There is no such thing as network reciprocity when human subjects play a Prisoner's Dilemma

Anxo Sánchez

Grupo Interdisciplinar de Sistemas Complejos
Departamento de Matemáticas,
Universidad Carlos III de Madrid

Instituto de Biocomputación y Física de Sistemas Complejos (BIFI), Universidad de Zaragoza

el dilema del PRISIONERO

Why study cooperation?

- Theoretically, cooperation is an evolutionary puzzle

Why study cooperation?

- Cooperation is a key player in the major transitions of evolution (Szathmary & Maynard-Smith, 1995)

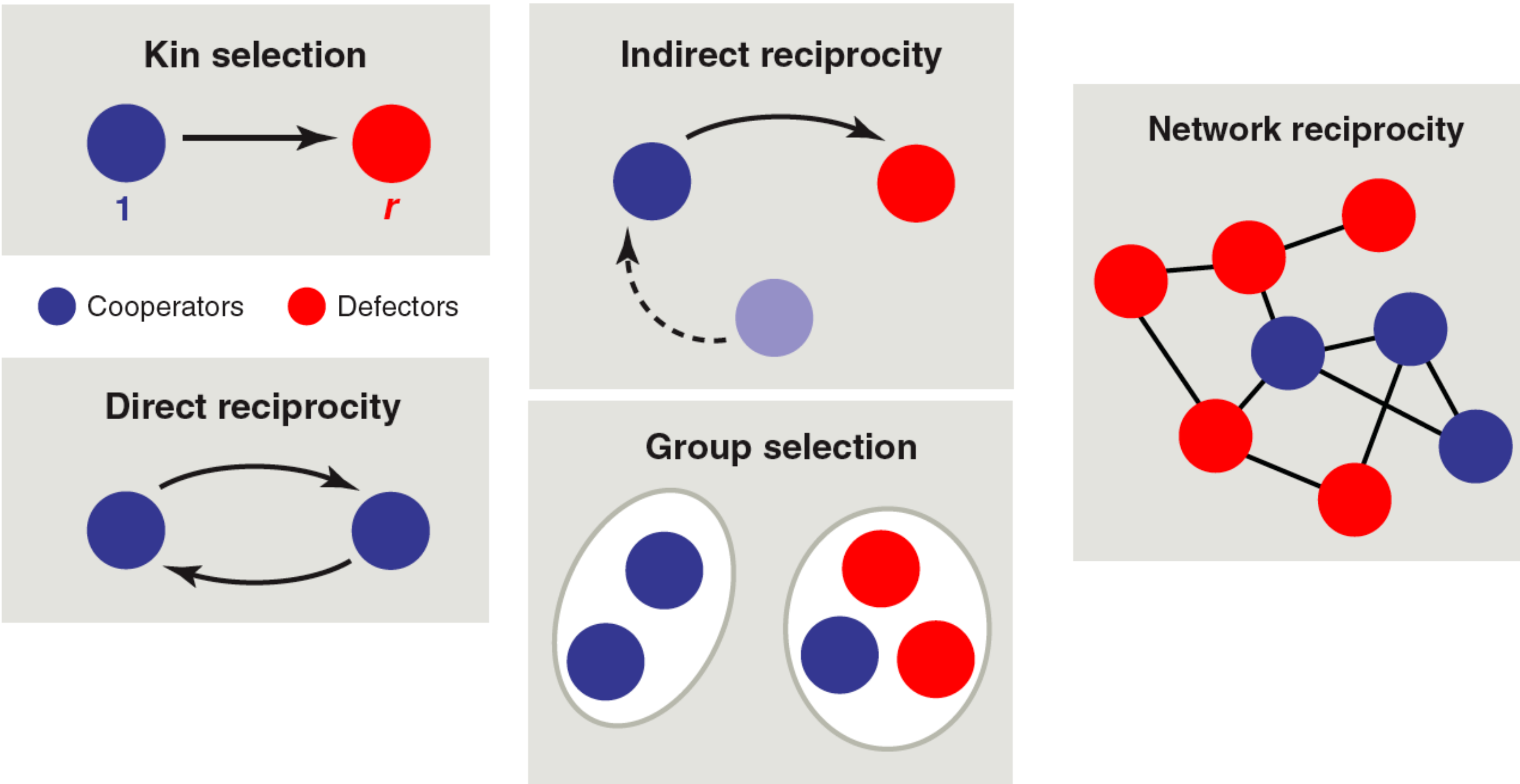
Replicating molecules → Populations of molecules
Independent replicators → Chromosomes
RNA → DNA
Prokaryotes → Eukaryotes
Asexual clones → Sexual populations
Protists → Animals, plants, fungi
Solitary individuals → Colonies
Primate societies → Human societies, **Language**

Why study cooperation?

- Collaborative endeavors are ubiquitous in human society

Five rules for the evolution of cooperation

Martin A. Nowak, *Science* **314**, 1560 (1992)



Prisoner's dilemma

- 2 players
- 2 actions: Cooperate or Defect

	C	D
C	1	S
D	T	0

$T > 1$: temptation to defect

$S < 0$: risk in cooperation

Evolutionary games

Well mixed population:

Individuals reproduce according to payoff accumulated after a round of games with everybody else

Replicator equation

$$\frac{\dot{x}_c}{x_c} = f_c - \bar{f} \quad \text{or} \quad \dot{x}_c = x_c(1 - x_c)(f_c - f_d)$$

Attractors are ESS which generally coincide with Nash equilibria

Prisoner's dilemma: convergence to full defection

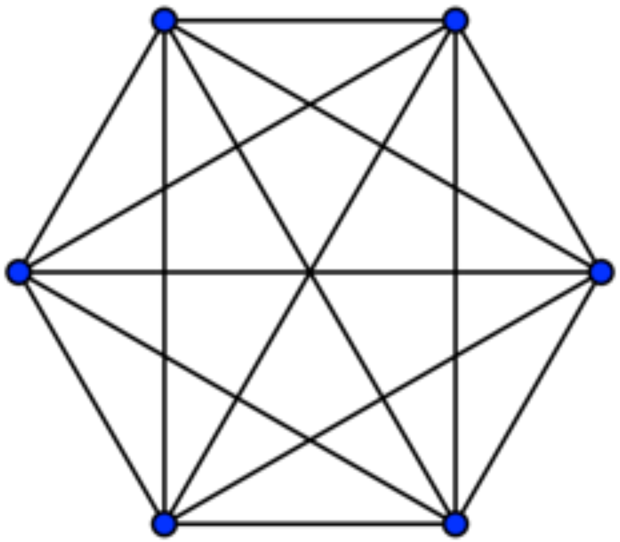
Network reciprocity

Martin A. Nowak and Robert M. May,
Nature **359**, 826 (1992)

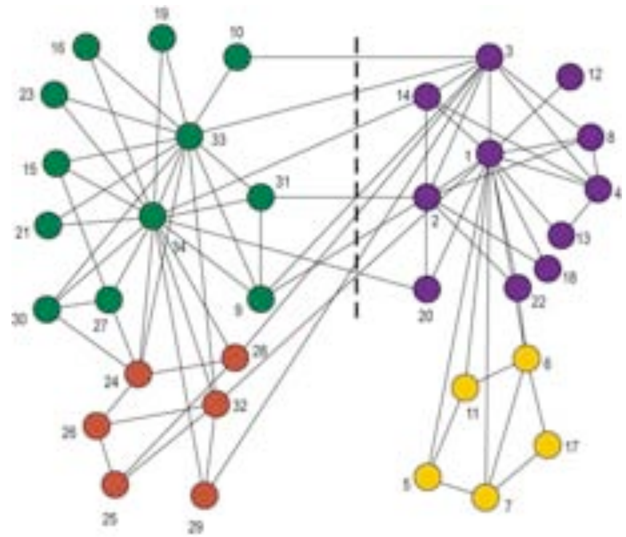
Spatial structure promotes cooperation in the Prisoner's Dilemma

Evolutionary games on networks

- Population structure: each player plays *only* with her neighbors, determined by the network



Well mixed / Complete graph



Social network

G. Szabó and G. Fáth, Evolutionary games on graphs, *Phys. Rep.* **446**, 97 (2007)

Evolutionary games on networks

- Population structure: each player plays *only* with her neighbors, determined by the network

Lattice

Inhomogeneous

G. Szabó and G. Fáth, Evolutionary games on graphs, *Phys. Rep.* **446**, 97 (2007)

Evolution: Update rule

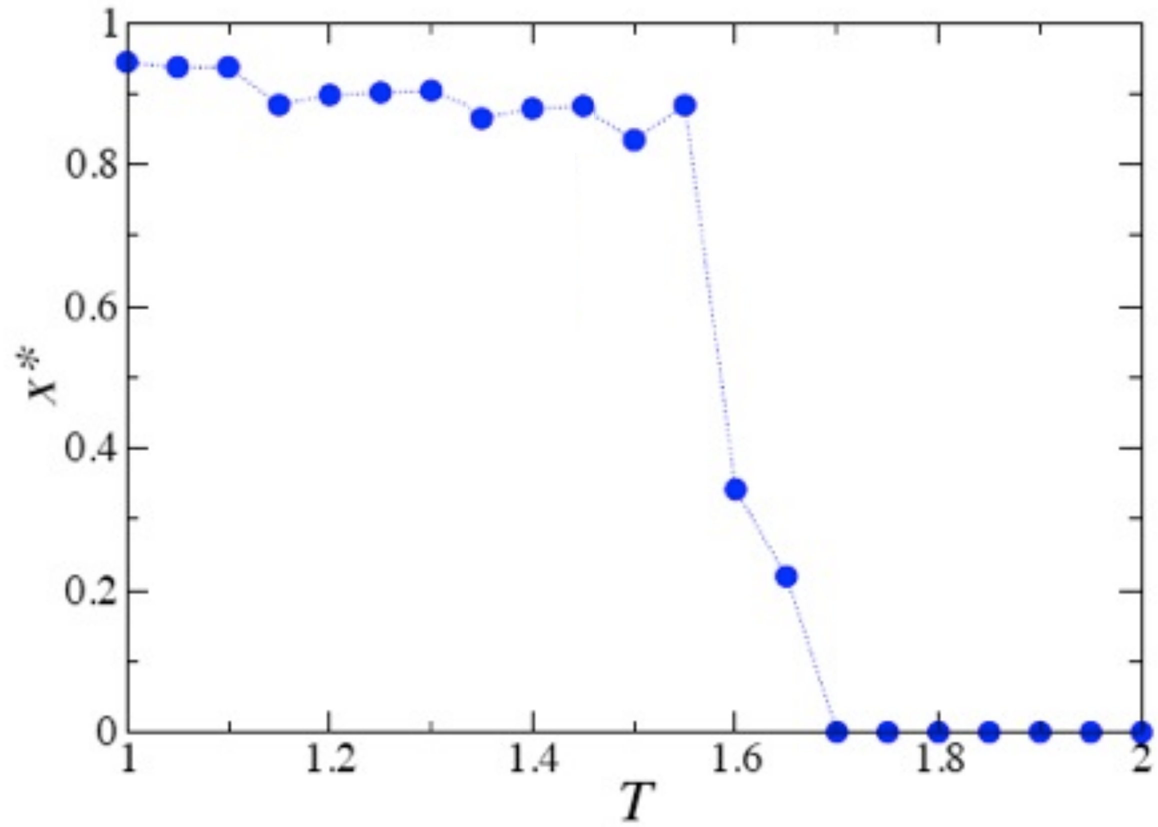
- Unconditional imitation: choose the strategy of the neighbor with the largest payoff if larger than yours

Seminal result on spatial structure

M. A. Nowak & R. M. May, *Nature* **359**, 826 (1992)

$$\begin{array}{ccc} & \text{C} & \text{D} \\ \text{C} & \multicolumn{2}{c}{\multirow{2}{*}{$\begin{pmatrix} 1 & 0 \\ T & \epsilon \end{pmatrix}$}} \\ \text{D} & & \end{array}$$

$$1 \leq T \leq 2 \qquad \epsilon \lesssim 0$$



Evolution: Update rules

- Unconditional imitation: choose the strategy of the neighbor with the largest payoff if larger than yours
- Proportional update:

$$\mathcal{P}\{s_i^{t+1} \leftarrow s_j^t\} = \begin{cases} (\pi_j^t - \pi_i^t)/\Phi\,, & \pi_j^t > \pi_i^t\,, \\ 0\,, & \pi_j^t \leq \pi_i^t\,, \end{cases}$$

Leads to replicator dynamics on the complete graph

- Best response: choose the strategy that would have yielded the largest payoff given the neighbors' strategies
- Pairwise comparison: $$\mathcal{P}\{s_i^{t+1} \leftarrow s_j^t\} = \frac{1}{1 + \exp\left(-\beta\left(\pi_j^t - \pi_i^t\right)\right)}$$

