Dissipation and information in stochastic processes

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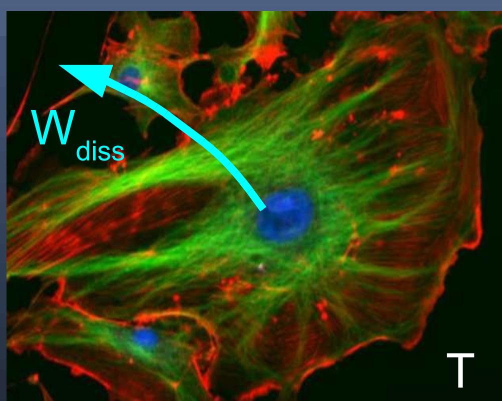
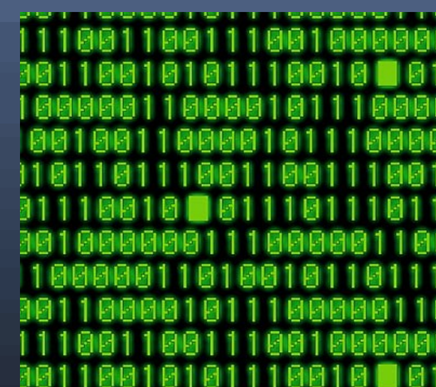
Dissipation and Irreversibility

Dissipation-irreversibility relationship in non-equilibrium stochastic processes :



Fluctuation Theorems



Energy dissipation

Information
Irreversibility revealed in the data

Fluctuation Theorems (FT)

J.M.R. Parrondo, C.V. Den Broeck and R. Kawai, New Journal of Physics **11**, 073008 (2009).

$$\langle \dot{W}_{diss} \rangle = \langle \dot{W} \rangle - \dot{\Delta F} = T\langle \dot{S} \rangle = \lim_{t\to\infty} \frac{kT}{t} D\left[p\left(\{x(\tau)\}_{\tau=0}^{t} \right) \middle|\middle| p\left(\{x(t-\tau)\}_{\tau=0}^{t} \right) \right]$$

Average dissipation of the process
Entropy production

Relative entropy between forward and backward processes

D = Relative Entropy (Kullback-Leibler distance)

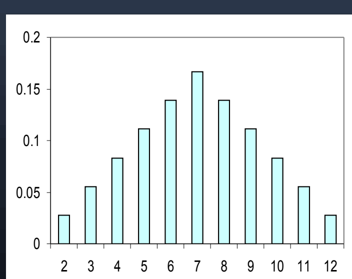
Relative Entropy or Kullback–Leibler distance

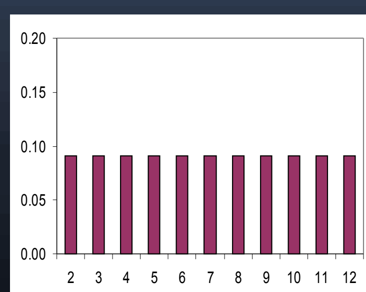
$$D(p||q) = \sum_i p_i \log\left(\frac{p_i}{q_i}\right)$$

5,7,12,9,6,2,5,2,9,2,12,8,10,4,....

OR

?



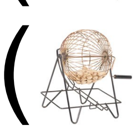


Stein's Lemma:

The probability of incorrectly guessing *p* from *n* data actually distributed as *q* is (asympt.):

$$2^{-nD(p||q)}$$

D( || ) = 0.185

D( || ) = 0.220

Fluctuation Theorems (FT)

$$\langle \dot{W}_{diss} \rangle = \langle \dot{W} \rangle - \Delta\dot{F} = T\langle \dot{S} \rangle = \lim_{t\to\infty} \frac{kT}{t} D\left[p\left(\{x(\tau)\}_{\tau=0}^{t} \right) \middle|\middle| p\left(\{x(t-\tau)\}_{\tau=0}^{t} \right) \right]$$

Stochastic discrete processes in stationary regime: $(x_1, x_2, \ldots, x_n)$

$$D_k(p_F || p_B) = \sum_{x_1,\ldots,x_k} p(x_1,\ldots,x_k) \log \frac{p(x_1,\ldots,x_k)}{p(x_k,\ldots,x_1)}$$

$$\frac{\langle \dot{S} \rangle}{k} = d(p_F || p_B) = \lim_{n\to\infty} \frac{1}{n} D_n(p_F || p_B)$$

→ partial information →

$$\frac{\langle \dot{S} \rangle}{k} \geq d(p_F || p_B)$$

Physics (Average)

Time series (single trajectory)

Even ignoring some physical details of the system we can estimate its dissipation !

