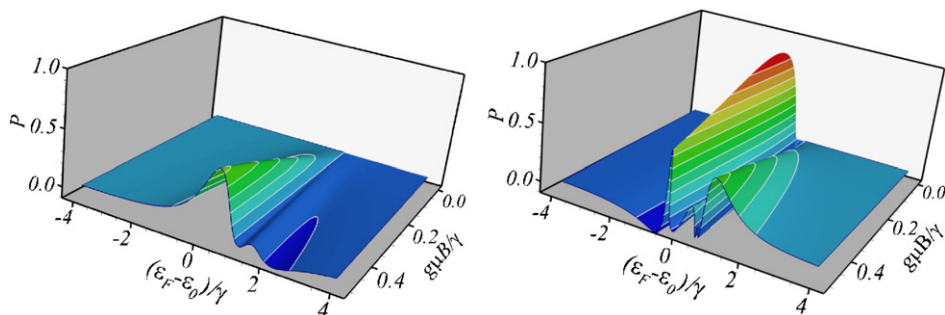
**Figure 3.** Schematic view of (a) design I (one quantum dot attached to a quantum wire) and (b) design II (T-shape double quantum dot).

Note that this definition takes into account not only the relative fraction of one of the spins, but also the contribution of those spins to the electric current. In other words, we will require not only the first term of the right-hand side of (4) to be of order of unity, but also the transmission probability  $T_\sigma(\omega)$ .

First of all, we realize that the weighted spin polarization  $P_\uparrow$  can be made optimum (100% polarization) at  $\omega = \varepsilon_0 + g\mu B$  when  $\Delta V = \pm 2g\mu B$ . Therefore, adjusting the gate voltage and the magnetic field, it becomes feasible to get optimum polarization of the electron current. Most important, the device can operate close to optimum polarization within a wide range of energy and magnetic values, as shown in figure 2. For GaAs-based devices with  $\gamma = 3$  meV [12], the magnetic field required to reach optimum polarization is about 4 T. However, this field is reduced to 40 mT in InAs-based devices for the same coupling constant  $\gamma$ . We then conclude that physical parameters of the semiconductors (Landé factor and Bohr magneton) are more relevant than the coupling between the quantum dots and the quantum wire. Furthermore, it is to be stressed that optimum performance, in the sense defined above, cannot be achieved so easily with some of the conventional designs, as we will show below.

After discussing our proposal for an efficient spin filter device, based on a quantum wire with *two* side-coupled quantum dots, we compare its performance to other designs studied in the literature. We focus on two other proposals that present spin filtering capabilities, namely a quantum wire with one side-coupled quantum dot (hereafter referred to as design I) [8] and a T-shape double quantum dot (hereafter referred to as design II) [5]. Figure 3 shows a schematic view of designs I and II.

We solve the transmission problem in a similar fashion as before, and for brevity we do not quote the final expressions. Figure 4 shows the weighted spin polarization as a function of the Fermi energy and magnetic field obtained with both designs. Remarkably, although design I shows a broader energy range for which polarization is significantly large, as compared to our design (see figure 2), the maximum



**Figure 4.** Weighted spin polarization for  $\sigma = \uparrow$ , as a function of the Fermi energy and magnetic field corresponding to design I (left plot) and design II (right plot), respectively.

polarization reaches only 50% for the same values of the magnetic field. To achieve a polarization close to the optimum one, the magnetic field must be increased by almost an order of magnitude. Therefore, our design demands much smaller magnetic fields to operate close to the optimum polarization, and in this sense II is easier to implement at the nanoscale.

