Capillary emptying and short-range wetting

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Alice L. Thorneywork
Dirk G.A.L. Aarts

Leganés, 8 Feb 2013

- **Tilting Capillaries**

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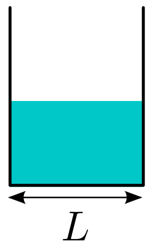
- $L \gg$



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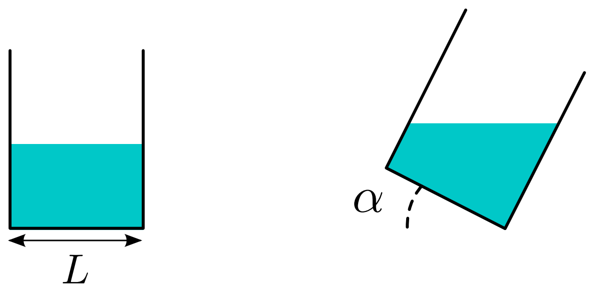

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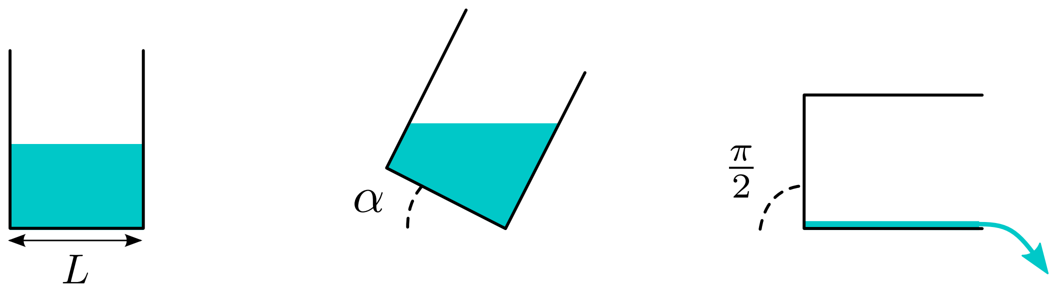

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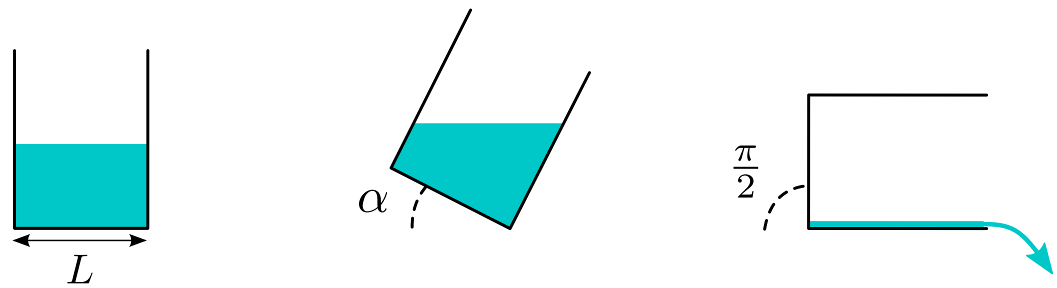

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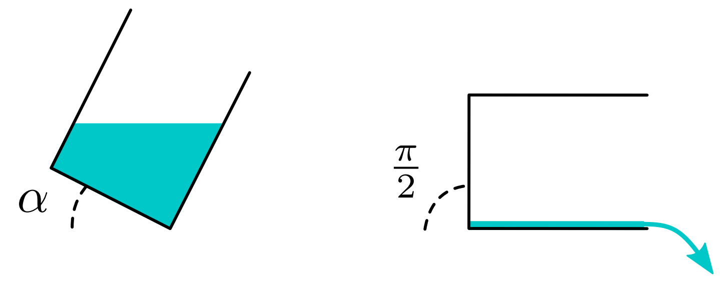

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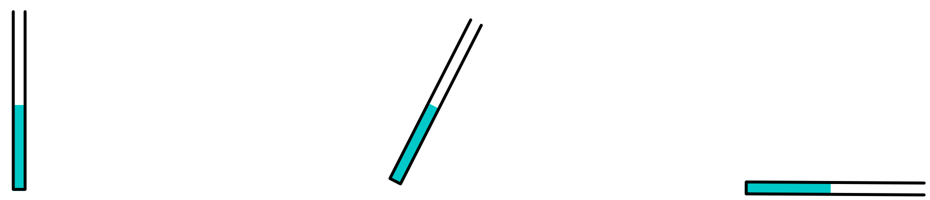