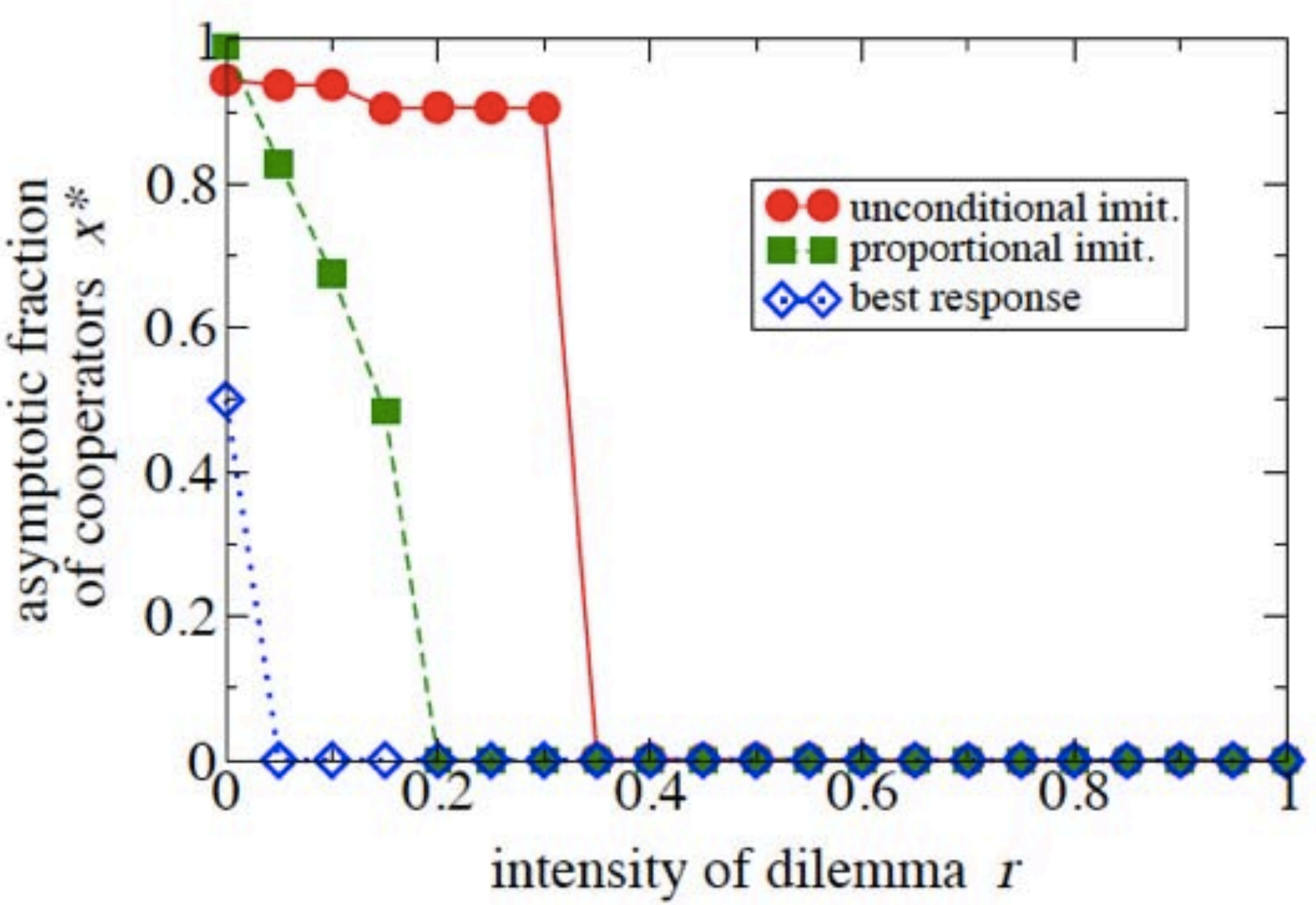
Note: All but BR based on payoff differences

Remark

Update rules are, in fact, strategies

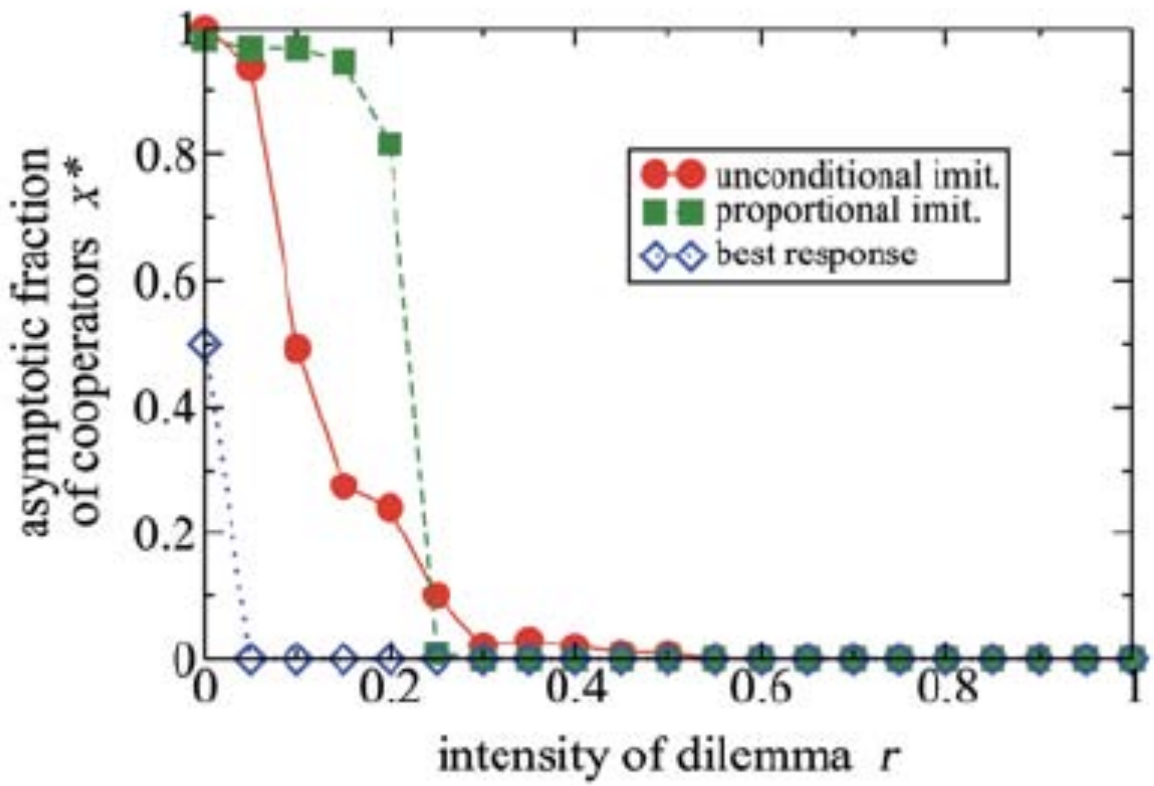
Strategies not to be confused with actions

Effect of the update rules



(Note different parametrization)

Effect of the type of network

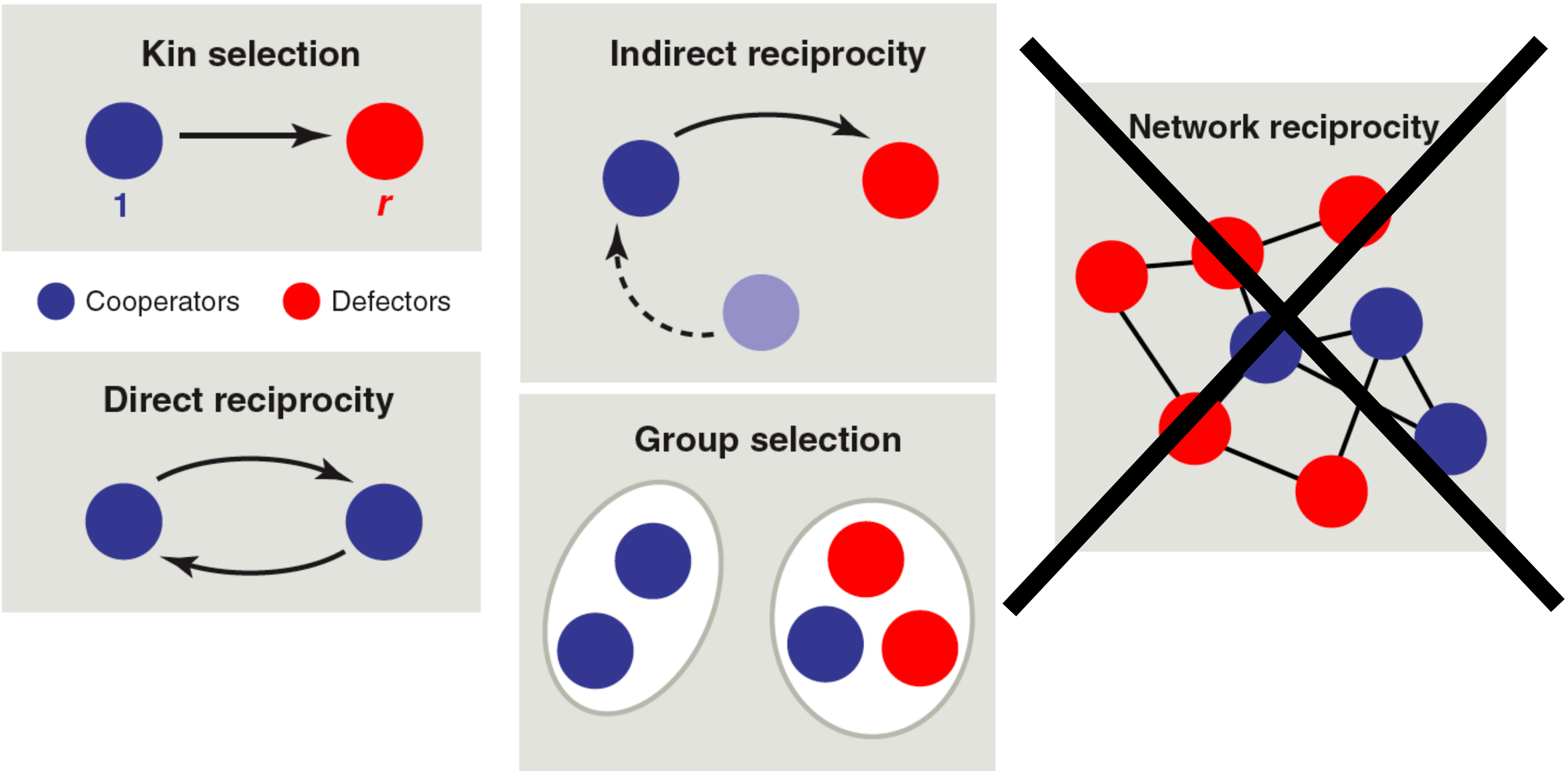


Barabási-Albert scale-free network

cf. F. Santos, J. M. Pacheco, T. Lenaerts, *Proc. Natl. Acad. Sci. USA* **103**, 3490 (2006)

Four! ~~Five~~ rules for the evolution of cooperation

Martin A. Nowak, *Science* **314**, 1560 (1992)

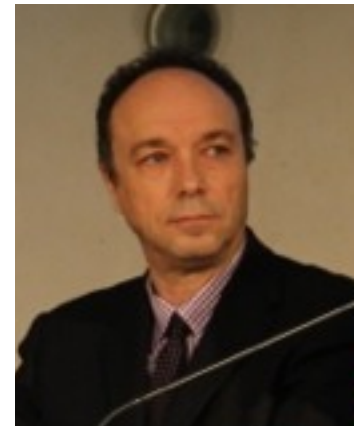


Thank you very much for your attention

Work done in collaboration with

José A. Cuesta

Jelena Grujić



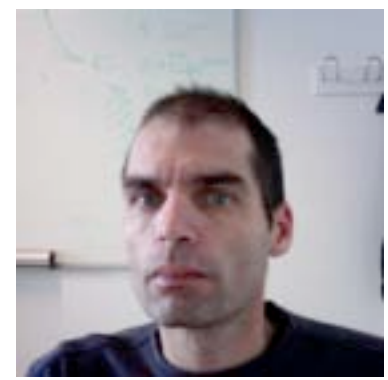
Alfonso Tarancón

Burcu Eke

Economics

Antonio Cabrales

Yamir Moreno



Carlos Gracia-Lázaro

Work done in collaboration with

Carlos P. Roca, José A. Cuesta, A.S., *Phys. Life Rev.* **6**, 208 (2009)

Evolutionary games on graphs are largely "everything-dependent"

Which update rules are relevant for modeling real human behavior?