Relative entropy estimators: Brute force

$$\frac{\langle \dot{S} \rangle}{k} \geq d(p_F || p_B)$$

Problem: Calculating D from a single trajectory
2,3,2,1,0,2,4,5,3,4,5,0,2,1,0,2,1,2,1, ...

1st estimator: Brute-force counting of k-th length strings

$$D_n(p_F || p_B) \equiv \sum_{x_1,\ldots,x_n} p(x_1,\ldots,x_n) \log \frac{p(x_1,\ldots,x_n)}{p(x_n,\ldots,x_1)}$$

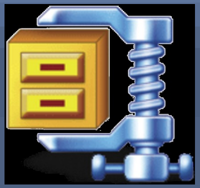
$$d_k \equiv D_k - D_{k-1}$$

$$d(p_F || p_B) = \lim_{n\to\infty} \frac{1}{n} D_n(p_F || p_B) = \lim_{k\to\infty} d_k$$

k-th order Markov chains: $d = d_{k+1}$

Relative entropy estimators: Ziv-Merhav

2nd estimator: Based on Lempel-Ziv compression

D (p || q) ~  (p || q)

$$d_{\mathrm{ZM}}(\mathbf{x}) \equiv \frac{1}{n}[c(\mathbf{x}|\mathbf{x}') \log n - c(\mathbf{x}) \log c(\mathbf{x})]$$

c(**x**) = Compression length of **x** using the Lempel-Ziv incremental parsing

c(**x**|**x'**) = Cross parsing length of **x** with respect to its reverse **x'**

Tends to relative entropy asymptotically.

Relative entropy estimators: Ziv-Merhav

c(z) , compression length of z

z=(0 1 1 1 1 0 0 0 1 1 0)

z=(0 | 1 | 1 1 | 1 0 | 0 0 | 1 1 0) c(z)=6

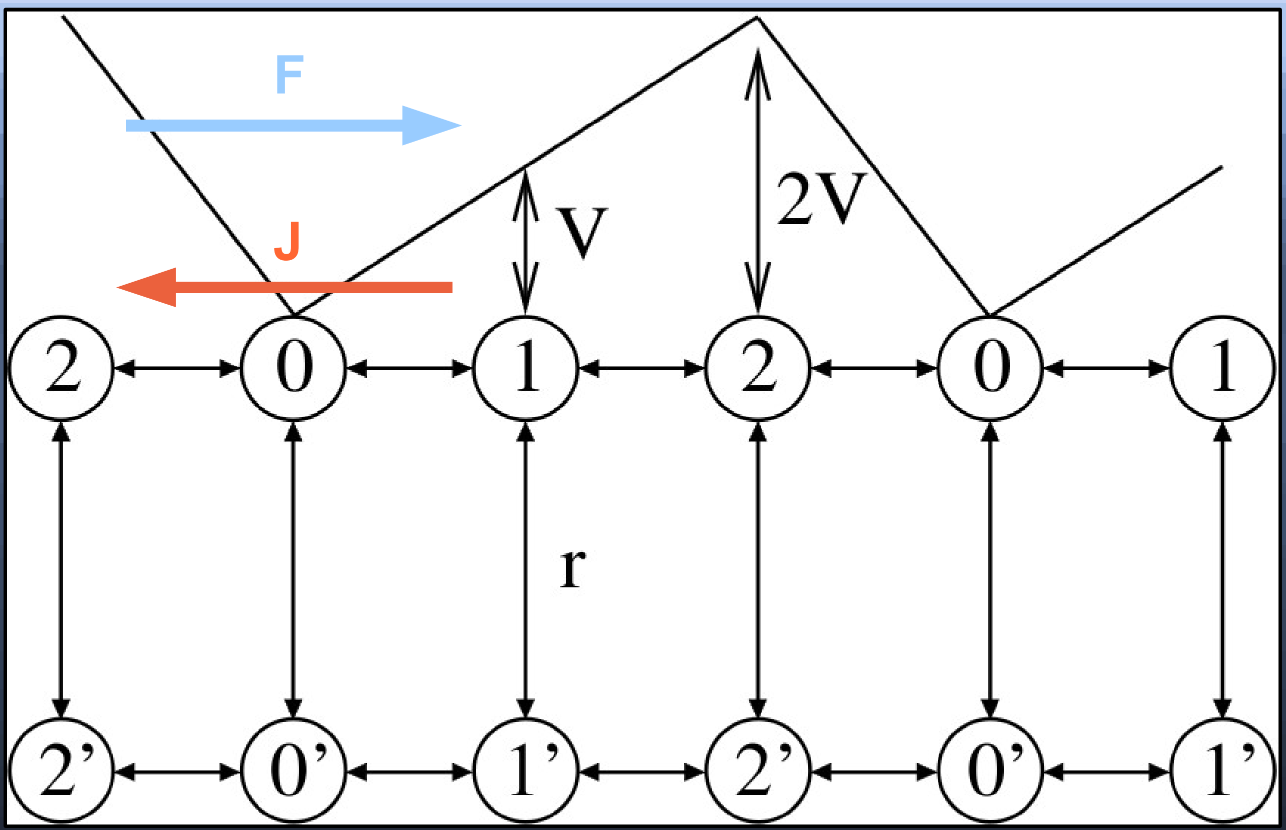
c(z|x) , Cross parsing length of z with respect to x

