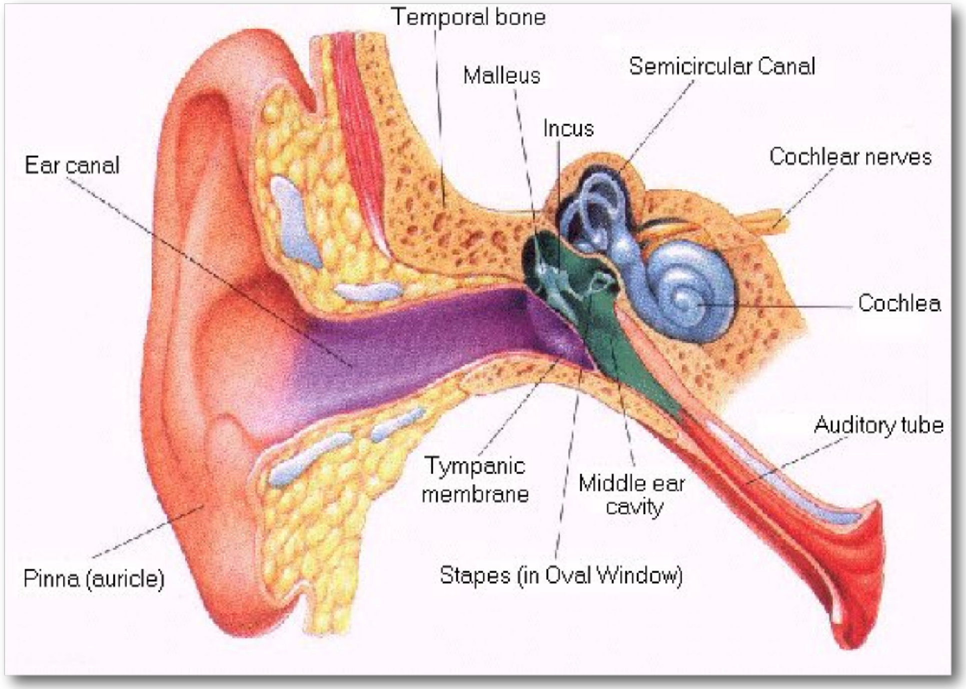
The hair-bundle:
Fluctuation dissipation for a noisy oscillator

L. Dinis, P. Martin, J. Barral, J. Prost and J.F. Joanny

Outline

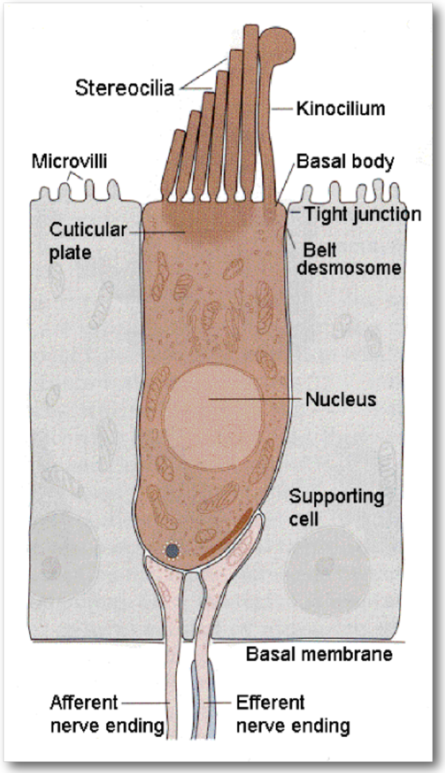
- The hair-bundle
- Fluctuation-Dissipation Theorem
- Generalization
- Checking the GFDT
- Conclusions