How do humans behave?

Experiments!

Theory!

I

“The largest experiment ever”

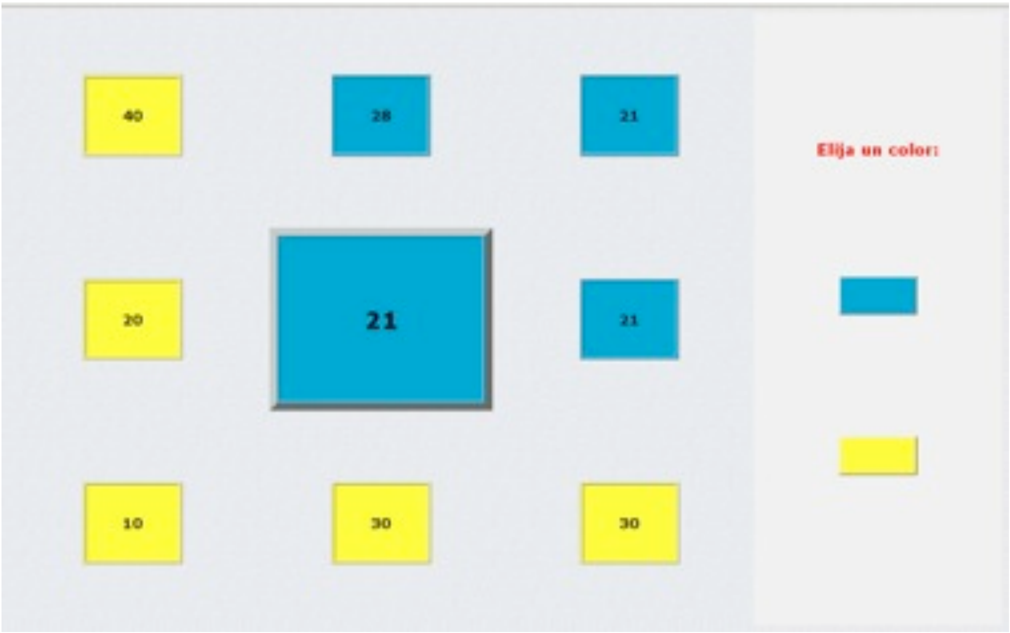
J. Grujić, “Prisoner’s Dilemma experiment”, Workshop GISC 2010

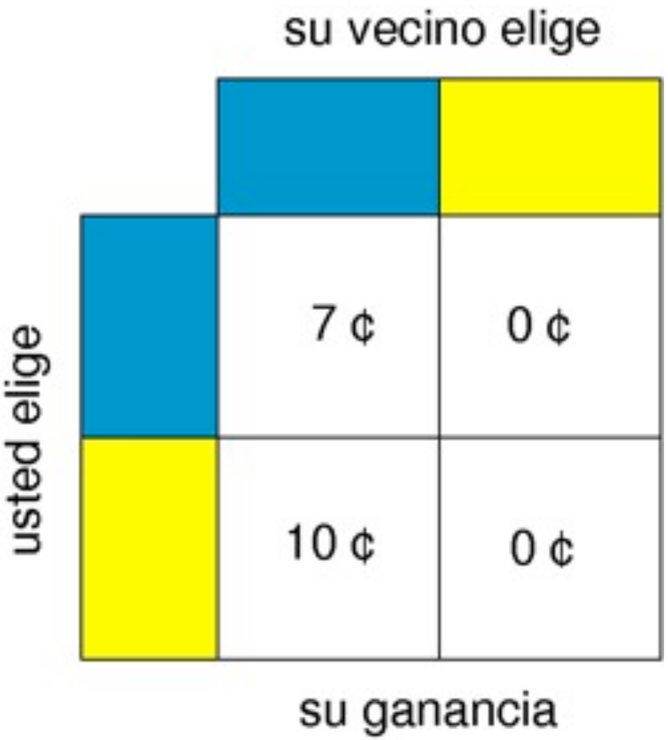
J. Grujić, C. Fosco, L. Araujo, J. A. Cuesta, A. S., *PLoS ONE* **5**, e13749 (2010)

Large-scale experiment

Lattice 13x13, periodic boundary conditions

Login → Tutorial → Experiment → Control → Experiment → Questionnaire





Large-scale experiment

169 players

Undergraduate students UC3M, Campus Leganes

40% female, 60% male

20 assistants,10 computer rooms, 2 buildings

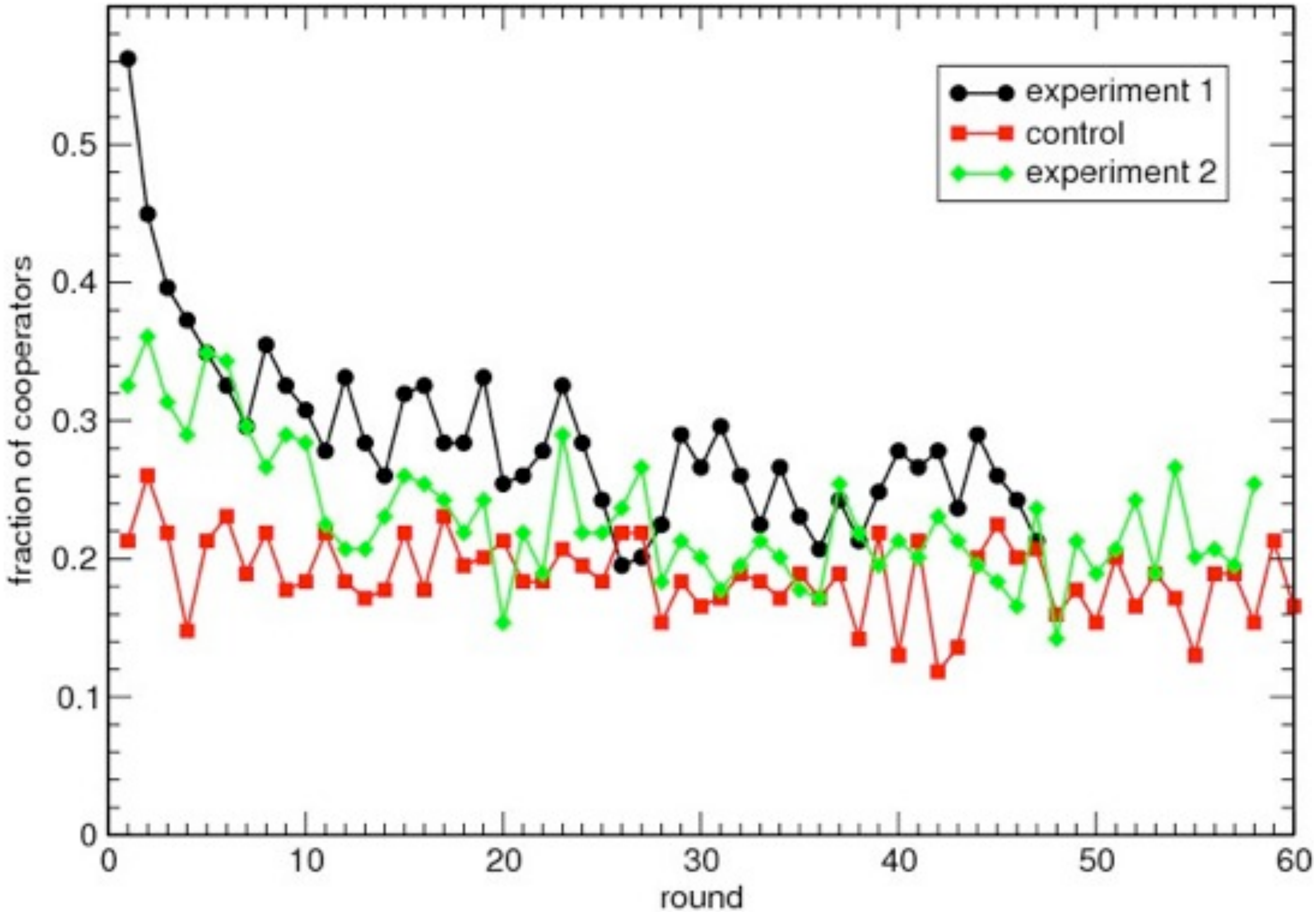
From 9 AM till 9 PM

About 8 000 €

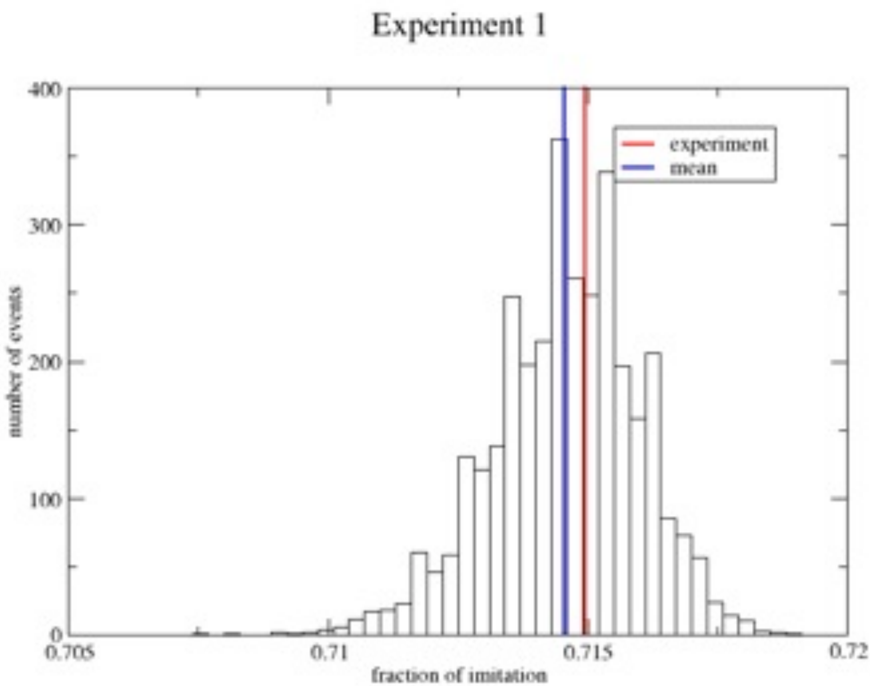
Carried out on April 25, 2009; largest size ever tried to date