z=(0 1 1 1 1 0 0 0 1 1 0)

x=(1 0 0 1 0 1 0 0 1 1 0)

z=(0 1 1 | 1 1 0 | 0 0 1 1 0) c(z|x)=3

Discrete ratchet

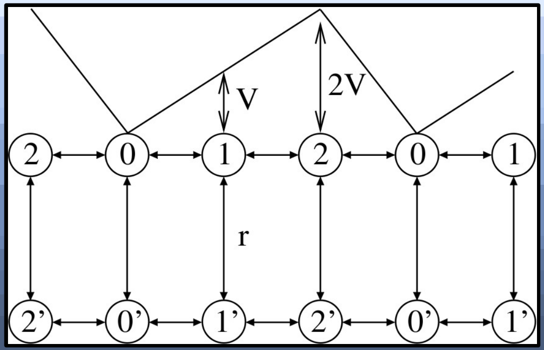


All rates obey detailed balance **except** switches → Irreversibility → **Dissipation**

$$\frac{k_{0\to 1}}{k_{1\to 0}} = e^{-\beta V} \cdot e^{\beta F L}$$

$$\frac{k_{1\to 1'}}{k_{1'\to 1}} = \frac{r}{r} = 1 \neq e^{\beta V}$$

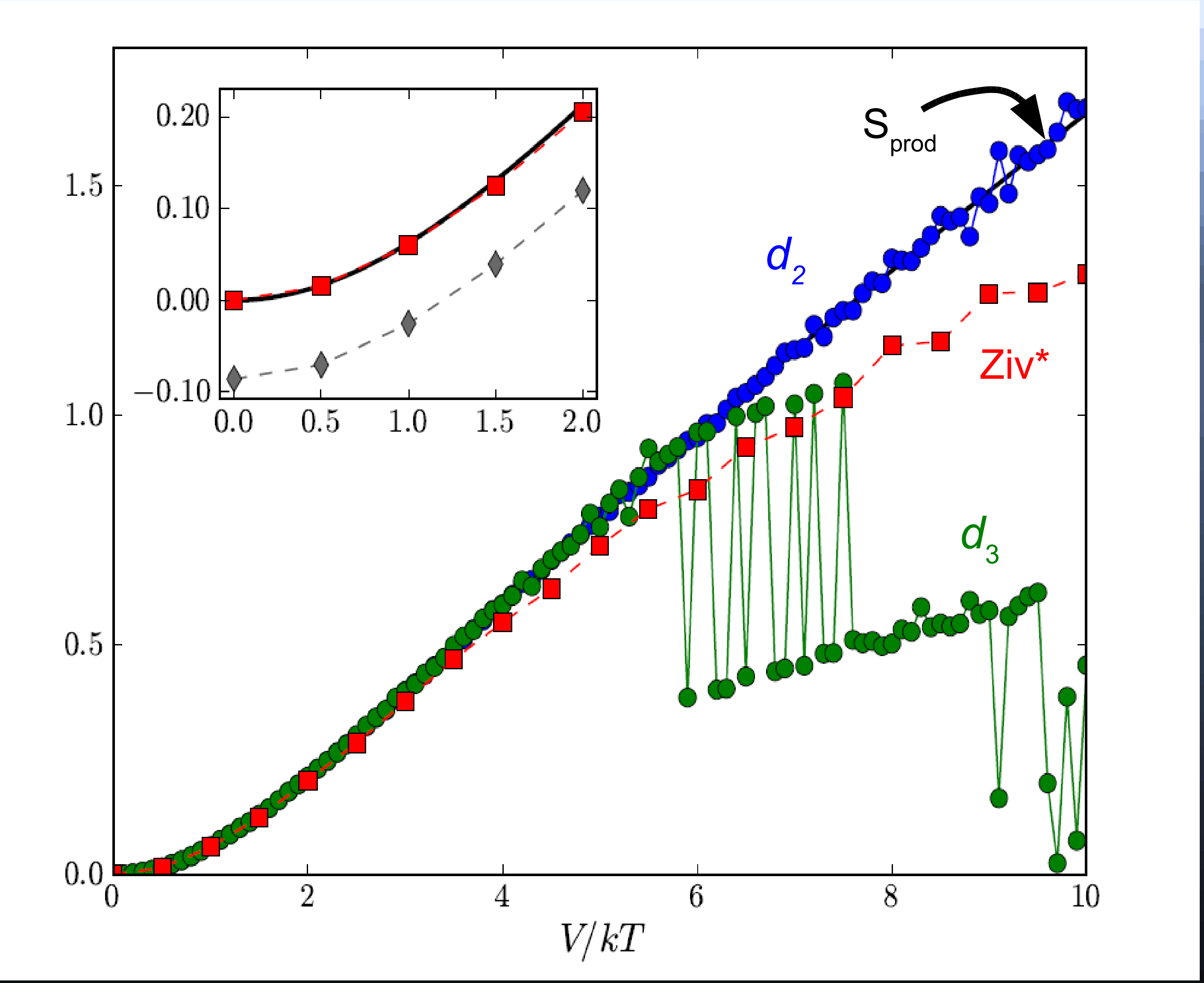
Discrete ratchet: full information



F=0
r finite