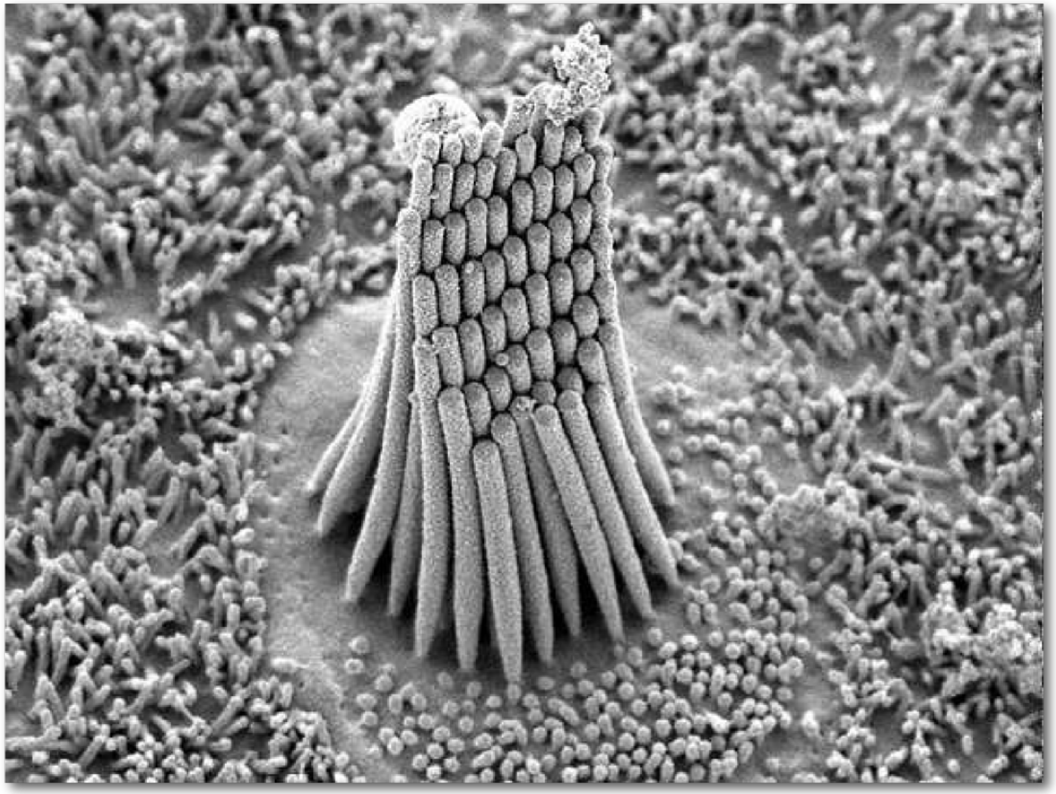
The hair-bundle



- *Rana catesbiana*
- Cochlea

The hair-bundle





- Hair-cell
- Hair-bundle

The hb is an amplifier

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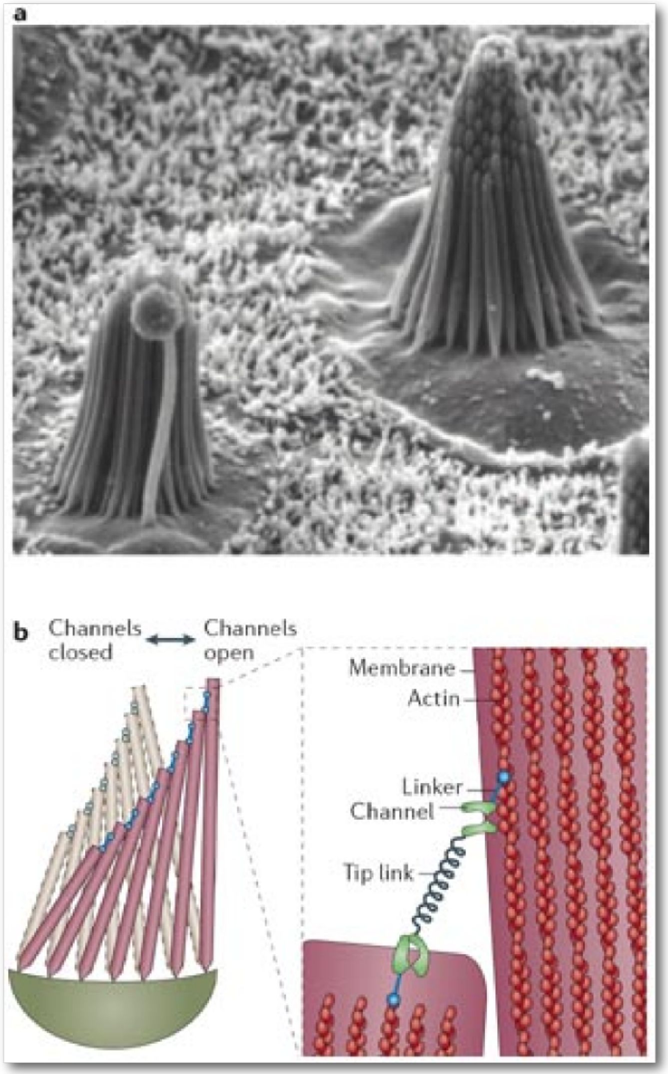


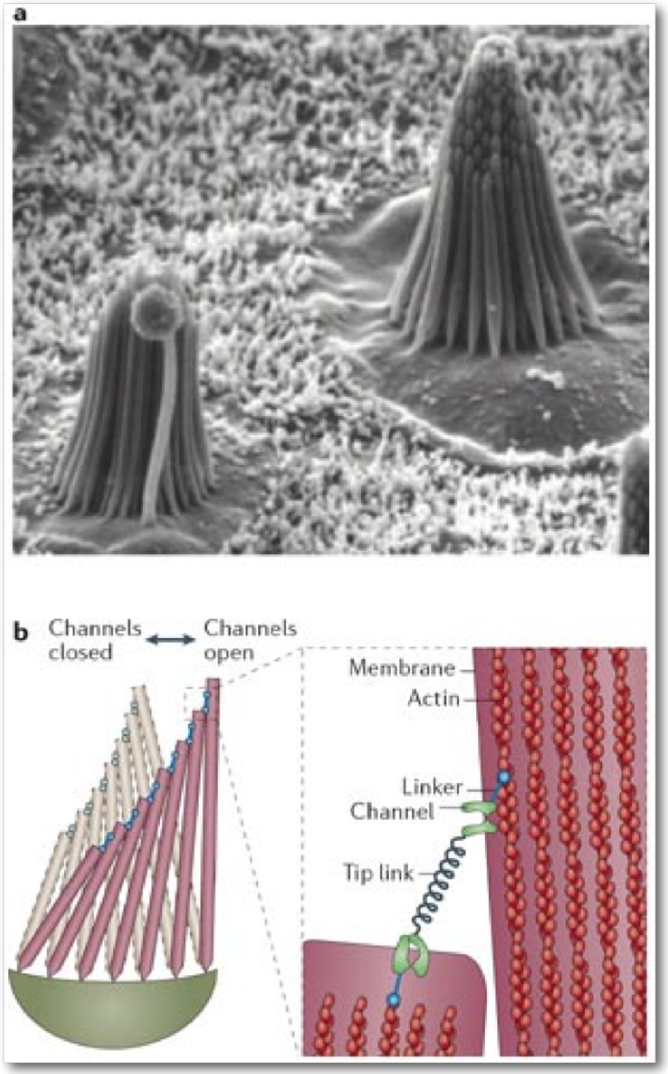
- Stereo cilia

- Mechano-electric transduction
- Uses energy to amplify sound stimuli
- Its amplification is frequency-specific
- Displays a compressive nonlinearity
- Otoacustic emission

Sensitivity

- Dynamic range: from 0 to 130 db
- Frequency range: from 20 Hz to 20 kHz
- Pitch resolution: tell 440Hz from 441Hz