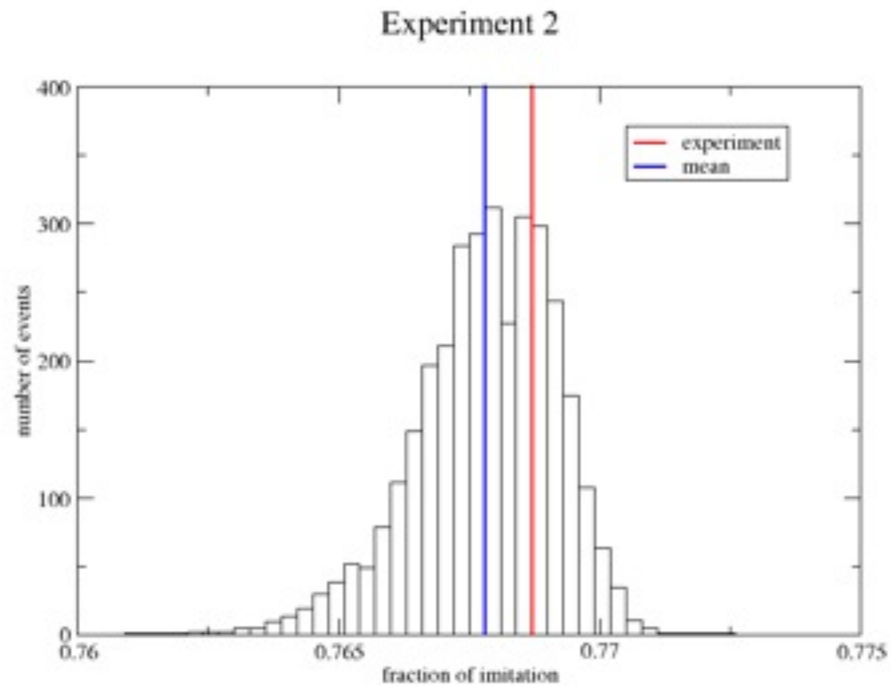
Experimental setup

Results: percentage of cooperation



Discussion: unconditional imitation



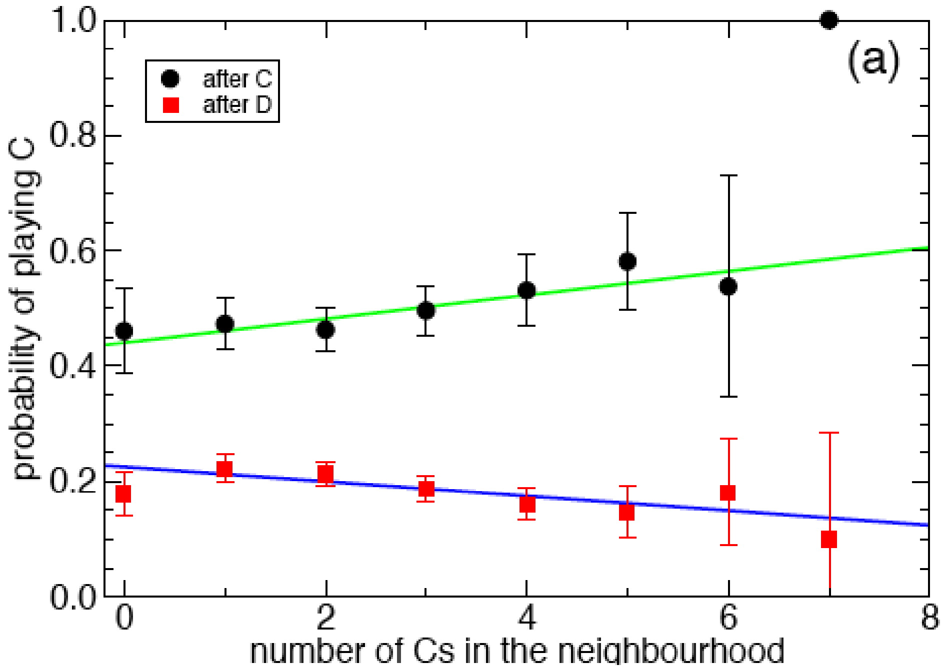


experimental = 0.7149
random = 0.7145 ± 0.0014
p-test = 0.425

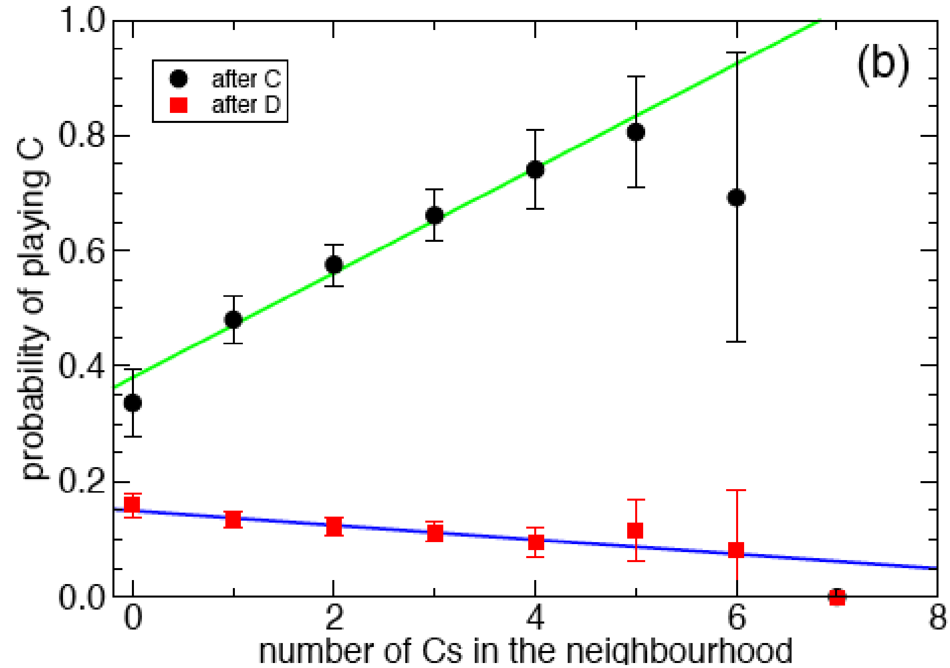
experimental = 0.7687
random = 0.7678 ± 0.0013
p-test = 0.282

Discussion: conditional cooperation

CC in public goods: Fischbacher, Gächter and Fehr, *Econ. Lett.* **71**, 397 (2001)



Experiment 1



Experiment 2

Conditional cooperation is ‘moody’!

Modeling: inhomogeneous agents

Type of player	Experiment 1	Control	Experiment 2
Pure defector	22	44	39
Mostly defector	38	40	37
Pure cooperator	2	1	6
Mostly cooperator	1	2	3
The Rest	106	82	84

Our model:

Pure defectors – always defect

Mostly defectors – D with P > 2/3

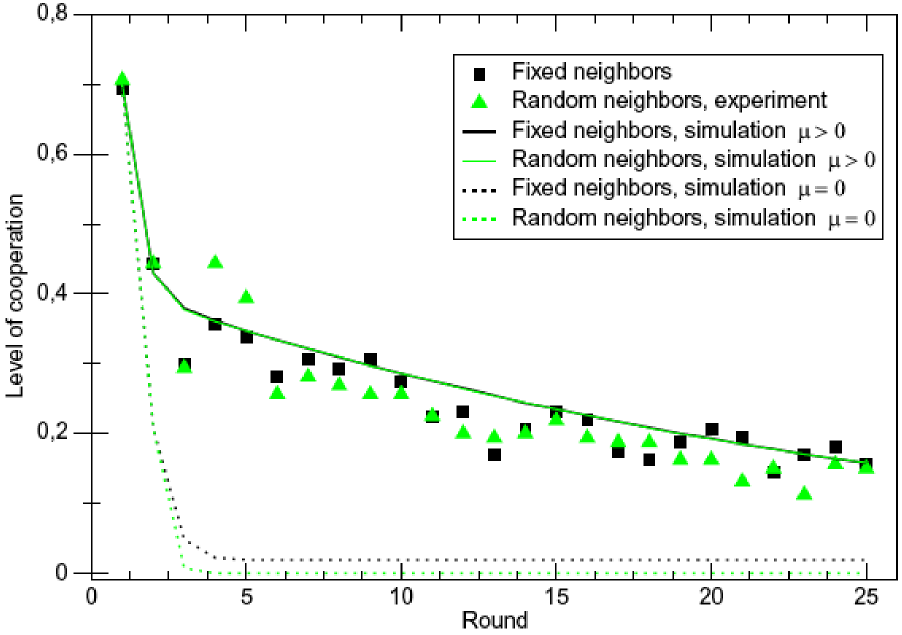
Pure cooperators – always cooperate

Mostly cooperators – C with P > 2/3

The rest – Moody conditional cooperators (aggregate)

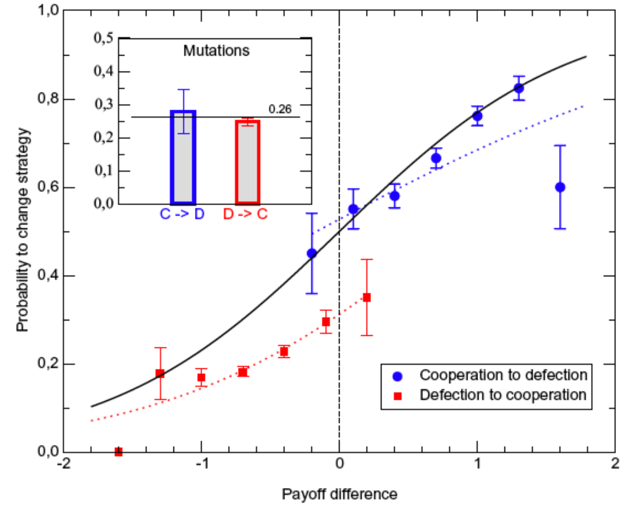
Agent-based model reproduces all features of the experiment!

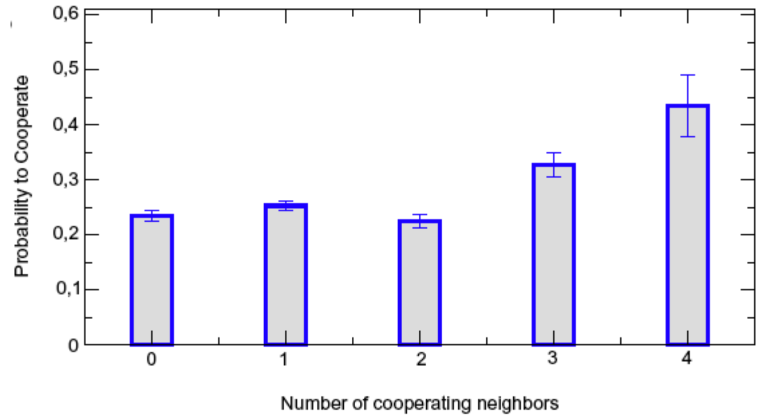
Discussion: other experiments



A. Traulsen, D. Semmann, R. D. Sommerfeld, H.-J. Krambeck & M. Milinski, *Proc. Natl. Acad. Sci. USA* **107**, 2962 (2010)

4x4 lattice, 4 neighbors, true Prisoner's Dilemma





Re-analysis of their data confirms moody conditional cooperation!

J. Grujić, A. Traulsen, M. Milinski, J. A. Cuesta, A. S., in preparation

Consequences for modeling

Get real

PLoS ONE doi:10.1371/journal.pone.0013749 (2010)

It's a dilemma — cooperate with a neighbour, or defect? Which course gives the better return? This common problem in game theory, known as the prisoner's dilemma, has formed the basis of many simulations of behaviour, including those for which a spatial dimension is added, with the 'players' distributed on a grid pattern.

But for Jelena Grujić and colleagues, the players are real. In their experiments, 169 volunteers, arranged as a 13 × 13 grid, played multiple rounds of the prisoner's dilemma game, each player interacting (virtually) with eight nearest neighbours. The first study of this size on real participants, it throws up some interesting features, particularly that behaviour turns out to be much more 'contextual' than has been seen in simulations — that is, many players were influenced by how much cooperation they observed, or by how they themselves behaved, in a previous round. At the same time, another significant fraction of the players created 'heterogeneity' in the system, by sticking round after round to the same strategy, be it defecting or cooperating.

Existing models do not describe well the results of this human study — results that should, say the authors, be a reference for more accurate modelling to come.

research highlights NATURE PHYSICS | VOL 7 | JANUARY 2011 | www.nature.com/naturephysics

II

"The coexistence theory"

J. Grujić, "Replicator dynamics for the iterated prisoner's dilemma with three types of playerst", Workshop GISC 2010

J. Grujić, J. A. Cuesta, A. S., submitted

Evolutionary explanation of coexistence

Three strategies:

- Pure cooperators (with probability p to make a mistake)
- Pure defectors (with probability p to make a mistake)
- Moody conditional cooperators
  - ★ Cooperate with probability q if defected in the previous round
  - ★ Otherwise, cooperate with probability $p_C(x) = (1-x)p_0 + xp_1$
    x being the density of cooperating neighbors

Interaction through infinitely iterated Prisoner's Dilemma

Dynamics: replicator equation

$$\frac{\dot{x}_c}{x_c} = f_c - \bar{f} \quad \Rightarrow \quad \dot{x}_i = x_i[(Wx)_i - x \cdot Wx]$$

Replicator dynamics

Strategies are random, so calculation is as follows (2 players):

Markov chain for pairs of **actions** occuring for pairs of **strategies**:

CC → CC, CC → CD, ...