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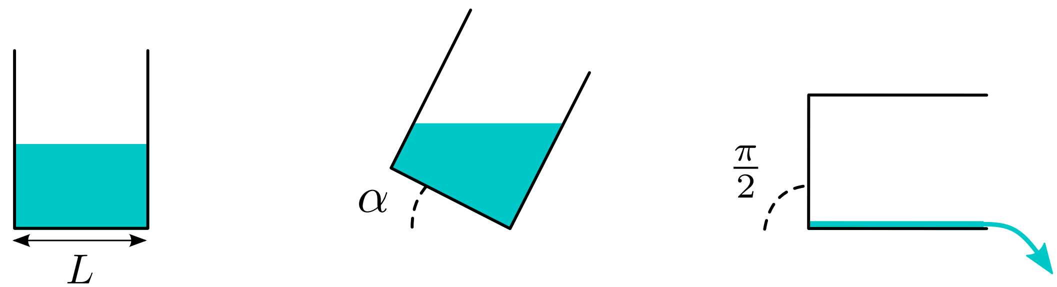

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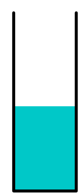

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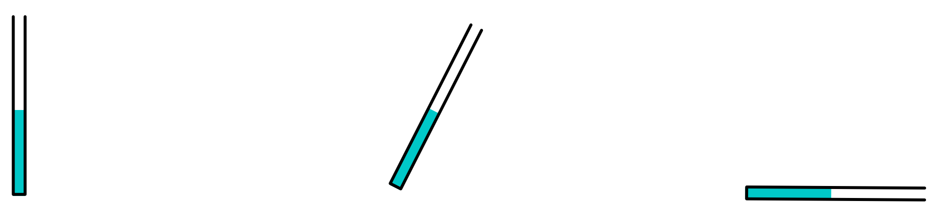

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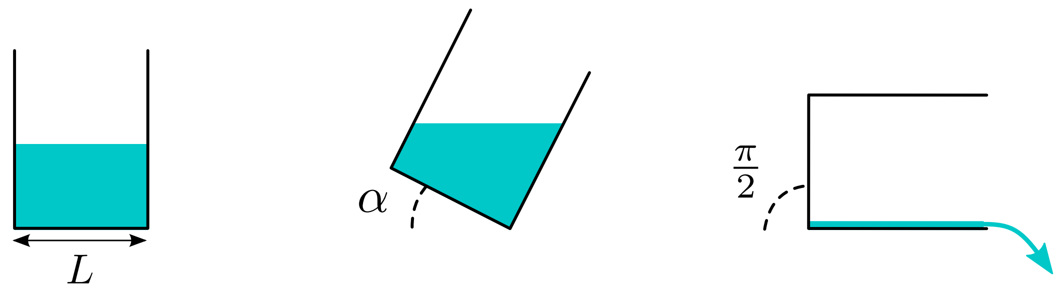

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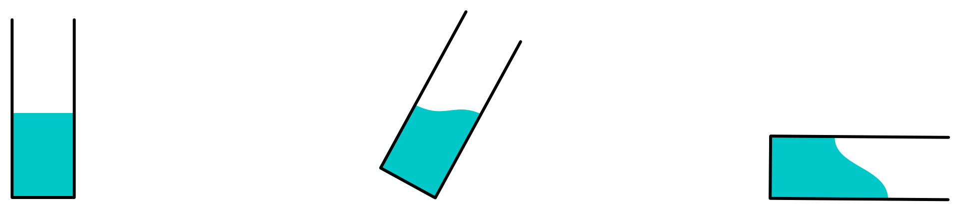

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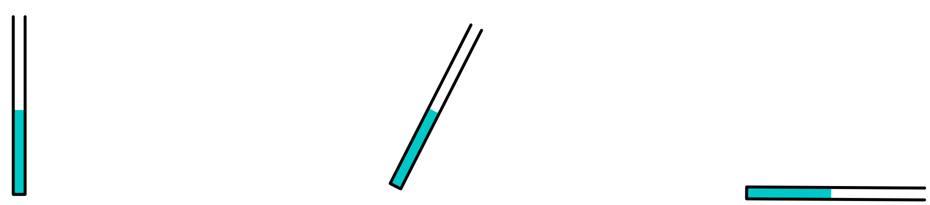

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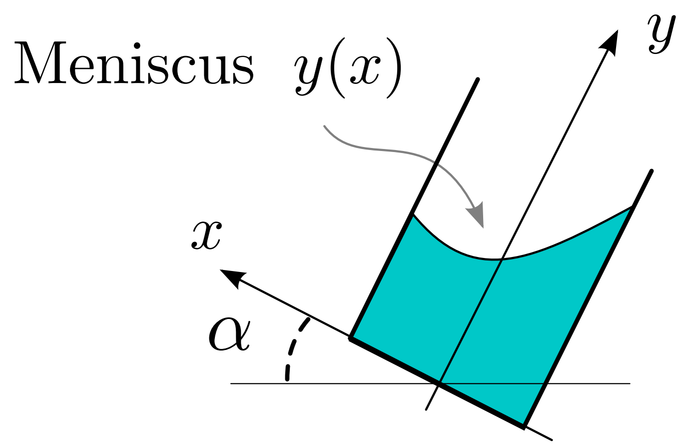