Estimate dissipation with single trajectories

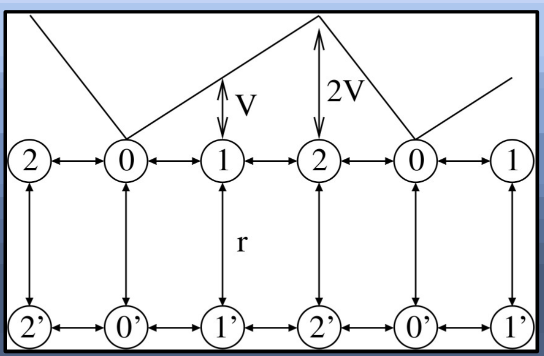
Full Information
x={0,1,2}
y={0,1}



$$\frac{\langle \dot{S} \rangle}{k} = d_2 = d_3 = d_4 = \dots$$

Markovian

Discrete ratchet: partial information

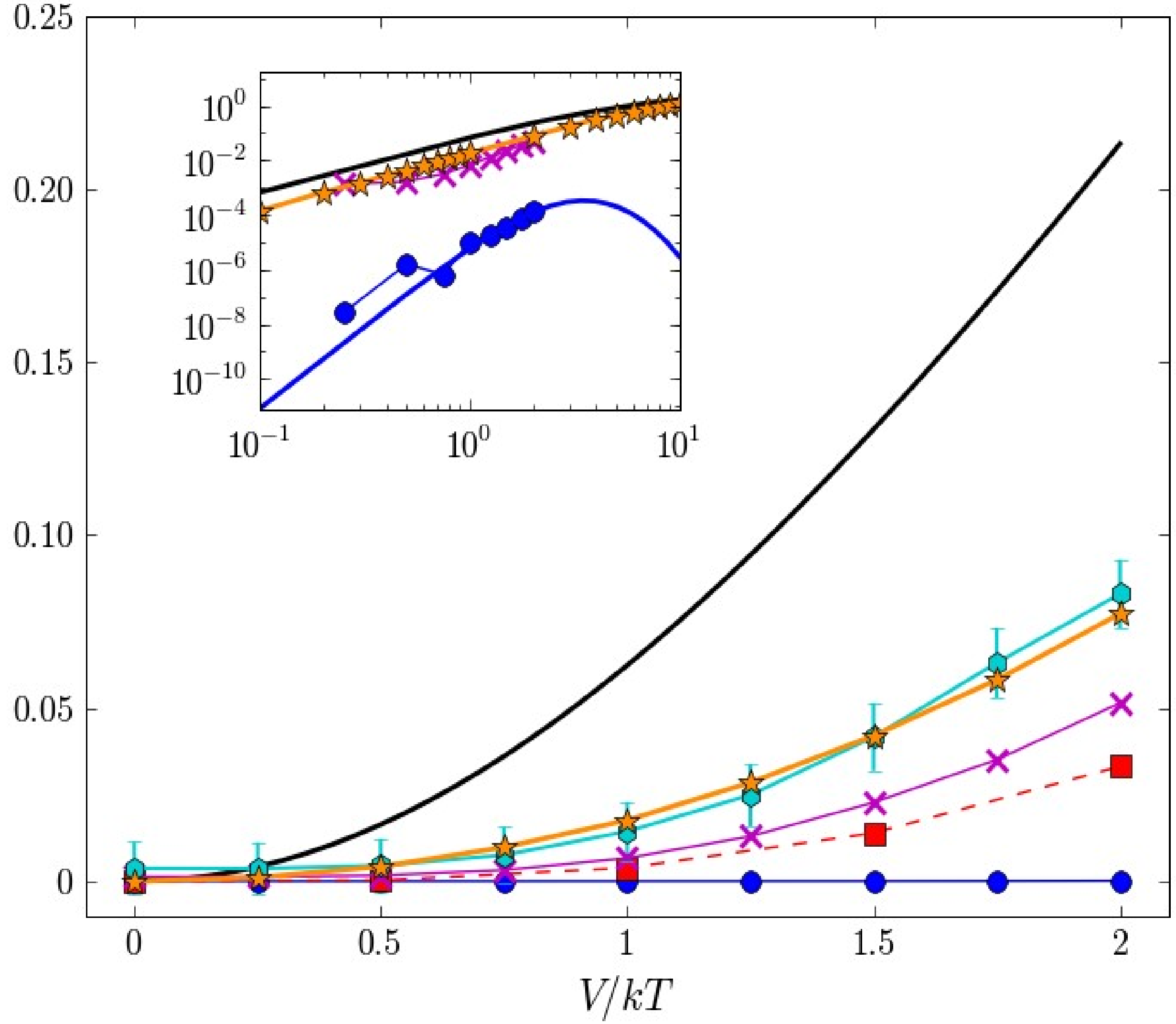


F=0
r finite

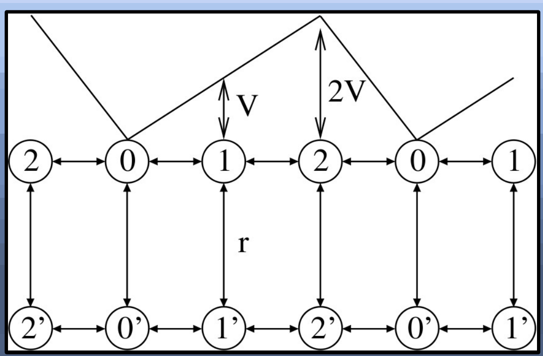
Partial Information
x={0,1,2}

$$d_k \equiv D_k - D_{k-1}$$

$$D_k = \sum_{x_1,\ldots,x_k} p(x_1,\ldots,x_k) \log \frac{p(x_1,\ldots,x_k)}{p(x_k,\ldots,x_1)}$$