Stationary probabilities

$$\pi = \pi M$$

Payoffs for the pairs of **strategies**

$$w_{ij} = R\pi_{\mathrm{CC}} + S\pi_{\mathrm{CD}} + T\pi_{\mathrm{DC}} + P\pi_{\mathrm{DD}}$$

$$\dot{x}_i = x_i[(Wx)_i - x \cdot Wx]$$

Replicator dynamics

Study of the replicator equation

$$\dot{x}_i = x_i[(Wx)_i - x \cdot Wx]$$

↓

Rest points

↓

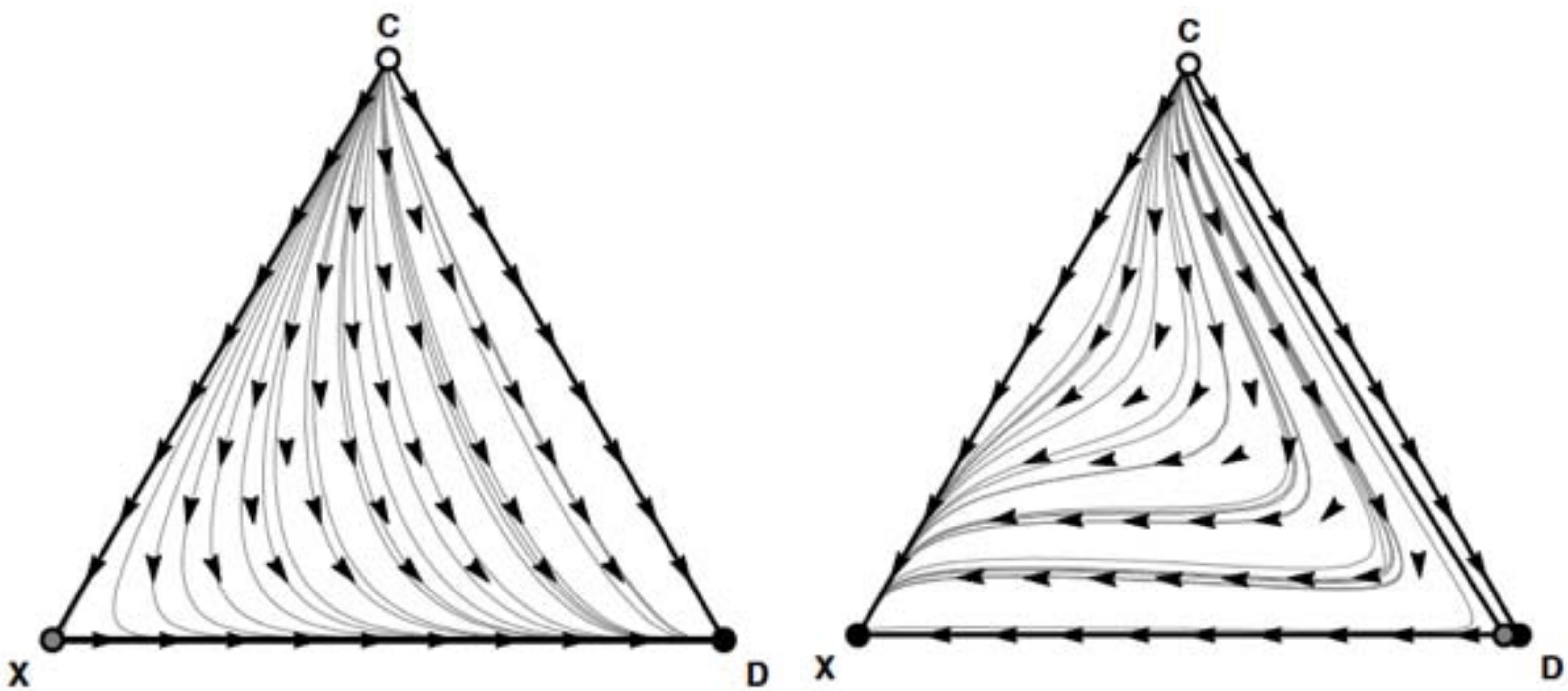
Phase portrait

↓

Keep in mind: results depend on the strategy parameters: p, q, p_{c0}, p_{c1}

Replicator dynamics

Two-player IPD: results for experimental parameters



No coexistence!

Replicator dynamics

Two-player IPD: results for parameters close to experiment

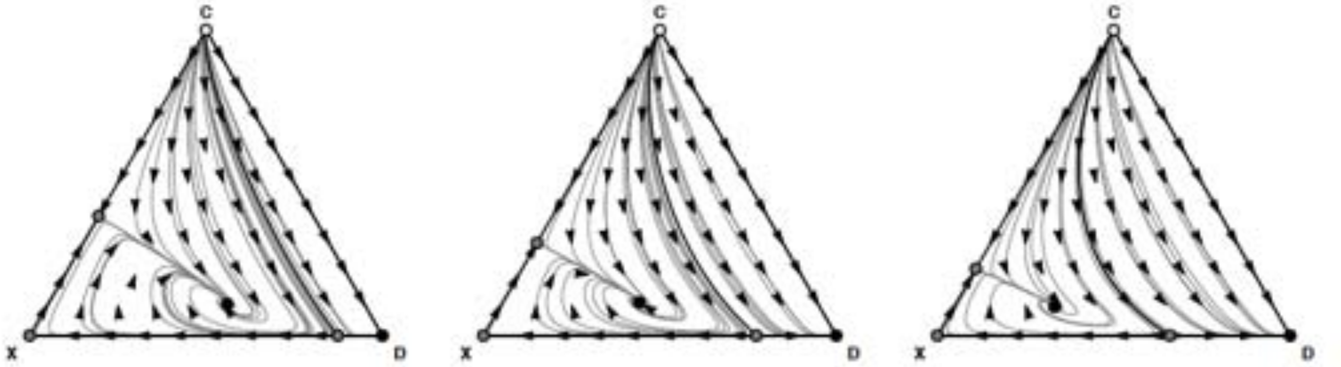


Figure 5: Phase portraits of the replicator dynamics for 2-players IMPD games with three strategies (C, D, and X) for different values of p_0: 0.20 (left), 0.40 (middle), and 0.50 (right). Other parameters: $p = 0.83$, $q = 0.2$, $p_1 = 0.8$. Rest points marked in the plot can be repellors (white), saddle points (grey) or attractors (black). In all three cases the inner point as well as the $x_D = 1$ point are the only attractors of the system.

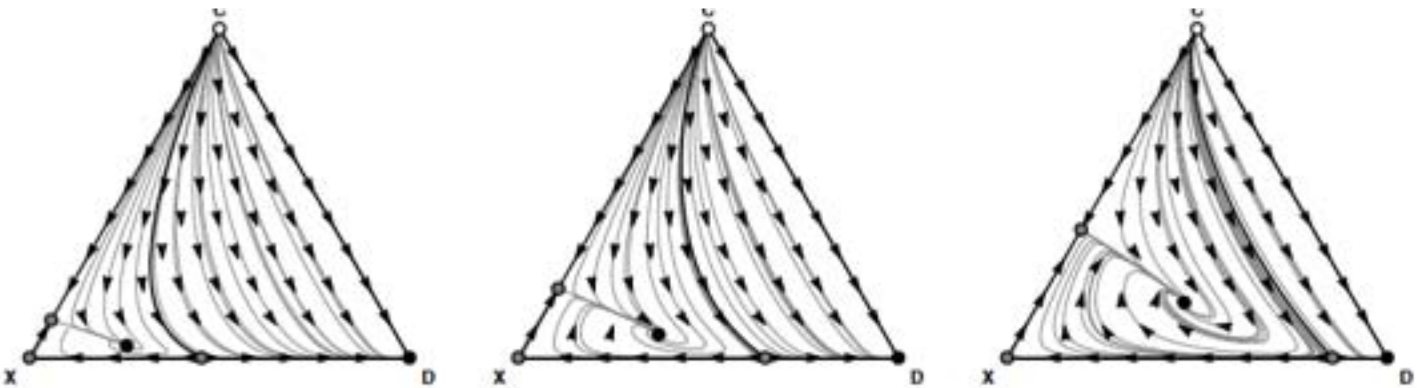
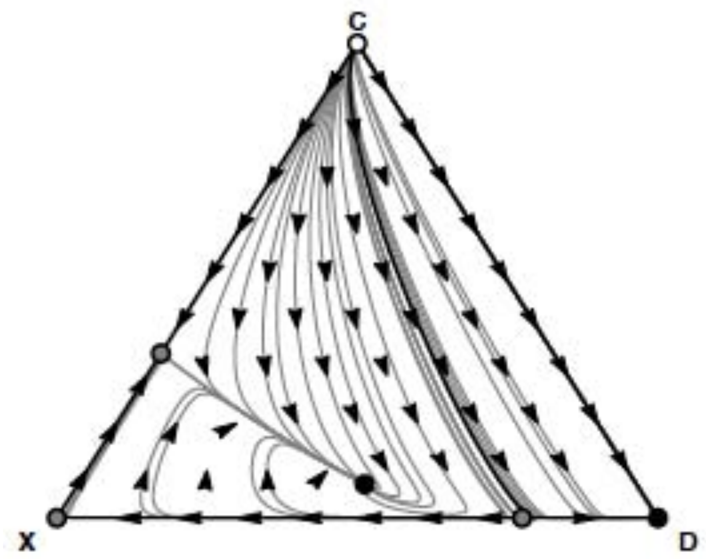


Figure 6: Phase portraits of the replicator dynamics for 2-players IMPD games with three strategies (C, D, and X) for different values of p_1: 0.70 (left), 0.75 (middle), and 0.85 (right). Other parameters: $p = 0.83$, $q = 0.2$, $p_0 = 0.4$. Rest points marked in the plot can be repellors (white), saddle points (grey) or attractors (black). In all three cases the inner point as well as the $x_D = 1$ point are the only attractors of the system.

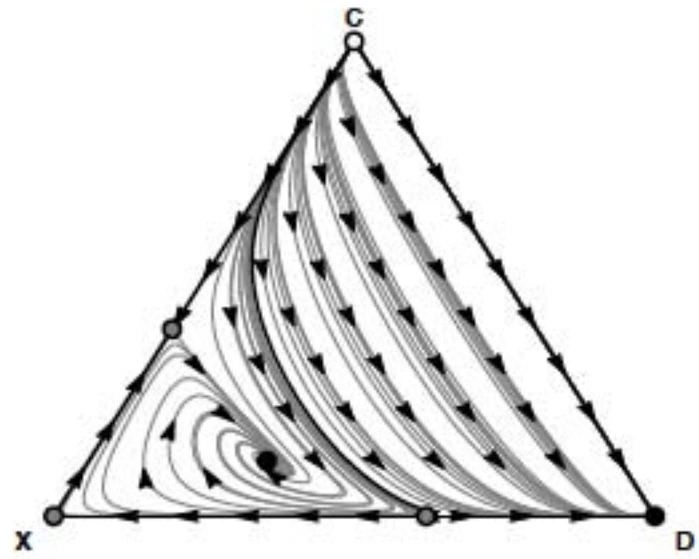
Coexistence possible!

Dependence on the number of players

Three players



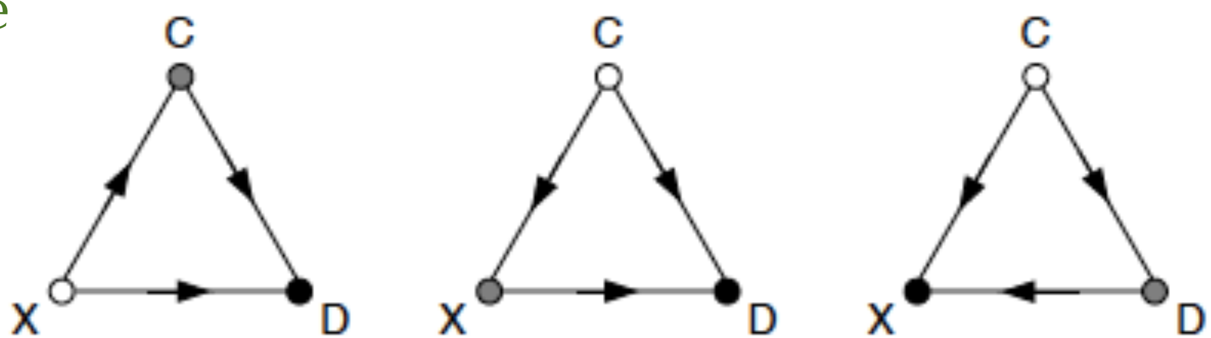
Four players



No coexistence for five players...

