Stability of (Super) Bloch Oscillations in the Presence of Time-Dependent Nonlinearities

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Collaborators

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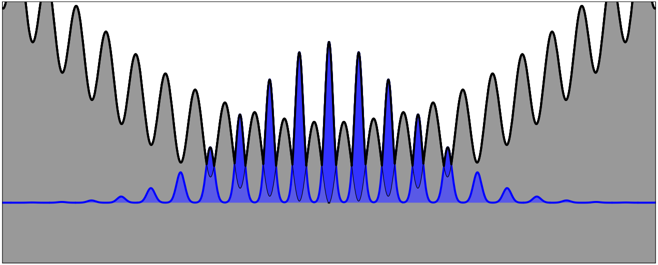
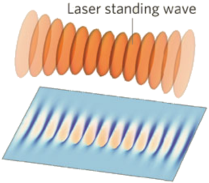
Publications

Phys. Rev. Lett., **102**, 255303 (2009)
Phys. Rev. A, **81**, 051607R (2010); ibid, **84**, 053627 (2011)

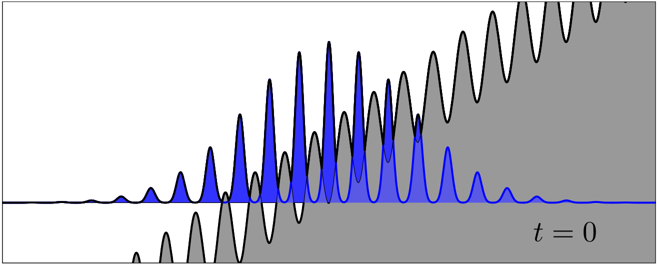
Intro – Ultracold atoms in optical lattices



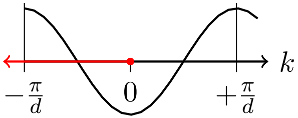


[Greiner *et al.*, nature (2008)]

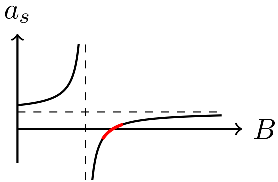
Intro – Ultracold atoms in optical lattices



- band structure
  effective mass $\sim 1/$curvature
  $\rightarrow$ Bloch oscillation



- Feshbach resonance
  $\rightarrow$ time-modulated interaction $g(t)$

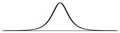


Overview

Tight-binding equation of motion

$$i\hbar\dot{\Psi}_n = -J(\Psi_{n-1} + \Psi_{n+1}) + g|\Psi_n|^2\Psi_n + [F_0 + \Delta F\cos(\omega t)]n\Psi_n$$

Scenarios

	nonlinearity g	tilt F_0	shaking ΔF	
0	$-g$			Soliton 
I		X		Bloch oscillation (BO)
II			X	?
III		X	X	?
IV	$g(t)$	X		?
V	$g(t)$	X	X	?

Analytical approach: Smooth-envelope equation of motion

Tight-binding equation of motion

$$i\hbar\dot{\Psi}_n = -J(\Psi_{n-1} + \Psi_{n+1}) + g|\Psi_n|^2\Psi_n + [\underbrace{F_0 + \Delta F\cos(\omega t)}_{= F(t)}]n\Psi_n$$

Smooth-envelope wave function $A(z,t)$

$$\Psi_n(t) = A(n - x(t), t)e^{ip(t)n + i\phi(t)}$$

kinematics: $\dot{p} = -F(t)$, $\dot{x} = 2\sin p \sim v_g = \frac{\partial(-\cos p)}{\partial p}$

$\Rightarrow$ wave function obeys

Nonlinear Schrödinger equation

$$i\dot{A} = -\cos p\, A'' + g(t)|A|^2 A \qquad \text{(NLSE)}$$

with time-dependent mass $\cos p = [2m(t)]^{-1}$

Scenario I: Linear Bloch oscillaion

Semiclassical equations of motion

$$\dot{p} = -F(t),\ \dot{x} = 2\sin p,\ i\dot{A} = -\cos p\, A'' + g(t)|A|^2 A$$

nonlinearity g	tilt F_0	shaking ΔF	
	X		Bloch oscillation (BO)

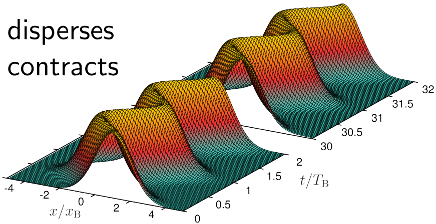
Bloch oscillation

- $p(t) = -F_0 t,\ x(t) = \frac{2}{F_0}\cos(F_0 t) \ \Rightarrow$ Bloch oscillation
- Mass term $\frac{1}{2m(t)} = \cos(F_0 t)$ oscillates