Discrete ratchet: partial information



F=0
***r* infinite**

Partial Information
x={0,1,2}

$$J_{ij} \equiv p_{ij} - p_{ji}$$

currents

$$J_{ij} \ll p_{ij}$$

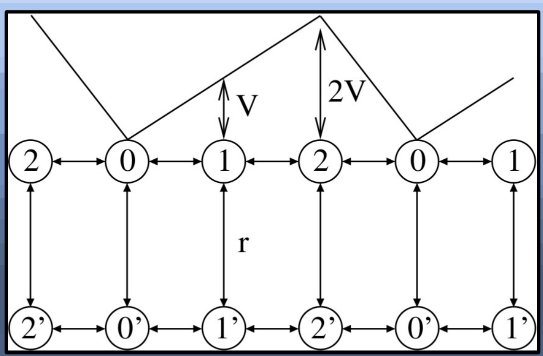
small current limit

$$d_2 \simeq \sum_{ij} \frac{(J_{ij})^2}{2p_{ij}} + O(J^3) \simeq \sum_{i<j} \frac{(J_{ij})^2}{p_{ij}} + O(J^3)$$

$$d_2 = d_3 = d_4 = \ldots = d = 0 < \frac{\langle \dot{S} \rangle}{k}$$

Markovianity is not enough!