The hb is an amplifier



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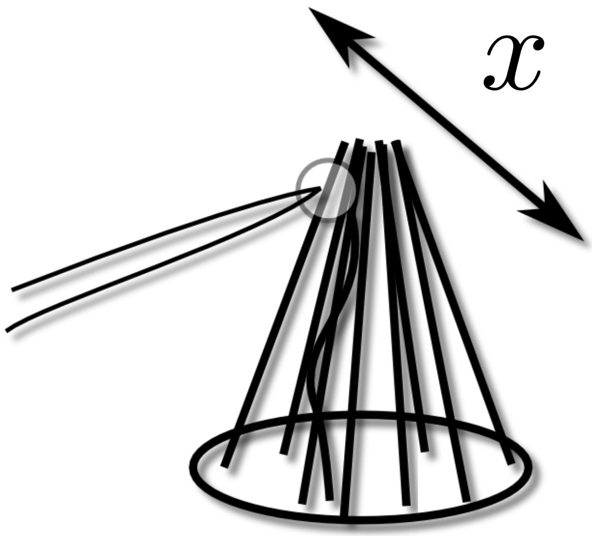
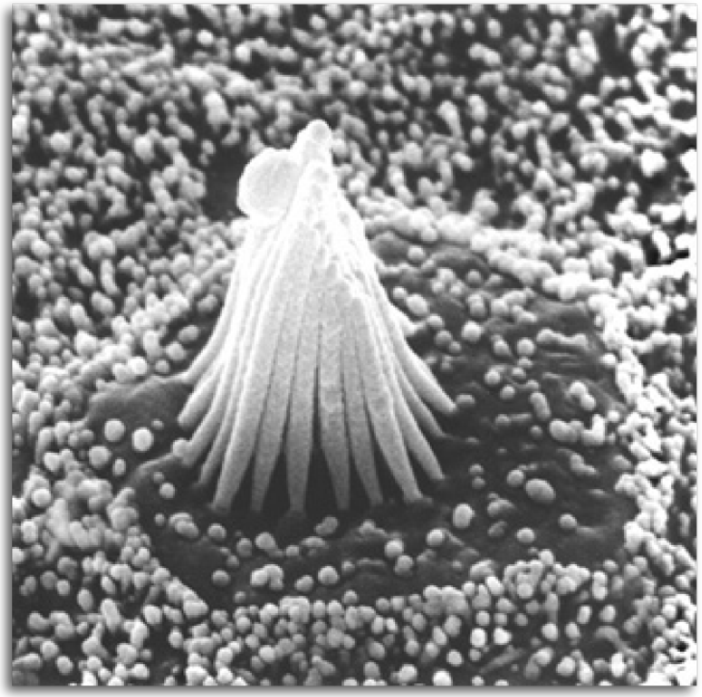
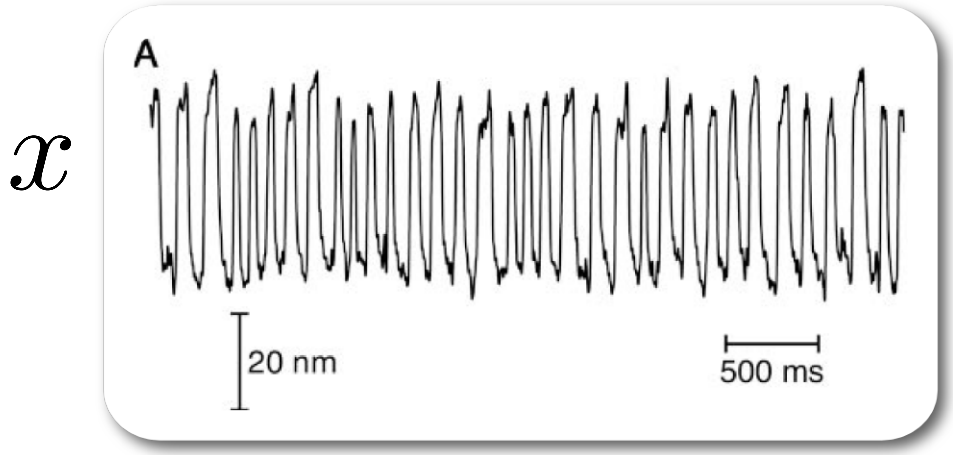
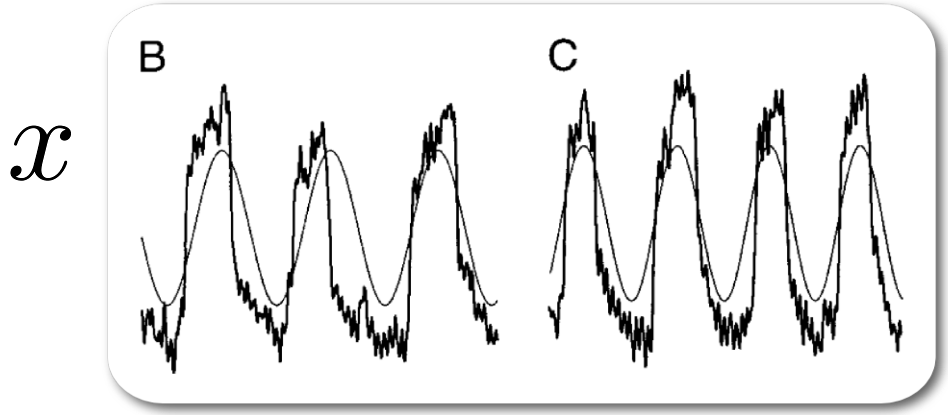
The experiment