  $m > 0$ wave packet disperses

  $m < 0$ wave packet contracts

  $\Rightarrow$ breathing



Scenario II: Dynamical localization

Semiclassical equations of motion

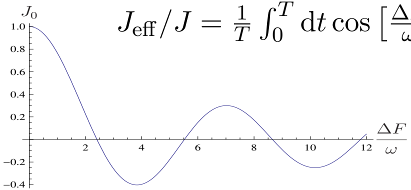
$$\dot{p} = -F(t),\ \dot{x} = 2\sin p,\ i\dot{A} = -\cos p\, A'' + g(t)|A|^2 A$$

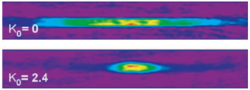
nonlinearity g	tilt F_0	shaking ΔF	
	X		Bloch oscillation (BO)
		X	Dynamic localization

Dynamical localization (simplified)

- $p(t) = \frac{\Delta F}{\omega} \sin(\omega t)$
- Time-averaged inverse mass (effective hopping):

$$J_{\text{eff}}/J = \frac{1}{T}\int_0^T \mathrm{d}t \cos\left[\frac{\Delta F}{\omega}\sin(\omega t)\right] = J_0(\Delta F/\omega)$$





[Dunlap & Kenkre, PRB 1986]

[Creffield et al., PRA 2010]

Scenario III: Super Bloch oscillation

Semiclassical equations of motion

$$\dot{p} = -F(t),\ \dot{x} = 2\sin p,\ i\dot{A} = -\cos p\, A'' + g(t)|A|^2 A$$

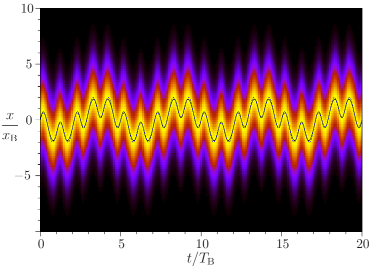
nonlinearity g	tilt F_0	shaking ΔF	
	X		Bloch oscillation (BO)
		X	Dynamic localization
	X	X	Super Bloch oscillation

Super Bloch oscillation

$$p(t) = -F_0 t + \frac{\Delta F}{\omega}\sin(\omega t)$$

$$x(t) \propto \int \mathrm{d}t' \sin p(t')$$

$\Rightarrow$ BO superposed with "super Bloch oscillation", beating $|\omega - F_0|$



Next level: Bloch oscillations with interactions

Linear BO (momentum representation)

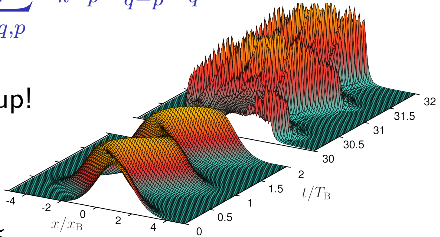
$$i\dot{A}_k = \cos(F_0 t) k^2 A_k \quad \Rightarrow \quad A_k(t) = \exp\left[-i\frac{k^2}{F_0}\sin(F_0 t)\right] A_k$$

- recurrence after $T_B = 2\pi/F_0 \Leftrightarrow$ Bloch oscillation
- independent evolution of A_k

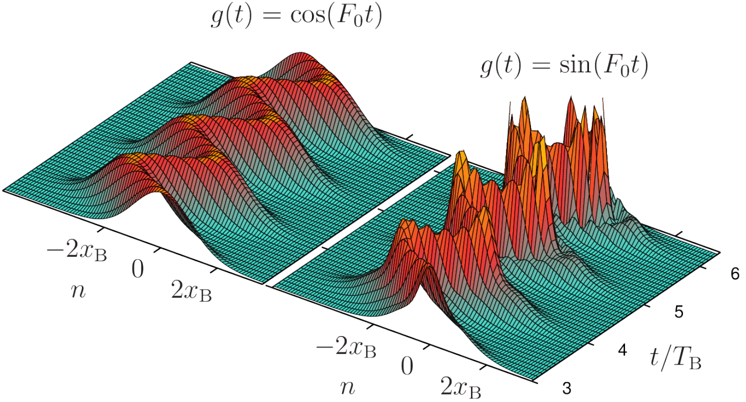
Interaction $|A(\boldsymbol{r})|^2 A(r) \to \sum_{q,p} A_{k-p} A^*_{q-p} A_q$

- mixes among k
- Bloch oscillation messed up!

Example:
constant interaction $g(t) = 0.5$



Harmonically modulated interaction



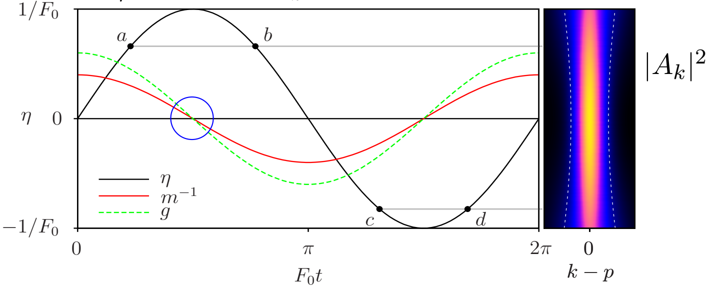
- Rapid decay vs. periodic breathing
- Sensitivity to relative phase