Discrete ratchet: partial information



If $F=F_{stall}$, the current vanishes but the system is out of equilibrium

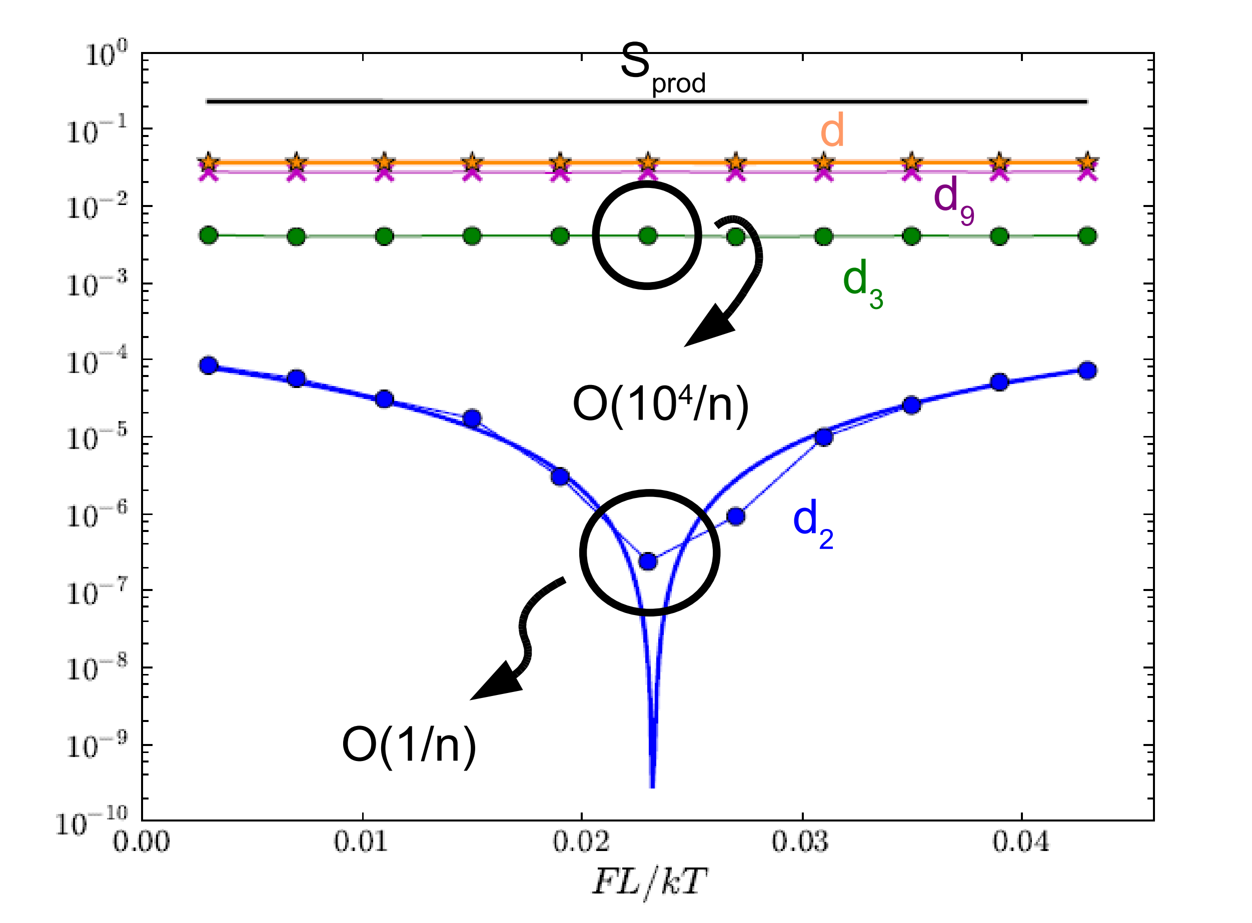
F=F

$F=F_{stall}$

***r* infinite**

Partial Information

x={0,1,2}



Three-time correlations (d_3)= A new footprint of irreversibility

Conclusion

- $d\,(p_F \,||\, p_B)$ proved to be a good estimator of S_{prod} even ignoring physical details of the system
- Brute force estimators >>> Ziv-Merhav estimator