Spontaneous movement

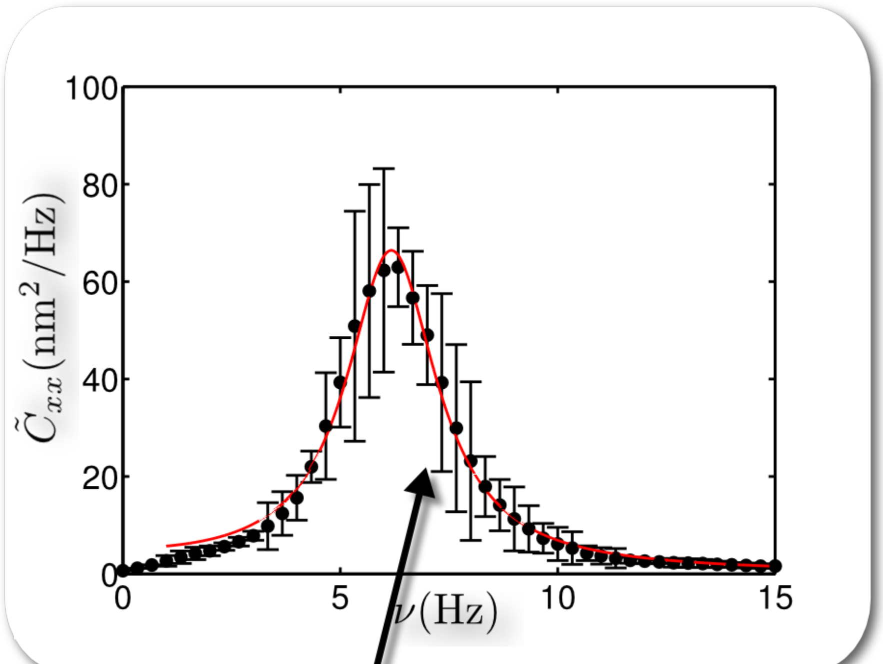


Forced oscillation

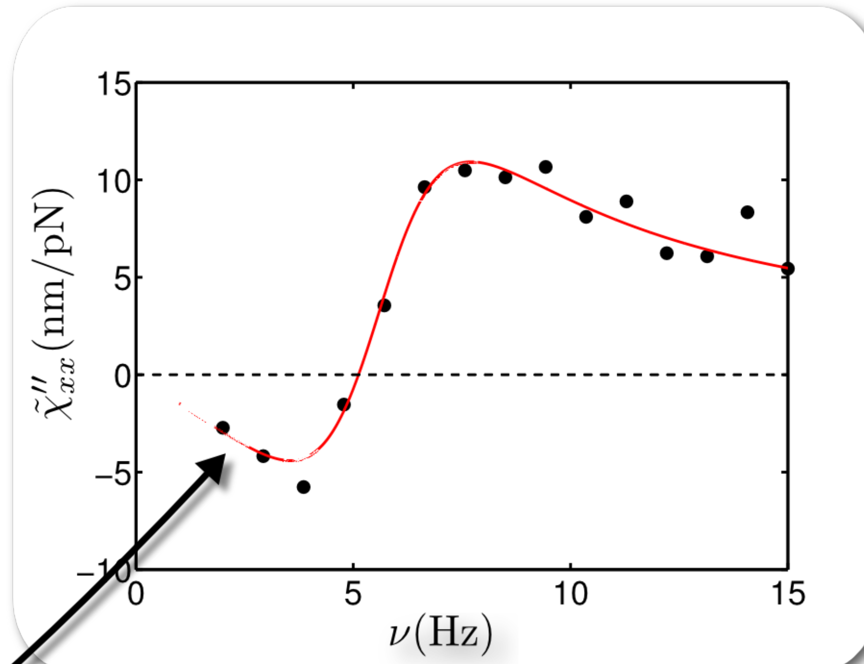
The FDT

The FDT

- power spectral density



- imag. part of response

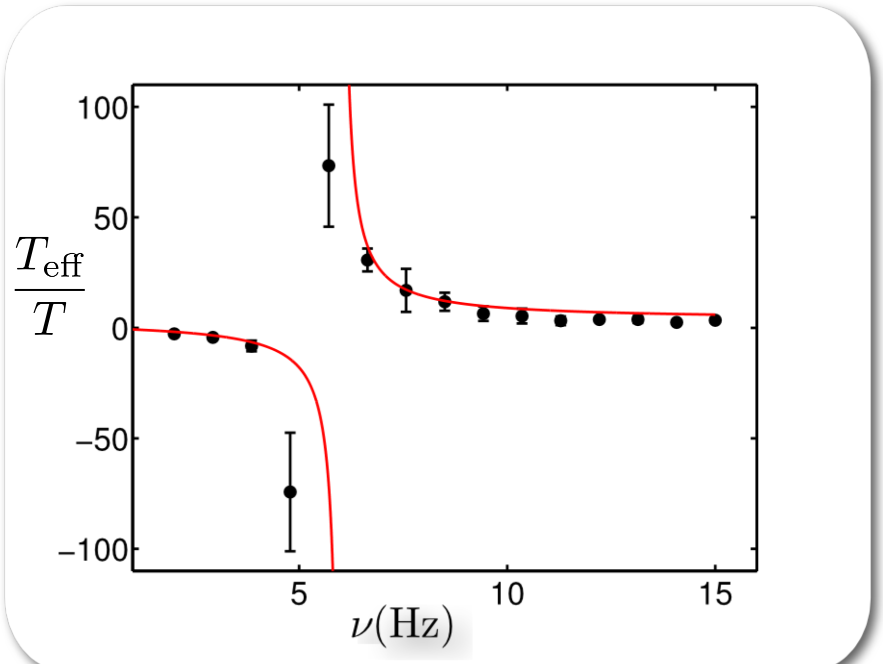


$$\tilde{C}_{xx}(\omega) = 2kT\frac{\tilde{\chi}''_{xx}(\omega)}{\omega} \quad \rightarrow \quad \frac{T_{\mathrm{eff}}}{T} = \frac{\omega\tilde{C}_{xx}(\omega)}{2kT\tilde{\chi}''_{xx}(\omega)}$$

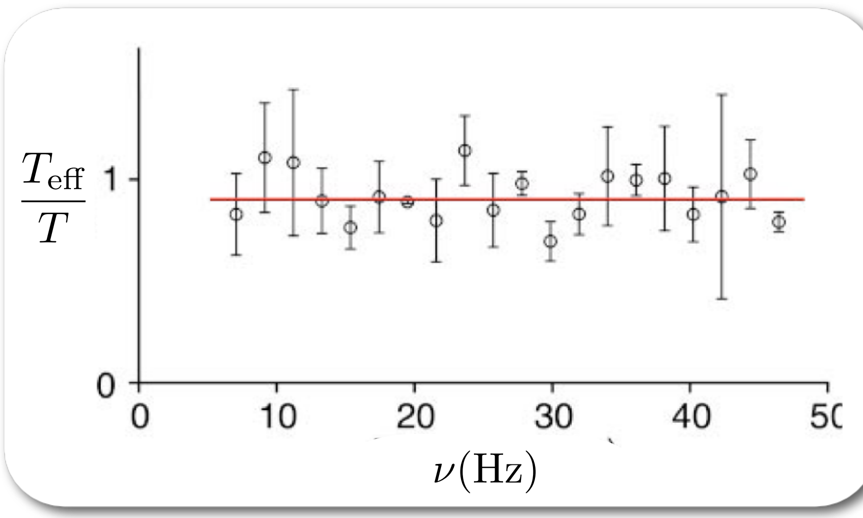
The FDT does not hold

- Effective temperature

$$\frac{T_{\text{eff}}}{T} = \frac{\omega \tilde{C}_{xx}(\omega)}{2kT\tilde{\chi}''_{xx}(\omega)}$$