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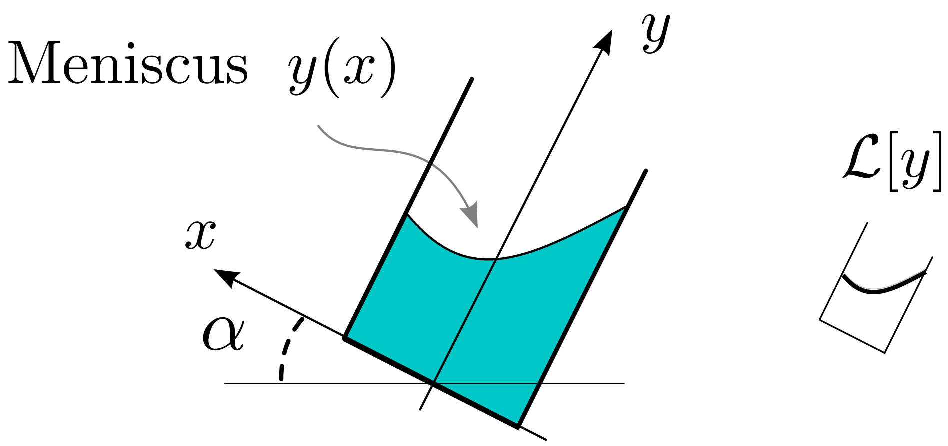

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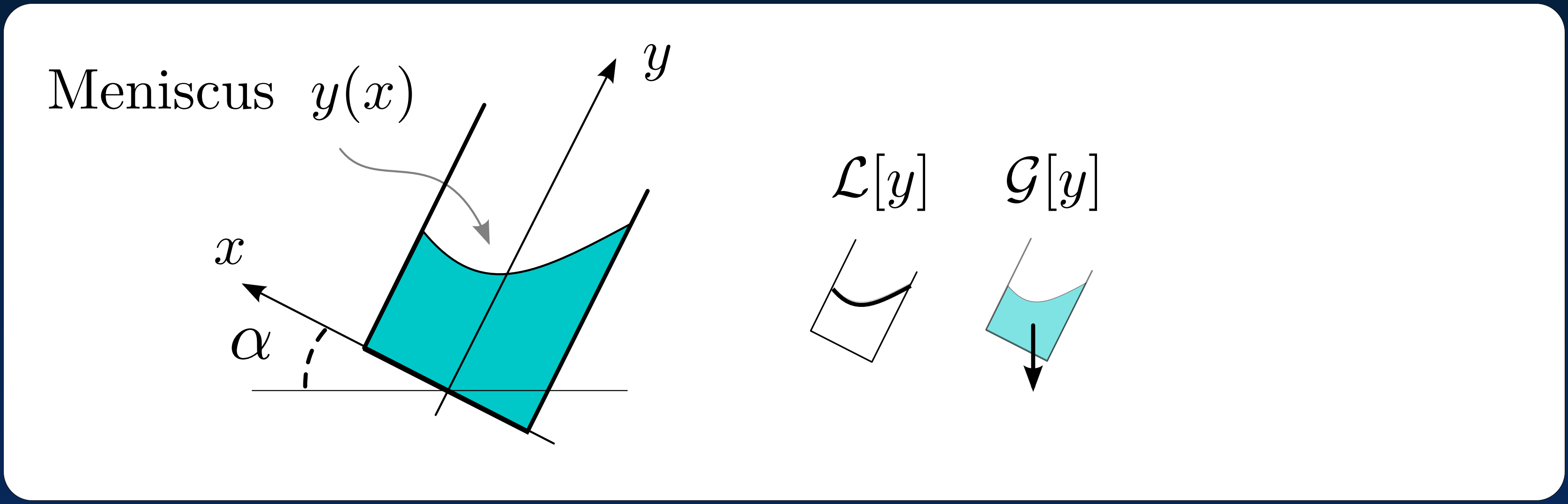