- F=0 and r finite: The technique works qualitatively. Ansatz found.
- F=0 and r infinite: Markovianity is not enough
- $F=F_{stall}$ and r finite: Three-time correlations (d_3) capture the irreversibility where the current is zero.

Future work

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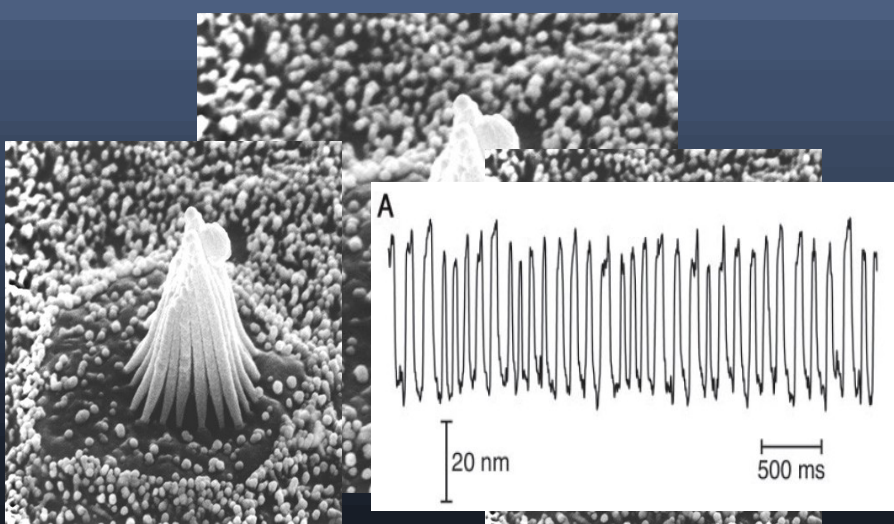
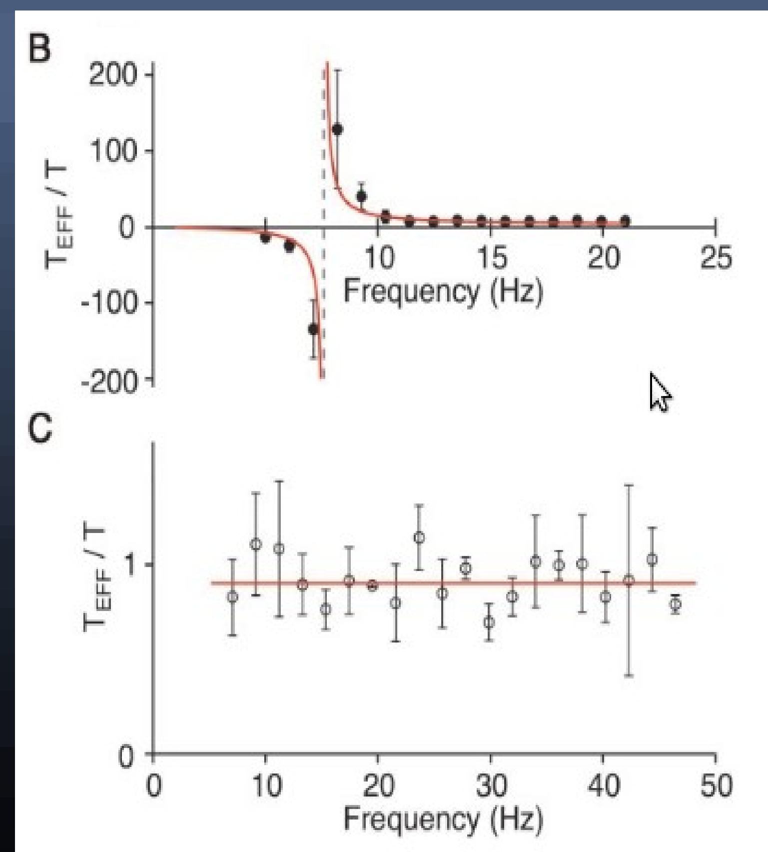
Max Planck Institute Für Physik Komplexer Systeme, Dresden (Germany)