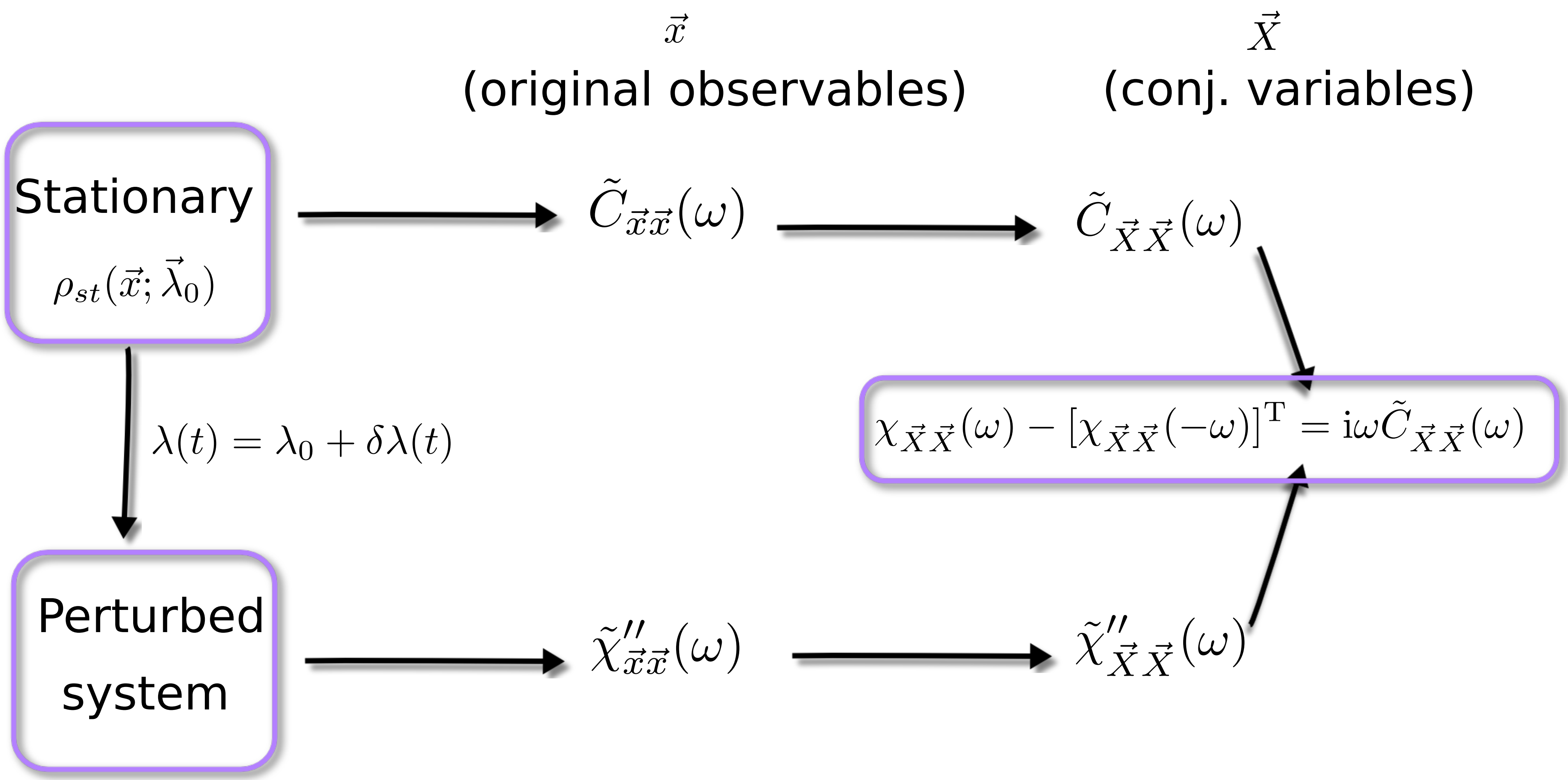
- Living cell



- Dead cell

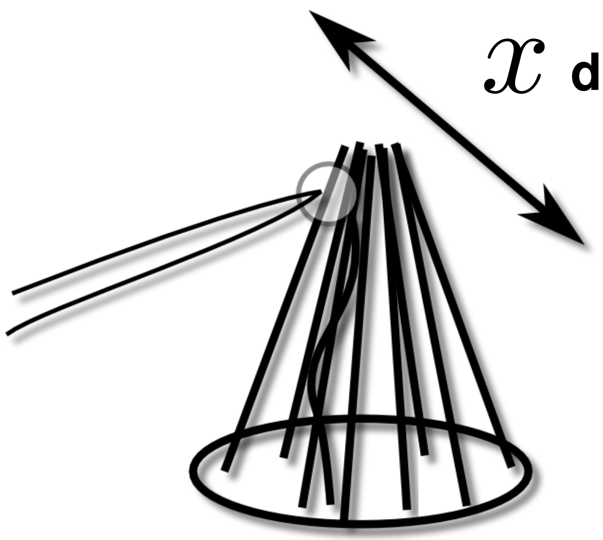
Generalized FDT

- Markovian systems in steady state



Modeling the system

- Linear part of a Hopf bifurcation system



x **displacement of the hair bundle**

y **active element force**

applied force

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -r & \omega_0 \\ -\omega_0 & -r \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} f_x \\ 0 \end{pmatrix} + \begin{pmatrix} \eta_x \\ \eta_y \end{pmatrix}$$

spontaneous frequency

Gaussian white noises

The GFDT for the hair-bundle

- GFDT for a linear system

$$\vec{x} = \begin{pmatrix} x \\ y \end{pmatrix} \xrightarrow{\text{Estimation from data}} A, \Sigma \xrightarrow{\text{c.v.}} \vec{X} = \left[A^{-1}\right]^t \Sigma^{-1} \vec{x}$$

GFDT ↓

$$\chi_{\vec{X}\vec{X}}(\omega) - [\chi_{\vec{X}\vec{X}}(-\omega)]^{\mathrm{T}} = \mathrm{i}\omega \tilde{C}_{\vec{X}\vec{X}}(\omega)$$

Effective Temperature ↓

$$\frac{T'_{\mathrm{eff}}}{T} = \frac{\omega \tilde{C}_{XX}(\omega)}{2\tilde{\chi}''_{XX}(\omega)} = 1$$

$$A \equiv \begin{pmatrix} -r & \omega_0 \\ -\omega_0 & -r \end{pmatrix}$$

$$\Sigma \equiv \begin{pmatrix} \langle x^2 \rangle & \langle xy \rangle \\ \langle xy \rangle & \langle y^2 \rangle \end{pmatrix}$$

stationary correlation matrix

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? (for y)

GFDT

$$\chi_{\vec{X}\vec{X}}(\omega) - \left[\chi_{\vec{X}\vec{X}}(-\omega)\right]^{\mathrm{T}} = \mathrm{i}\omega \tilde{C}_{\vec{X}\vec{X}}(\omega)$$

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stationary correlation matrix