Infinite number of players

Theorem: the only possible phase portraits for an infinite number of players are



Proof (sketch)

$$\Pr\{k_C, k_D, k_X | l_C, l_D, l_X\} = \binom{n_C}{k_C}\binom{n_D}{k_D} p^{n_D - k_D + k_C}(1-p)^{n_C - k_C + k_D}$$
$$\times \sum_{j=0}^{k_X} \binom{l_X}{j}\binom{n_X - l_X}{k_X - j} p_C(x)^j [1 - p_C(x)]^{l_X - j}$$
$$\times q^{k_X - j}(1-q)^{n_X - l_X - k_X + j}, \qquad x \equiv \frac{l_C + l_D + l_X - 1}{n-1}$$

$$n \to \infty$$

$$\kappa \equiv p x_C + (1-p) x_D + r_X x_X$$

$$W_C(x) = p\kappa R + p(1-\kappa)S + (1-p)\kappa T + (1-p)(1-\kappa)P,$$
$$W_D(x) = (1-p)\kappa R + (1-p)(1-\kappa)S + p\kappa T + p(1-\kappa)P,$$
$$W_X(x) = r_X \kappa R + r_X(1-\kappa)S + (1-r_X)\kappa T + (1-r_X)(1-\kappa)P.$$

Replicator dynamics does not explain the observed results

III

“Get real”

J. Grujić, B. Eke, A. Cabrales, J. A. Cuesta, A. S., in preparation

Experiments on the number of players

Definite prediction on the lack of coexistence

Experiments on size dependence

Iterated Multi-Player (2, 3, 4, 5) Prisoner's Dilemma

Planned 10 groups per size, 228 subjects in total showed up

Payoffs to obtain similar earnings in all sizes

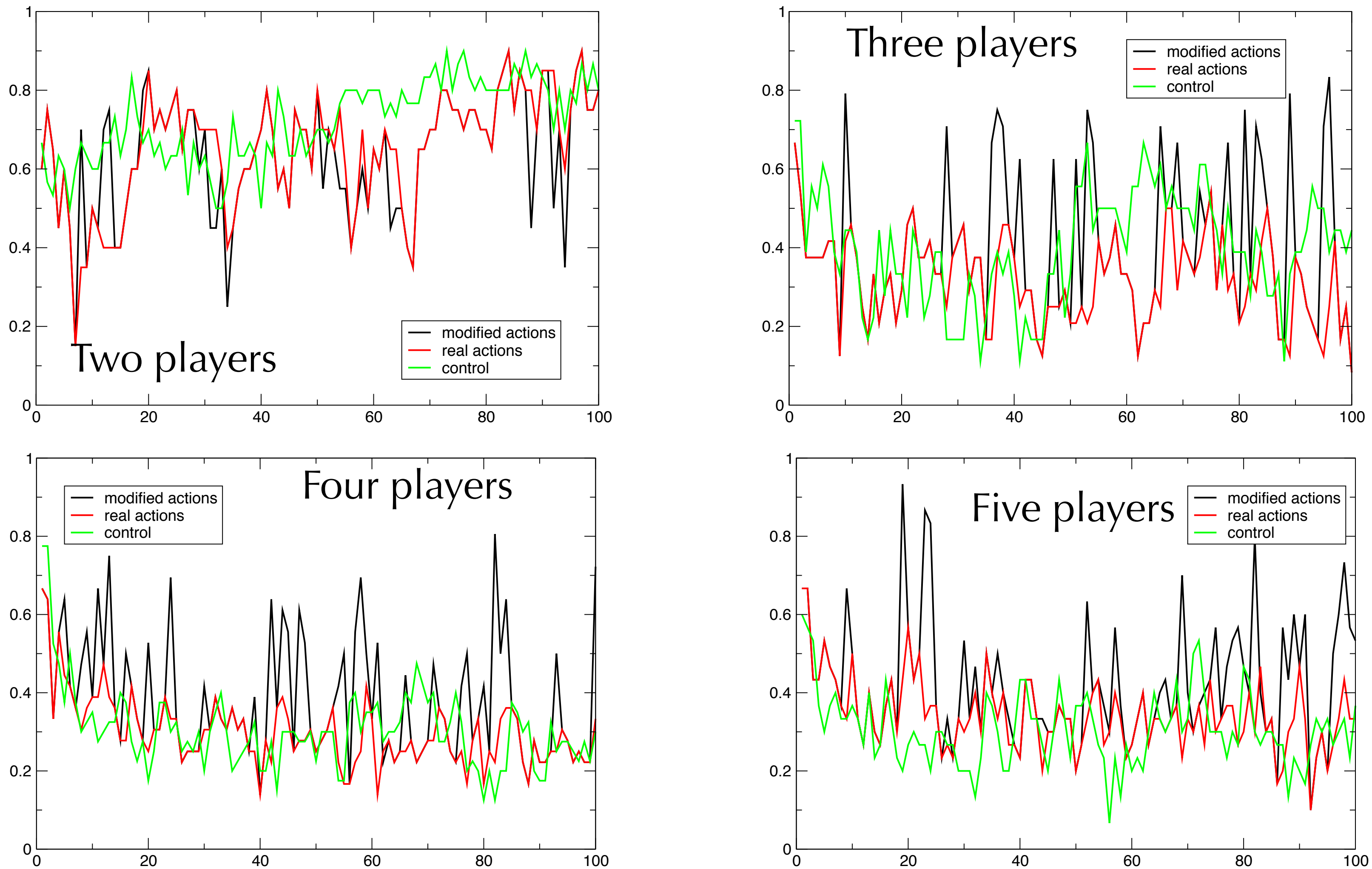
Computer intervention in half of the groups to explore higher cooperative contexts better

1 assistants,1 computer rooms, 1 building

45 min sessions at 10h, 12h, 14h and 16h, 3 days after Easter 2011

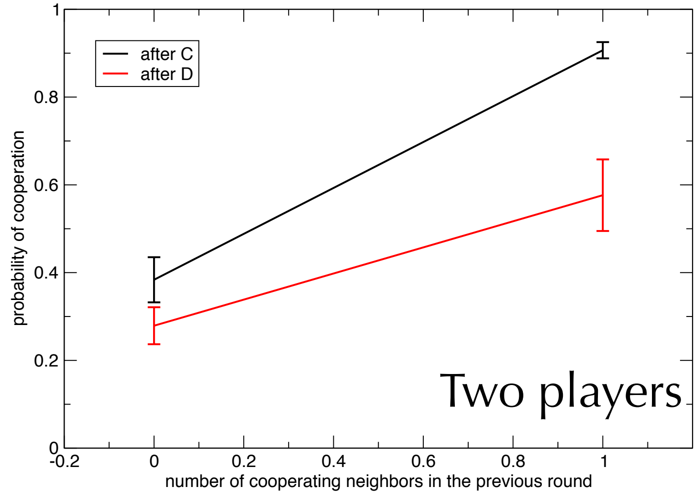
About 5 000 €

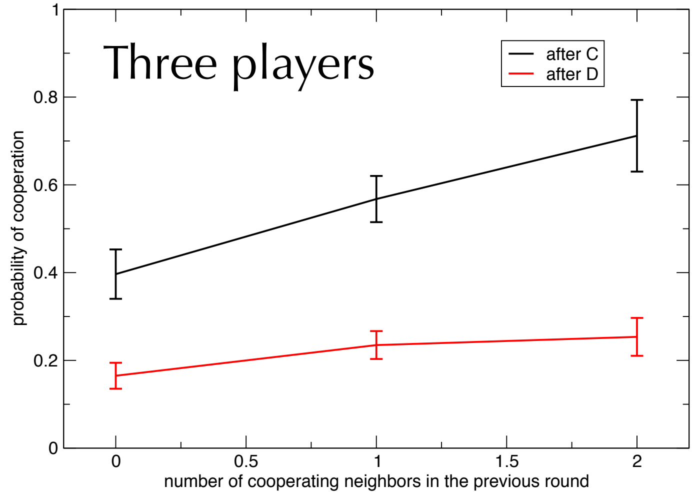
Results: percentage of cooperation

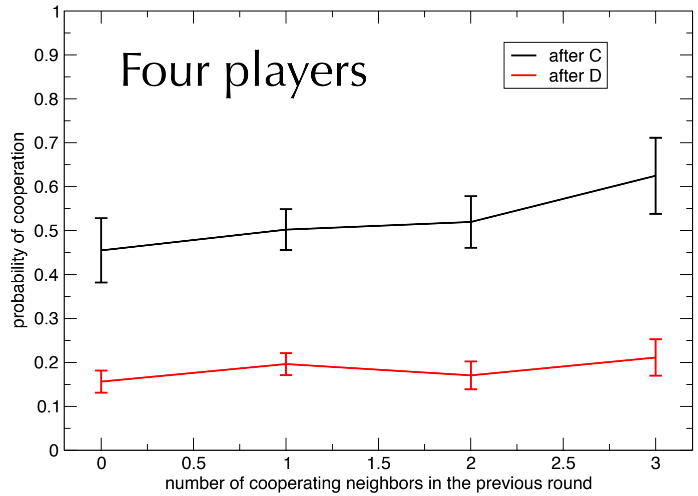


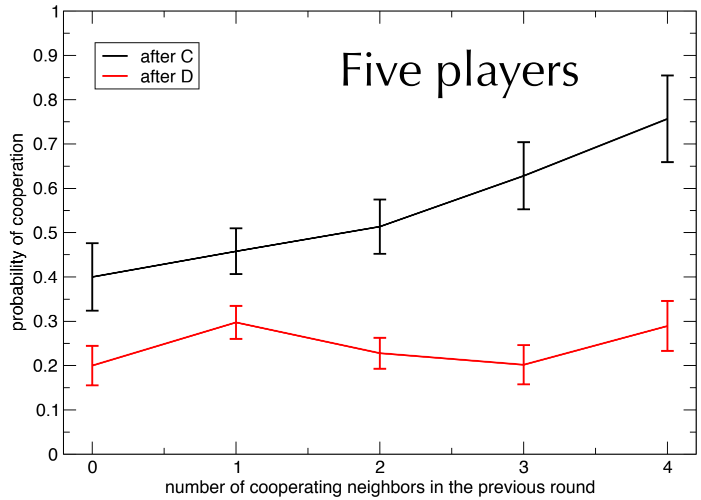
Size 2 different from the others

Results: moody conditional cooperation









Analysis: Generalized Linear Mixed Model

Let $\mathbf{y}$ be an $(n \times 1)$ vector of binary responses. Let $\boldsymbol{\alpha}$ be an $(r \times 1)$ vector of random effects. Then the generalized liner mixed model (GLMM) assume that $E[\mathbf{y}|\boldsymbol{\alpha}] = g^{-1}(\mathbf{x}\boldsymbol{\beta} + \mathbf{z}\boldsymbol{\alpha})$ where $\mathbf{x}$ is the matrix of covariates, $\mathbf{z}$ is a design matrix for the random effects, and $g(.)$ is a differentiable monotonic link function and $g^{-1}()$ is its inverse. The linear predictor then becomes $\eta = \mathbf{x}\boldsymbol{\beta} + \mathbf{z}\boldsymbol{\alpha}$. In the case of binary response, the most common link function is the logit function. That is, $\mathrm{logit}(\eta) = \ln \frac{\eta}{1-\eta}$, and

$$\begin{aligned} \mathrm{logit}^{-1}(\eta) &= \frac{\exp(\eta)}{1+\exp(\eta)} \\ &= \frac{\exp(\mathbf{x}\boldsymbol{\beta} + \mathbf{z}\boldsymbol{\alpha})}{1+\exp(\mathbf{x}\boldsymbol{\beta} + \mathbf{z}\boldsymbol{\alpha})} \end{aligned}$$

Incorporates time dependence through an AR(1) process

Analysis: Our Model

$$\begin{aligned} \mathrm{logit}(p_{ijt}) &= \sum_{l=1}^{4} \beta_l \chi(\mathrm{size}_{il}) + \sum_{l=1}^{4} \beta_{4+l} \left(\mathrm{LagCoop} \times \chi(\mathrm{size}_{il})\right) + \\ &+ \sum_{l=1}^{2} \beta_{8+l} \chi(\mathrm{major}_{li}) + \beta_{11} \mathrm{genderf}_i + \sum_{l=1}^{5} \alpha_{il} \chi(q = l)_i \end{aligned}$$

size_{il} accounts for the size of the group in which the subject played

LagCoop accounts for the fractions of cooperators in the previous round

major_{il} accounts for the subject's studies (economics/engineering)

genderf accounts for the gender

α_{il} accounts for the cooperativeness of each subject

Analysis: Our Model

Table 2: Results for the Parameter Estimates

Effect	Estimate	p−value
$\chi(\text{size}_2)$	0.5512	0.0576
$\chi(\text{size}_3)$	-0.7964	0.0066
$\chi(\text{size}_4)$	-0.8287	0.0017
$\chi(\text{size}_5)$	-1.0401	0.0007
(LagCoop $\times$ $\chi(\text{size}_2)$)	1.0264	<0.0001
(LagCoop $\times$ $\chi(\text{size}_3)$)	0.4828	<0.0001
(LagCoop $\times$ $\chi(\text{size}_4)$)	0.2681	<0.0001
(LagCoop $\times$ $\chi(\text{size}_5)$)	0.2239	<0.0001
genderf	-0.3064	0.0784
$\chi(\text{major}_1)$	-0.6624	0.0091
$\chi(\text{major}_2)$	-0.2798	0.3285
Type 3 Tests		
	F−value	p−value
Size	14.45	<0.0001
LagCoop $\times$ size	184.16	<0.0001
genderf	3.13	0.0784
major	3.62	0.0285
Contrast Analysis for Size		
	t-value	Bonferroni adjusted p-value
2 vs. 3	25.27	<0.0001
2 vs. 4	34.26	<0.0001
2 vs. 5	26.11	<0.0001
3 vs. 4	0.02	0.8955
3 vs. 5	0.57	0.4529
4 vs. 5	0.62	0.4314

Discussion of the experiment

- Moody conditional cooperation confirmed
- People does behave as though groups consisted either of two or many opponents: connection with explanation of coexistence
- GLMM shows in addition that humans are similar in reciprocity (slope of the cooperation probability) but diverse in generosity (initial propensity to cooperate)

IV

“Back to modeling”

C. Gracia-Lázaro, J. A. Cuesta, A. S., Y. Moreno, submitted

Mean field theory

Mean field approach for the stationary state (different from previous analysis, no dynamics)