Ear hair bundles



spontaneous oscillations

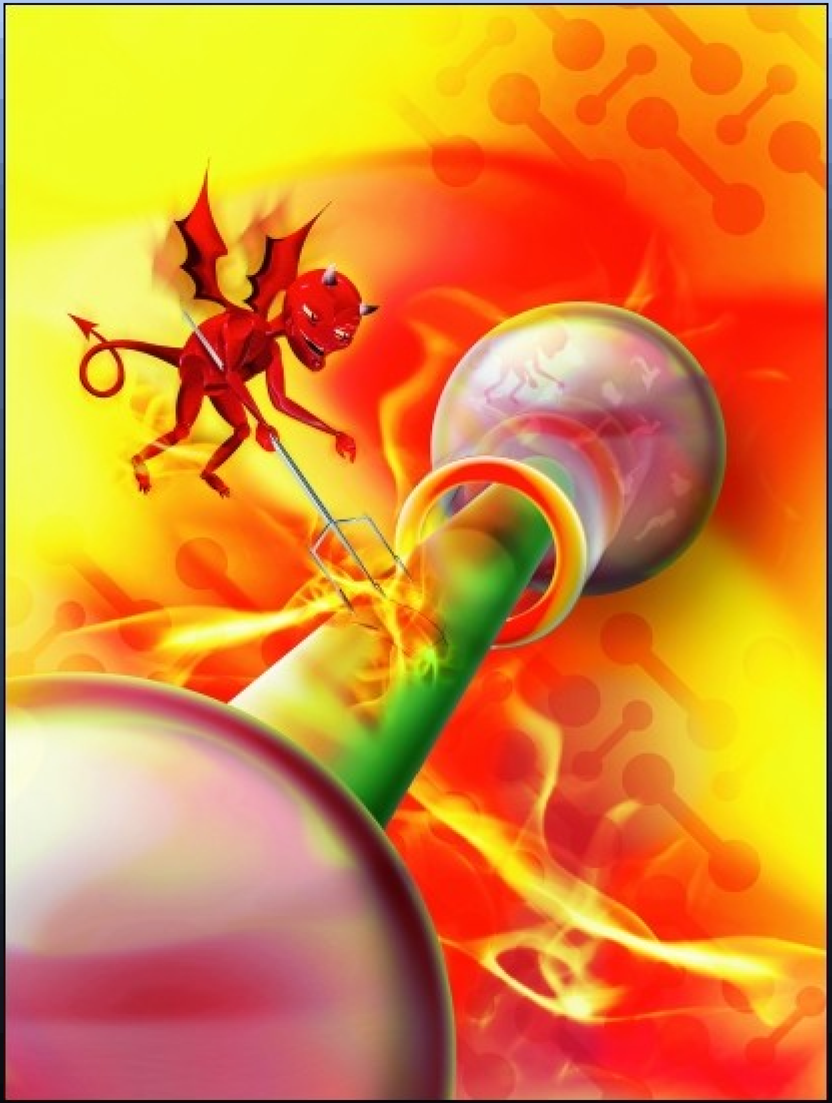
forced oscillations



Active cells

Passive cells

P. Martin, A.J. Hudspeth, and F. Jülicher, PNAS **98**, 14380 (2001).



Thanks for your attention