Estimation from data

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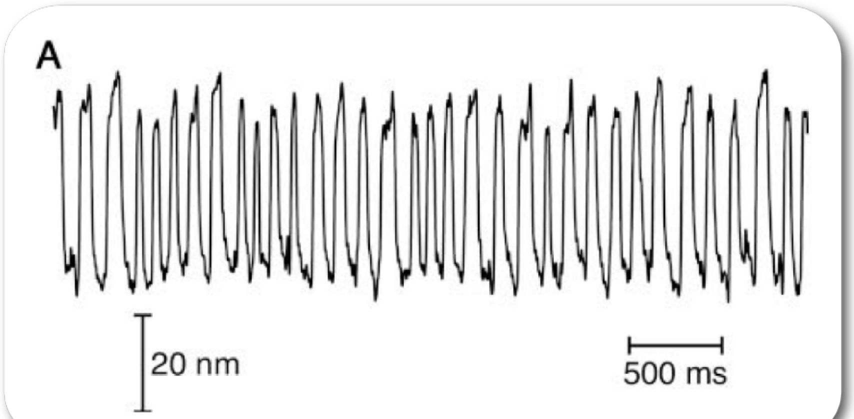
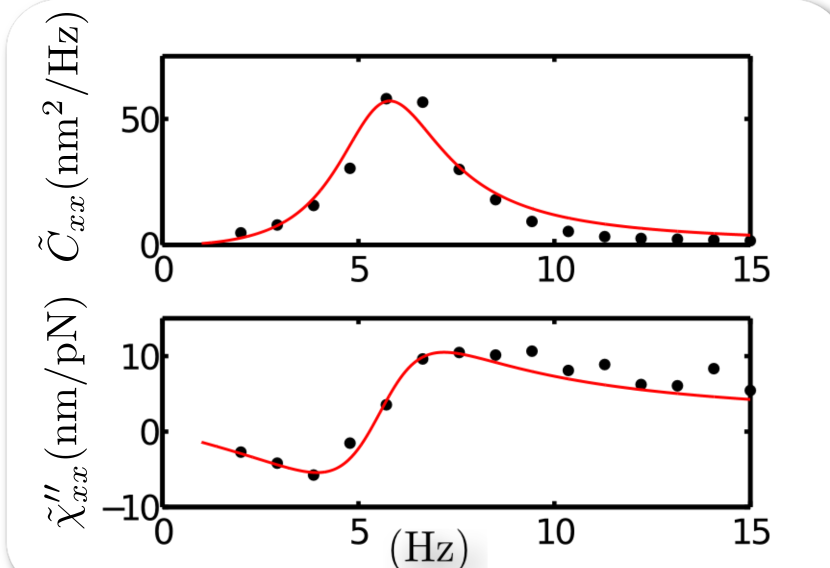
$x(t)$



spontaneous motion



power spectral density and response function

$\vec{x} = \begin{pmatrix} x \\ y \end{pmatrix}$ → **Fit PS and Response** → A → (ν) **c.v.** → $\vec{X} = \left[A^{-1}\right]^t \Sigma^{-1} \vec{x}$ → **GFDT**

? (y)

↓ **c.v.** $z = -rx + \omega_0 y$

$\dot{x} = z + \eta_x$ → **Denoising by running average** → z → **c.v.** → y, Σ ↑ (to $\vec{X}$)

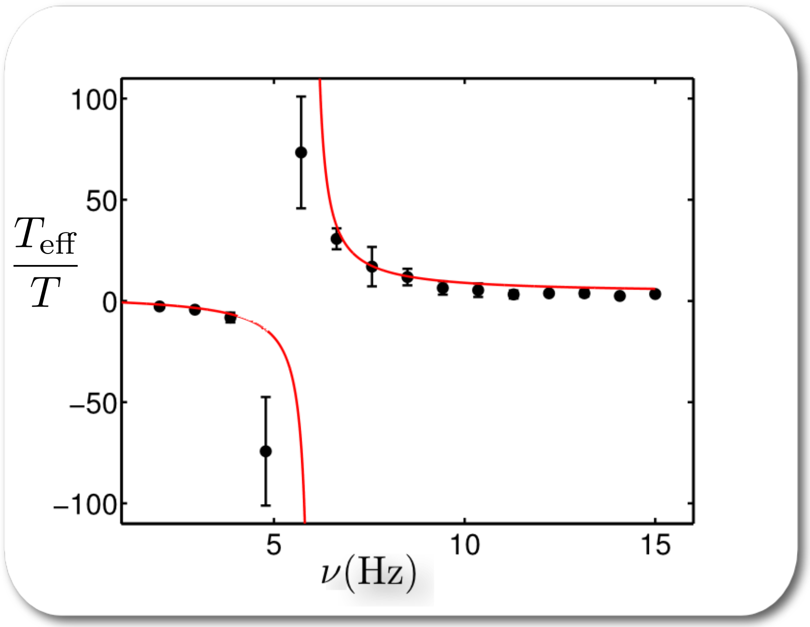
Results 1

- Experimental data. Filtering

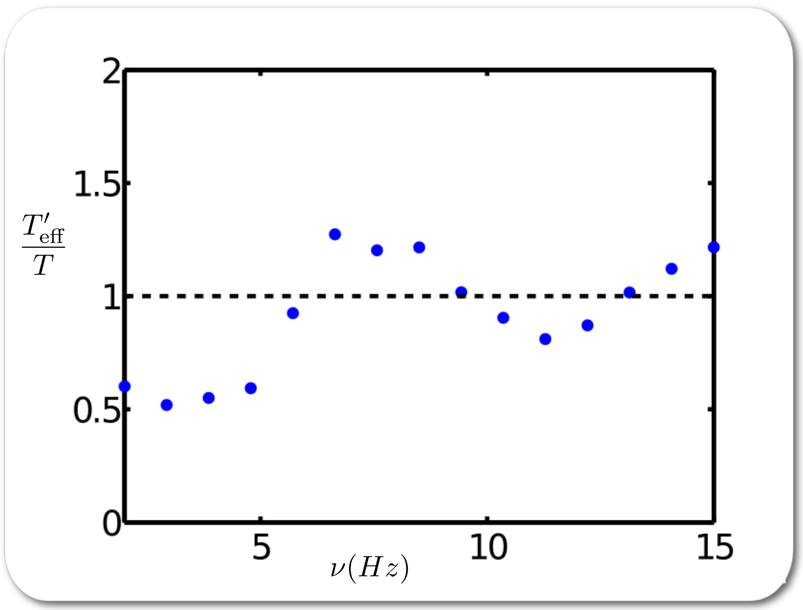
$$\chi_{\vec{X}\vec{X}}(\omega) - [\chi_{\vec{X}\vec{X}}(-\omega)]^{\mathrm{T}} = \mathrm{i}\omega\tilde{C}_{\vec{X}\vec{X}}(\omega)$$

Effective Temperature

$$\frac{T'_{\mathrm{eff}}}{T} = \frac{\omega\tilde{C}_{XX}(\omega)}{2\tilde{\chi}''_{XX}(\omega)} = 1$$