P(A) is the cooperation probability for strategy *A=C, D, X*

$$\langle c\rangle = \rho_C P(C) + \rho_D P(D) + \rho_X P(X)$$

$$P_{t+1}(X) = (d+\gamma\langle c\rangle_t)P_t(X)+(a+\beta\langle c\rangle_t)[1-P_t(X)]$$

$$\lim_{t\to\infty} P_t(X) = P(X), \qquad \lim_{t\to\infty}\langle c\rangle_t = \langle c\rangle$$

Therefore, for a given value of mean cooperation $\langle c\rangle = c_0$

$$\boxed{A\rho_C + B\rho_D = 1}$$

Results

Comparison with simulations

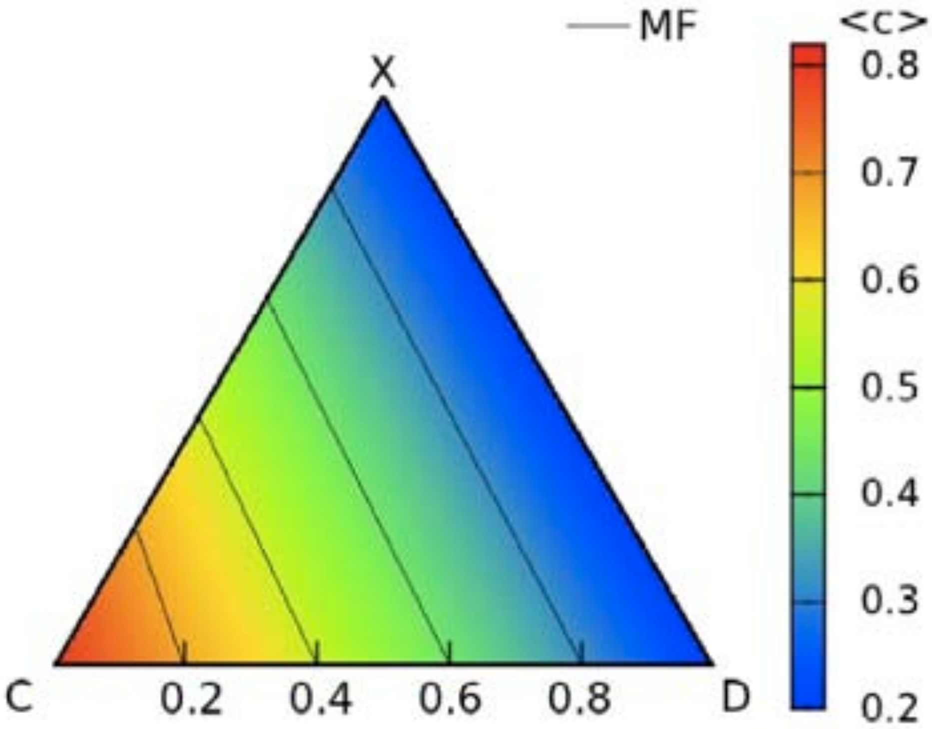
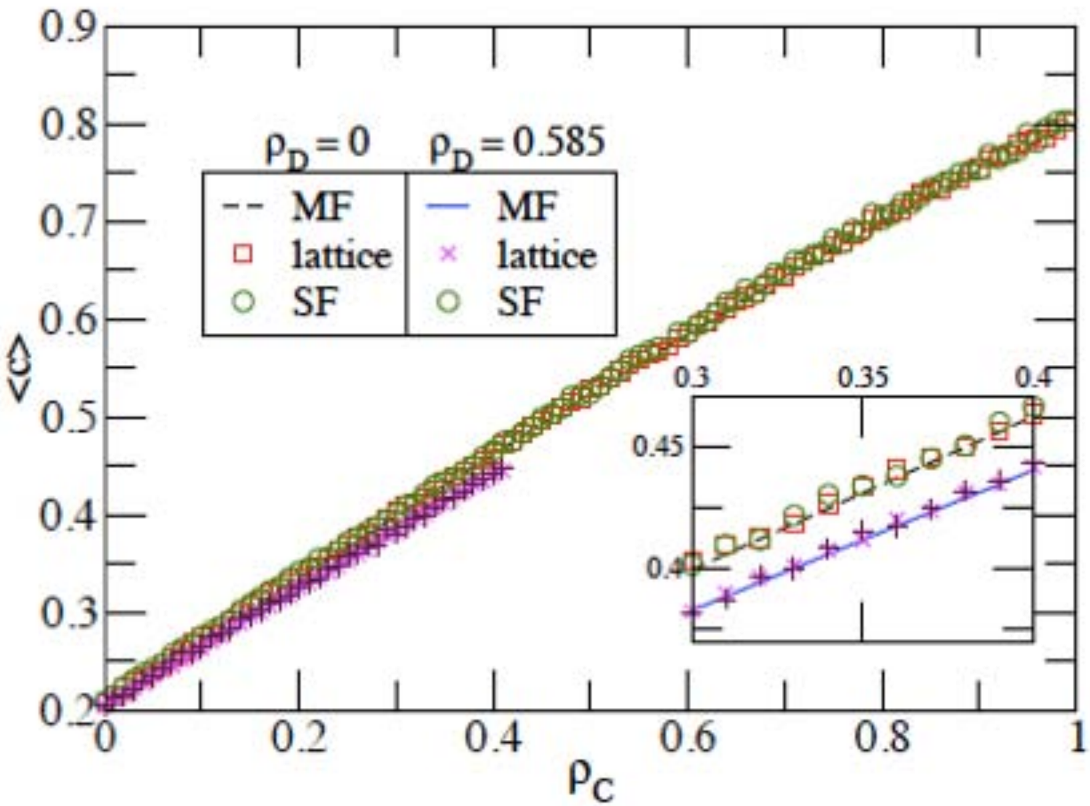


FIG. 1. (color online) Density plot of the average level of cooperation in the stationary state, $\langle c\rangle$, as a function of the fractions of the three strategies (mostly cooperators, C, mostly defectors, D, and moody conditional cooperators, X). The plot corresponds to a Barabási-Albert network of contacts ($\langle k\rangle = 6$), but the corresponding plot for an Erdős-Renyi graph or a regular lattice is indistinguishable from this one. The system is made up of $N = 10^4$ players and the rest of parameters, taken from [14], are: $d = 0.38$, $a = 0.15$, $\gamma = 0.62$, $\beta = -0.1$. The thin lines represent the mean-field estimations [c.f. Eq. (5)] for $\langle c\rangle = 0.32, 0.44, 0.56, 0.68$. They very accurately match the contour lines of the density plot corresponding to those values of $\langle c\rangle$, thus proving that the same outcome is obtained in a complete graph (mean-field). Simulation results have been averaged over 200 realizations.

Results

Comparison with simulations



Discussion

- Mean-field agrees with the cooperation level on lattices or on scale-free networks
- Makes sense as payoffs are not important for the action update anymore (note action, not strategy)
- Connection with previous results on Best Response dynamics
  [C. P. Roca, J. A. Cuesta, A. S., *Eur. Phys. J. B* **71**, 587 (2009)]
- Prediction is empirically testable

V

"Not again!"

C. Gracia-Lázaro, A. Tarancón, J. A. Cuesta, A. S., Y. Moreno, in preparation

Large-scale experiment

1129 players (625, lattice; 604, heterogeneous)