The right plot of figure 4 displays the weighted spin polarization for  $\sigma = \uparrow$  of design II as a function of the Fermi energy and magnetic field, when the coupling between both quantum dots is  $0.3\gamma$ . We observe that design II is capable of reaching optimum polarization ( $\sim 100\%$ ), but in a much narrower energy interval as compared to our proposal. Therefore, small changes in the Fermi energy or thermal fluctuations could leave the device out of resonance, thus losing its spin filtering capabilities. The reason for this narrower spin-polarized resonance arises from the peculiarities of design II. It is based on Rashba coupling [5], which usually is not large in most semiconductors. In addition, the way of achieving the maximum polarization is by varying the coupling between the two quantum dots. Therefore, this leaves only one free parameter, namely the magnetic field. In contrast, in our design the resonance width is controlled by  $\Delta V$  and this can be made large. As a second advantage, a number of semiconductors could be used to build the device proposed in this work since the control parameter  $\Delta V$  is independent of the material and can be varied at will. We thus claim that the design proposed in this work should present a more stable performance as compared to design II. Thus, we conclude that our proposal presents a good balance meeting both requirements (low magnetic field and high stability), and in this sense is better than the other two designs.

As charging effects could become important in electronic devices based on quantum dots, we now discuss the effects of Coulomb interaction in the landscape presented above. In the case of finite  $U$ , it is known that electron–electron interaction mainly enlarges the separation between the peak-dip features (as shown in figure 2) by an amount of the order of  $U$ . On the other hand, within the so-called Hubbard I approximation, it is straightforward to show that for spin-up electrons ( $\sigma = \uparrow$ ) in a Coulomb blockade peak ( $\varepsilon_F = \varepsilon_0 + g\mu B + U$  and  $n_\downarrow = 1$ ), the transmission probability becomes unity ( $T_\uparrow(\varepsilon_F) = 1$ ). In other words, in a Coulomb blockade peak the conductance becomes  $G_\uparrow = e^2/h$ . Thus we consider the case with  $\sigma = \uparrow$ ,  $n_\downarrow = 1$  and  $n_\uparrow = 0$  at  $\omega = \varepsilon_F = \varepsilon_0 + g\mu B + U$ . For this case,

the transmission probability for spin-down electrons is

$$T_\downarrow(\varepsilon_F) = \frac{\mathcal{K}(\Delta V, -U)\mathcal{K}(-\Delta V, -U)}{\mathcal{K}(\Delta V, -U)\mathcal{K}(-\Delta V, -U) + 4\gamma^2\mathcal{K}(\Delta V, -U)},$$

where  $\mathcal{K}(x, y) \equiv (x + y + 2g\mu B)^2$ . Therefore adjusting  $\Delta V$  such that  $\Delta V = \pm(2g\mu B - U)$ , the transmission probability for spin-down electrons vanishes. Consequently, we obtain optimum polarization, namely  $P_\uparrow = 1$  and  $P_\downarrow = 0$  when  $\varepsilon_F = \varepsilon_0 + g\mu B + U$ . Finally, we note that this result is important not only for device design, but also for basic physics. By varying the Fermi energy through gate voltages we can obtain the magnitude of  $U$ , a parameter that is usually difficult to measure. To this end, one only needs to vary the Fermi energy until maximum conductance for spin-up electrons is reached for a given magnetic field, since  $U = \varepsilon_F - \varepsilon_0 - g\mu B$  at resonance.

In summary, we have presented a novel spin filter device based on a quantum wire with two side-coupled quantum dots. Assuming perfectly coherent transport, we demonstrated that spin-polarization filtering capabilities can be made optimum ( $\sim 100\%$ ) within a wide range of parameters (Fermi energy and applied magnetic field). This value is twice the largest polarization obtained with other designs previously studied in the literature (design I). Also, it might be more stable under normal operating conditions as compared to similar designs presenting optimum spin polarization (design II). To deal with electron–electron interactions, the device must operate at a Coulomb blockade peak. Finally, we proposed an experiment to directly measure the Coulomb repulsion parameter  $U$ .

## Acknowledgments

Work at Madrid was supported by MCyT (MAT2003-01533). PAO is thankful for the financial support from Milenio ICM P02-054-F and FONDECYT under grants 1060952 and 7020269. Additionally, PAO would like to thank the Departamento de Física de Materiales (UCM) for the hospitality during his visit.

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