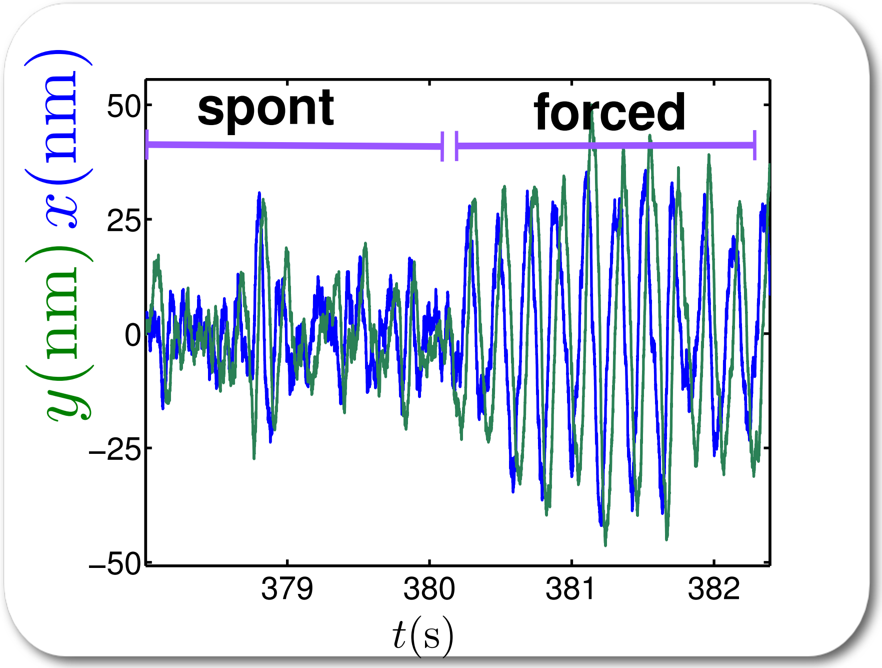
- Measured *x*

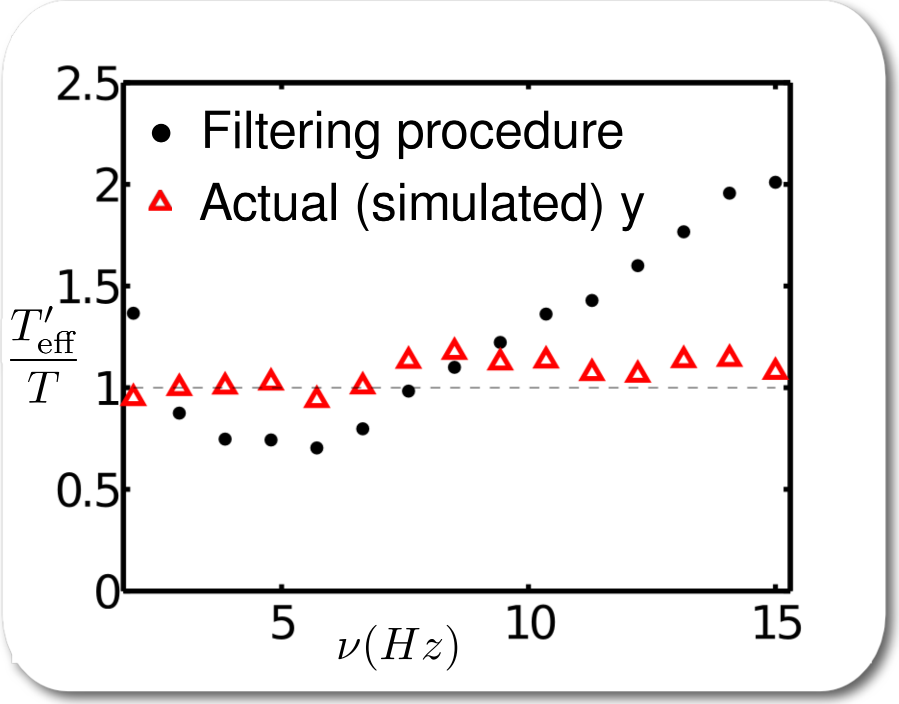


- New variables *X*

Simulation Results

- Simulation results. Filtering and actual *y*





Alternative y estimation

Alternative y estimation

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -r & \omega_0 \\ -\omega_0 & -r \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \eta_x \\ \eta_y \end{pmatrix}$$

↓

$$x_{n+1} = x_n + (-r x_n - \omega_0 y_n)\Delta t + z_x\sqrt{2D_x \Delta t}$$

$$y_{n+1} = y_n + (-r y_n + \omega_0 x_n)\Delta t + z_y\sqrt{2D_y \Delta t}$$

Discretization

↓

$$x_{n+1} - x_n + (r x_n + \omega_0 y_n)\Delta t \sim N(0, \sqrt{2D_x \Delta t})$$

$$y_{n+1} - y_n - (-r y_n + \omega_0 x_n)\Delta t \sim N(0, \sqrt{2D_y \Delta t})$$