Last year high school students

44% male, 56% female

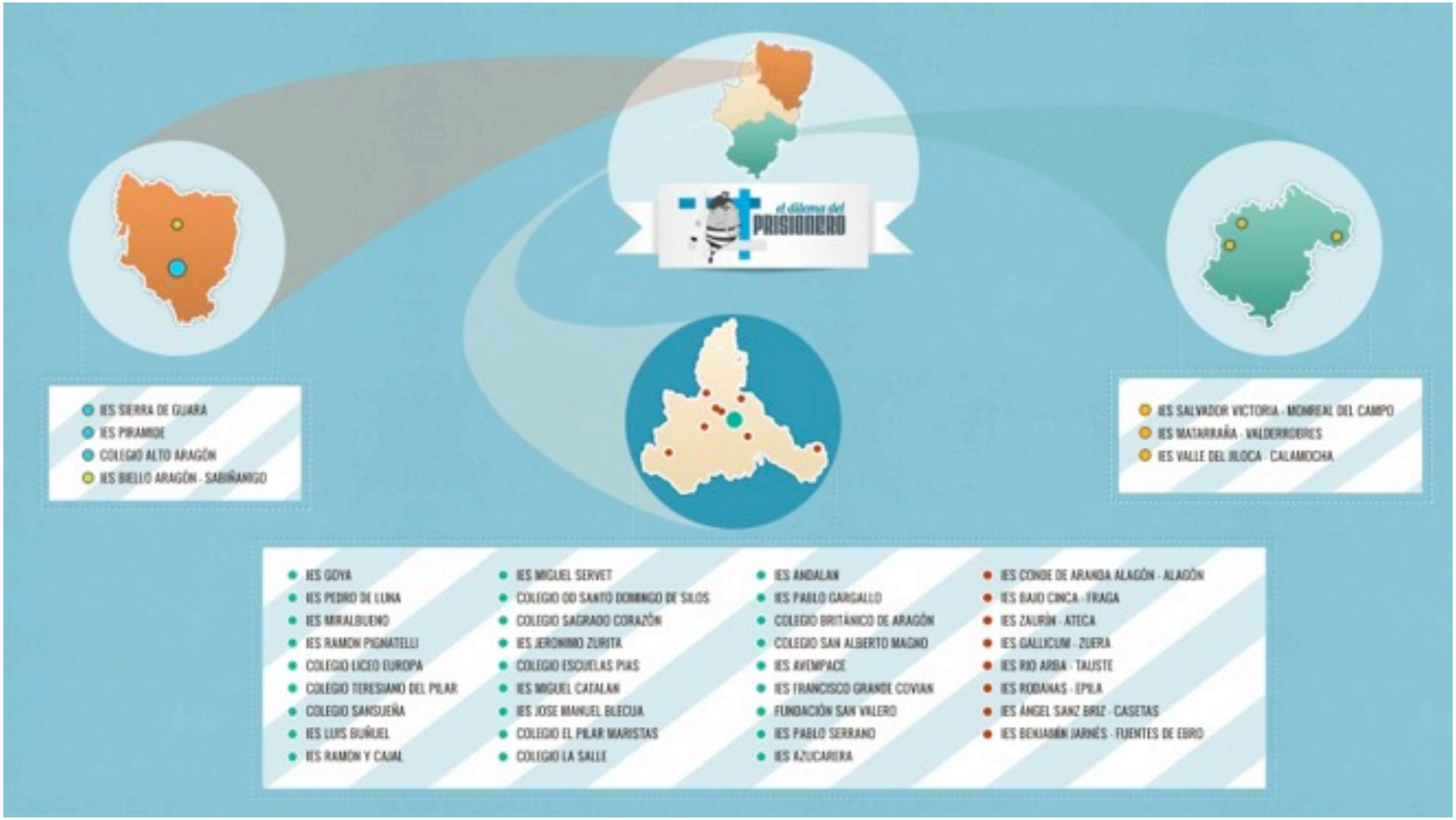
42 high schools in Aragón

From 10 AM till noon

About 10 000 €

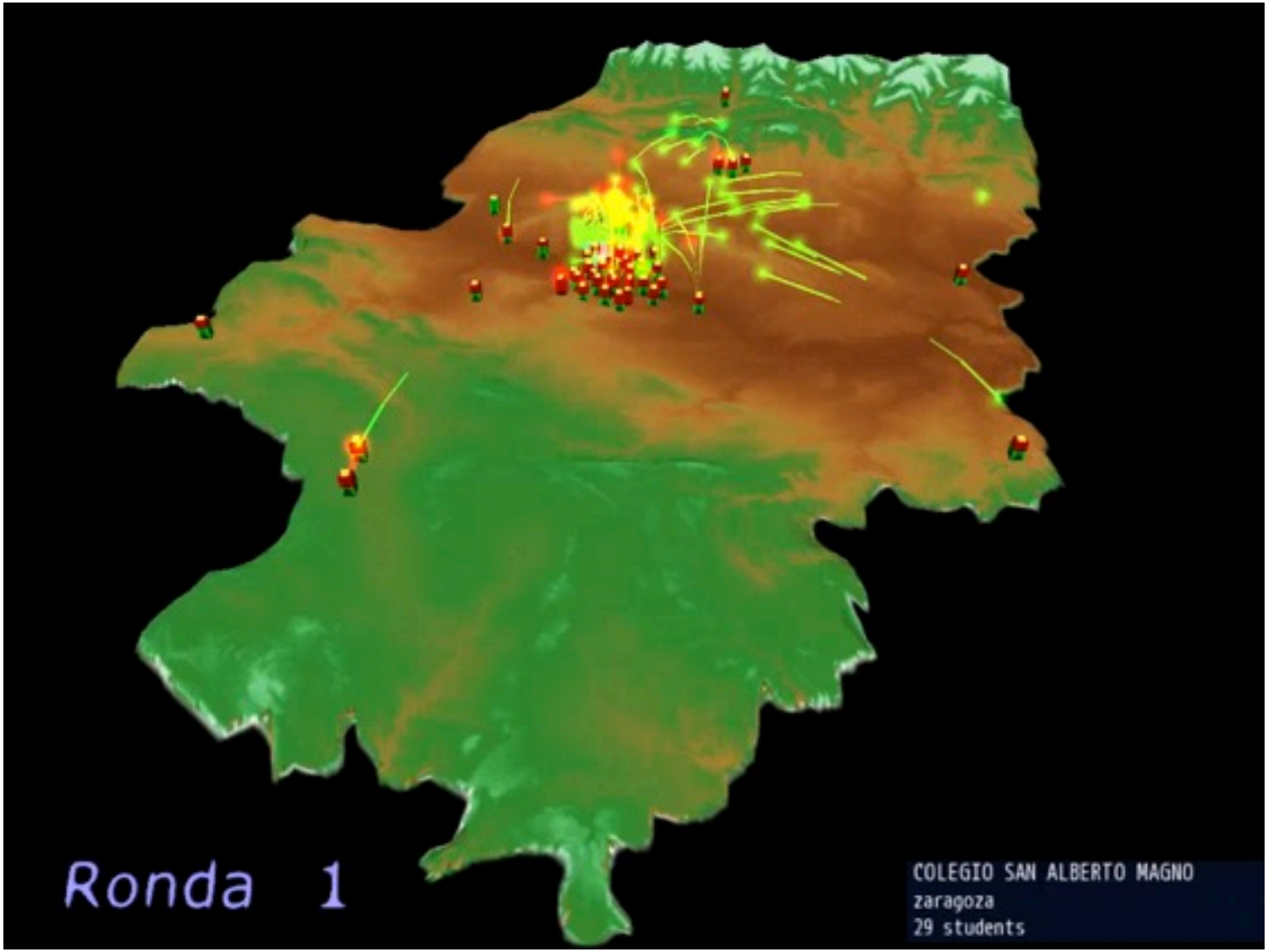
Carried out on December 20, 2011; largest size ever

The largest experiment ever, redux

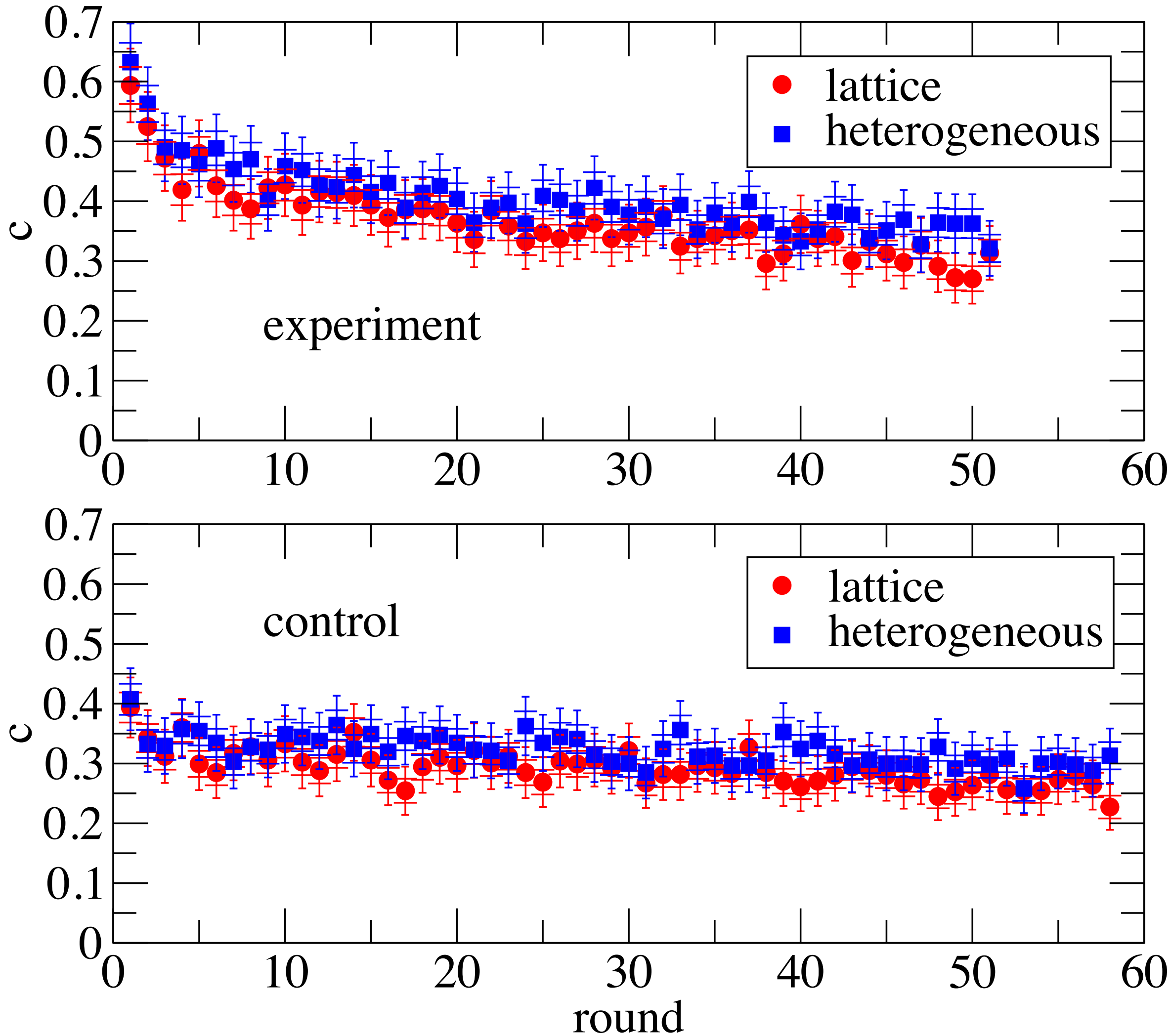


The largest experiment ever, redux

The largest experiment ever, redux

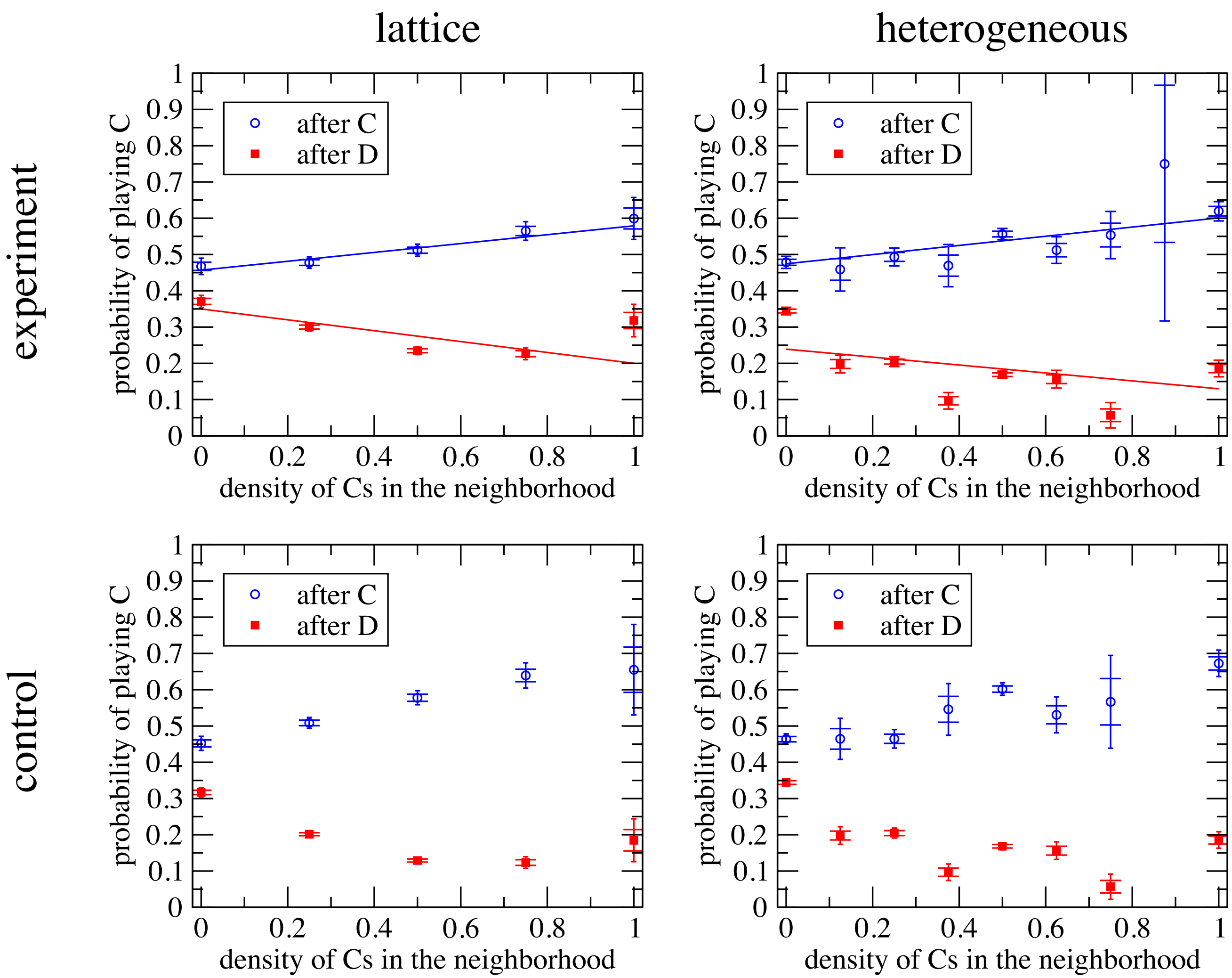


Results: percentage of cooperation



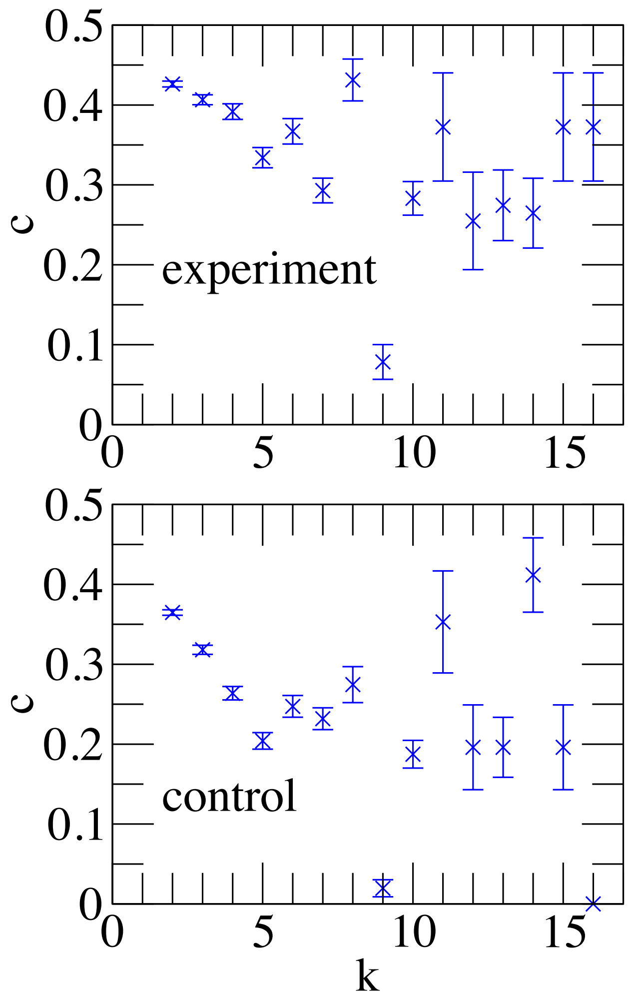
No network reciprocity

Results: moody conditional cooperation



Yet another confirmation

Results: degree dependence



Possible decay with degree when low

Conclusions

- **No network reciprocity**

Conclusions

- **No network reciprocity**
- About behavior
    - Moody conditional cooperation: independence of payoff
    - Groups of two vs many
    - Universality and diversity in human behavior
    - Degree dependence of cooperation?
    - Gender, type of studies...

- **Note**: Only for Prisoner's Dilemma, only for humans

Further data analysis coming

Thank you very much for your attention