Cyclic-time argument

NLSE $$i\frac{\partial}{\partial t}A(z,t) = -\cos(F_0 t)\frac{\partial^2}{\partial z^2}A(z,t) + g(t)|A|^2 A(z,t)$$

- Consider $g(t) = g_0 \cos(F_0 t)$
- Cyclic time $\eta := \sin(F_0 t)/F_0 \quad \Rightarrow \quad \frac{\partial}{\partial t} = \cos(F_0 t)\frac{\partial}{\partial \eta}$
- $\cos(F_0 t)$ cancels

$$i\partial_\eta \tilde{A}(z,\eta) = -\partial_z^2 \tilde{A}(z,\eta) + g_0|\tilde{A}|^2 \tilde{A}(z,\eta)$$

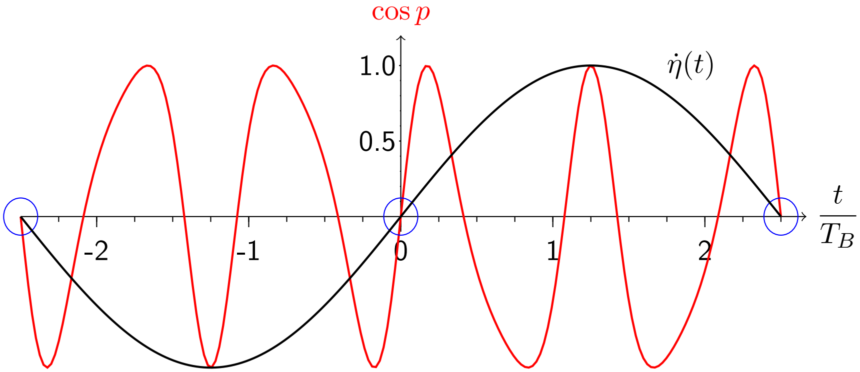


Crucial:
- $g(t)$ and $m^{-1} \sim \cos(p)$ commensurate
- common node of $g(t)$ and $\cos(p)$
- $g(t)$ and $\cos(p)$ odd with respect to that point

Super Bloch oscillations with time-modulated interaction?

Crucial: $g(t)$ with respect to $\cos(p)$

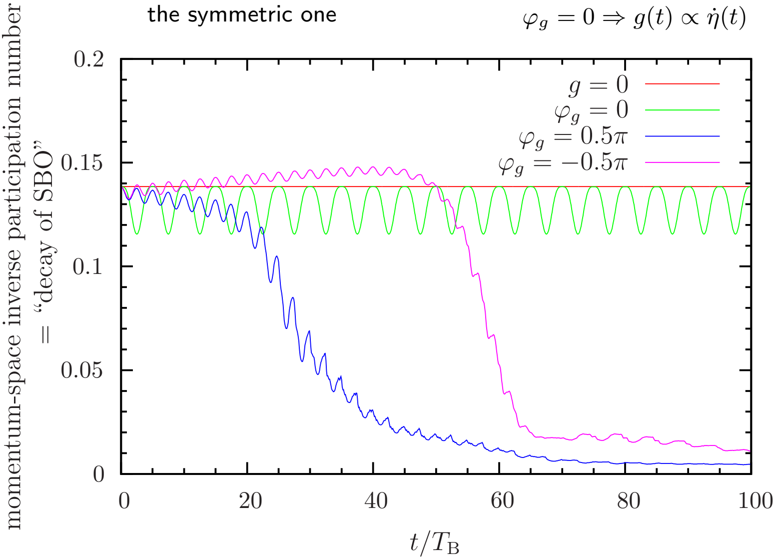
$$\dot{p} = -F(t), \qquad p(0) = \frac{\pi}{2}$$



- $F(t) = F_0 + \Delta F \sin(4F_0 t/5) \rightarrow$ No symmetry point.
- $F(t) = F_0 + \Delta F \cos(4F_0 t/5) \rightarrow$ Symmetry points.
  - Cyclic time $\eta(t)$, e.g. $g(t) \propto \dot{\eta}(t)$

Super Bloch oscillations with time-modulated interaction?

$$\underbrace{F(t) = F_0 + \Delta F \cos(4F_0 t/5),}_{\text{the symmetric one}} \qquad \underbrace{g(t) = \sin(F_0 t/5 + \varphi_g)}_{\varphi_g = 0 \Rightarrow g(t) \propto \dot{\eta}(t)}$$



Summed up

- One can do all kind of stuff with ultracold atoms:

	nonlinearity g	tilt F_0	shaking ΔF	
I		X		Bloch oscillation
II			X	Dynamic localization
III		X	X	Super Bloch oscillation
IV	$g(t)$	X		Phase sensitivity
V	$g(t)$	X	X	"Super phase sensitivity"

- Usally, interaction (=nonlinearity) destroys coherent Bloch oscillations (and super Bloch oscillations as well).
- Cyclic-time argument: adjust phase of $g(t)$ for periodic dynamics
- High sensitivity to that phase

Refs. Gaul, Díaz, Domínguez-Adame, Müller & Lima,
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