Gaussian white noises

↓

$$\rho(\{x_n, y_n\})$$

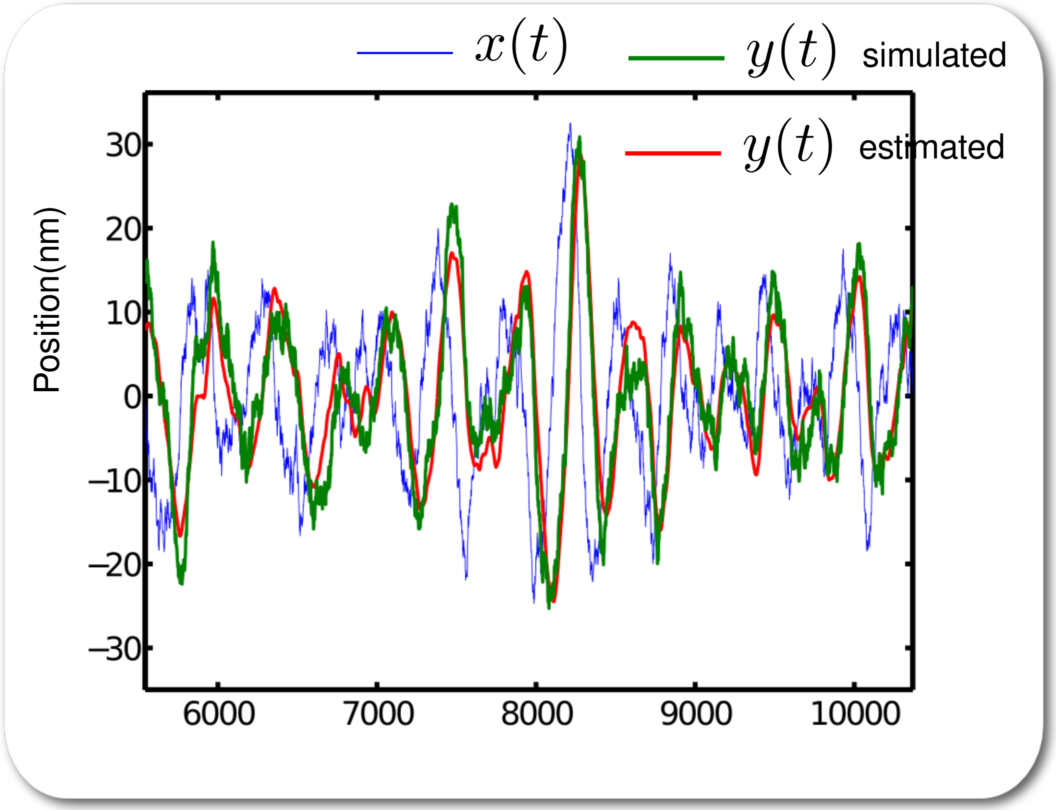
Probability of a trajectory

↓

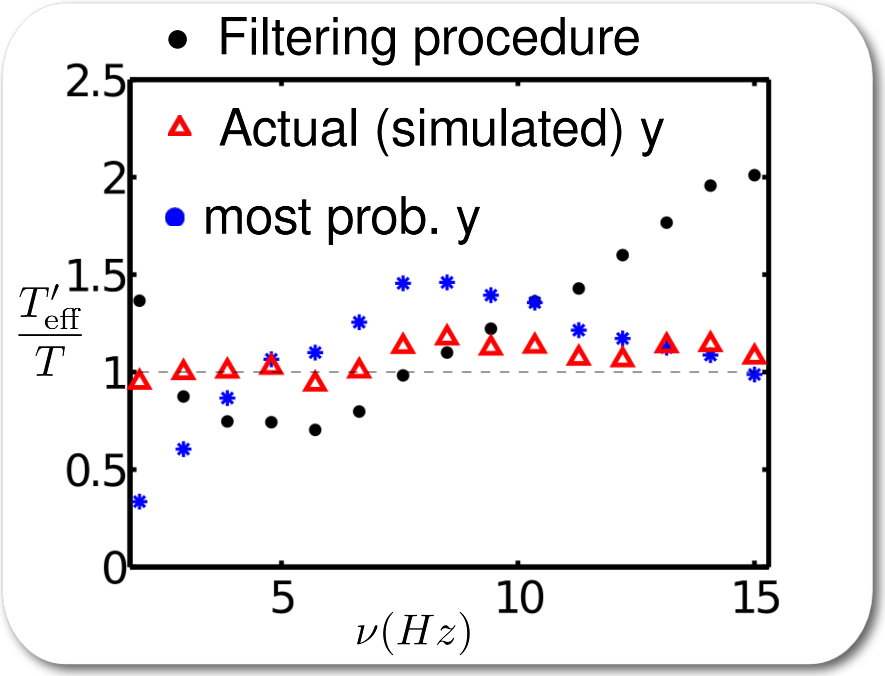
$$\{y_n\}$$

Maximize with respect to $\{y_n\}$

Simulation Results 2



- Simulations and y estimation

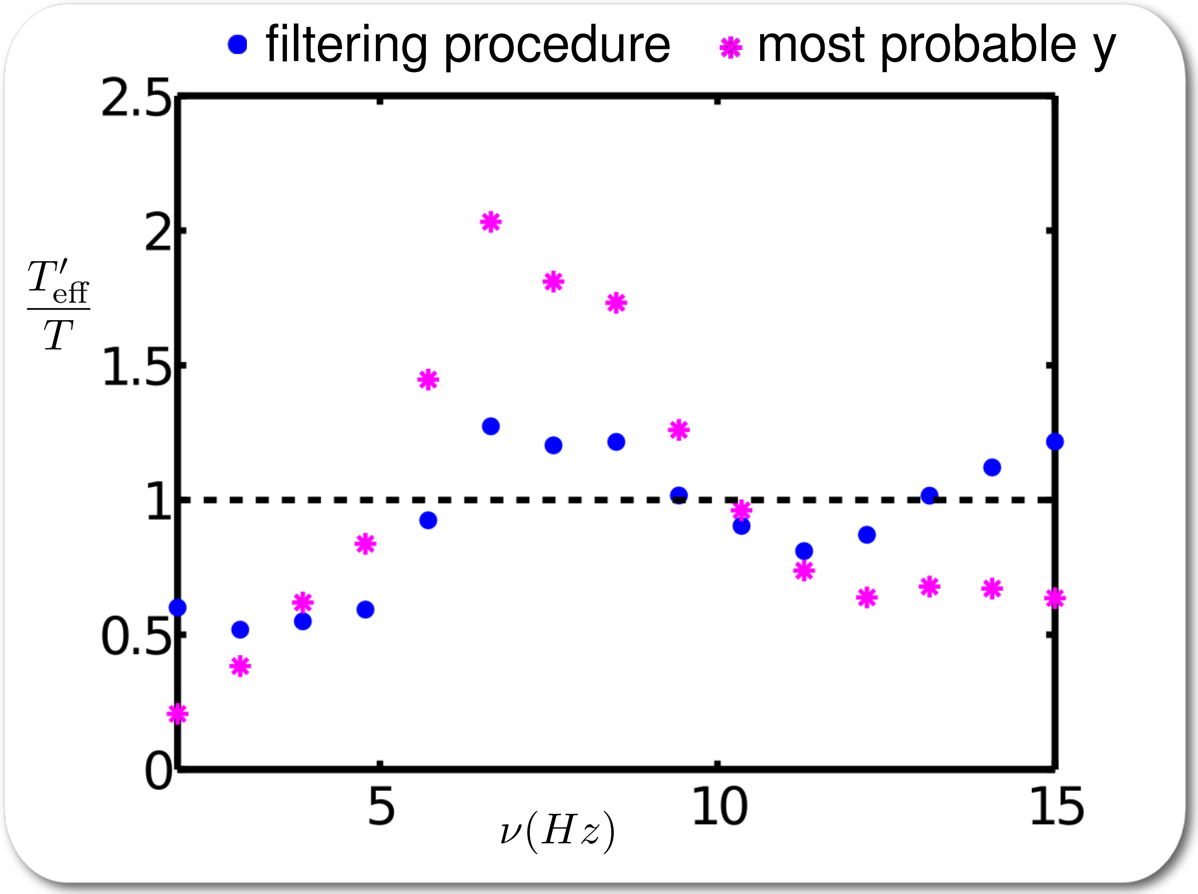


- Effective temperature

Results 2

Results 2

- Experimental data. Filtering and most probable y



Conclusions

- Fluctuation-dissipation is recovered in the new variables
- The divergence of effective temperature disappears
- Estimations work better for frequencies close to the peak.
- Hair-bundle can be described as an active noisy oscillator close to a Hopf-bifurcation with Markovian dynamics