$$E[y] \; = \; \sigma\,\mathcal{L} \; + \; g\,\delta\rho\,\mathcal{G}$$

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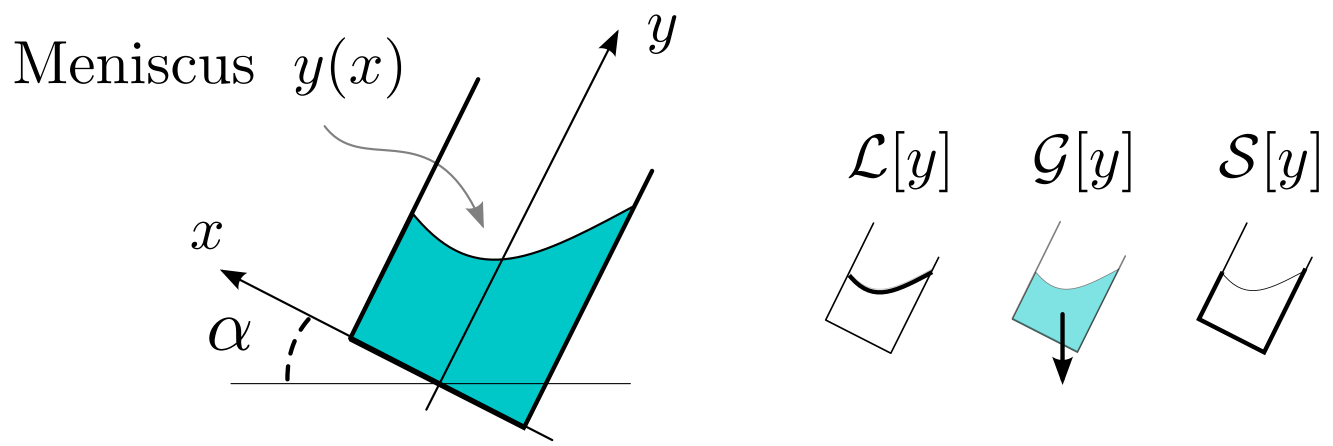

$$E[y] \;=\; \sigma\,\mathcal{L} \;+\; g\,\delta\rho\,\mathcal{G} \;+\; (\sigma_{wl} - \sigma_{wg})\,\mathcal{S}$$

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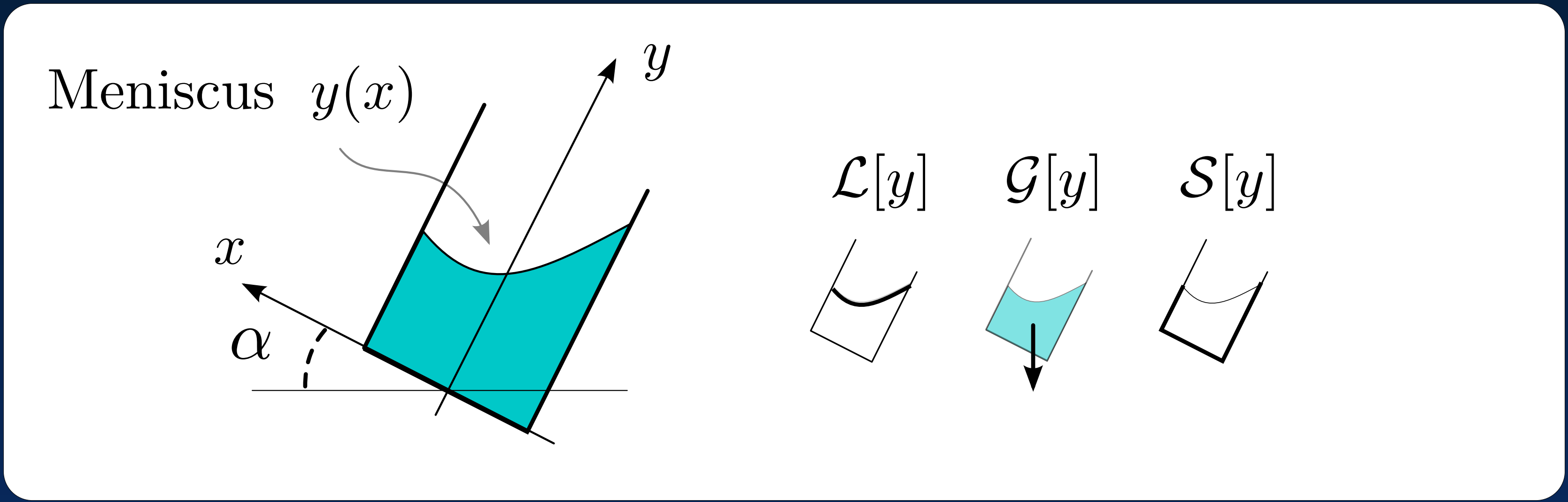

$$E[y] = \sigma\,\mathcal{L} + g\,\delta\rho\,\mathcal{G} - \sigma\cos\theta\,\mathcal{S}$$

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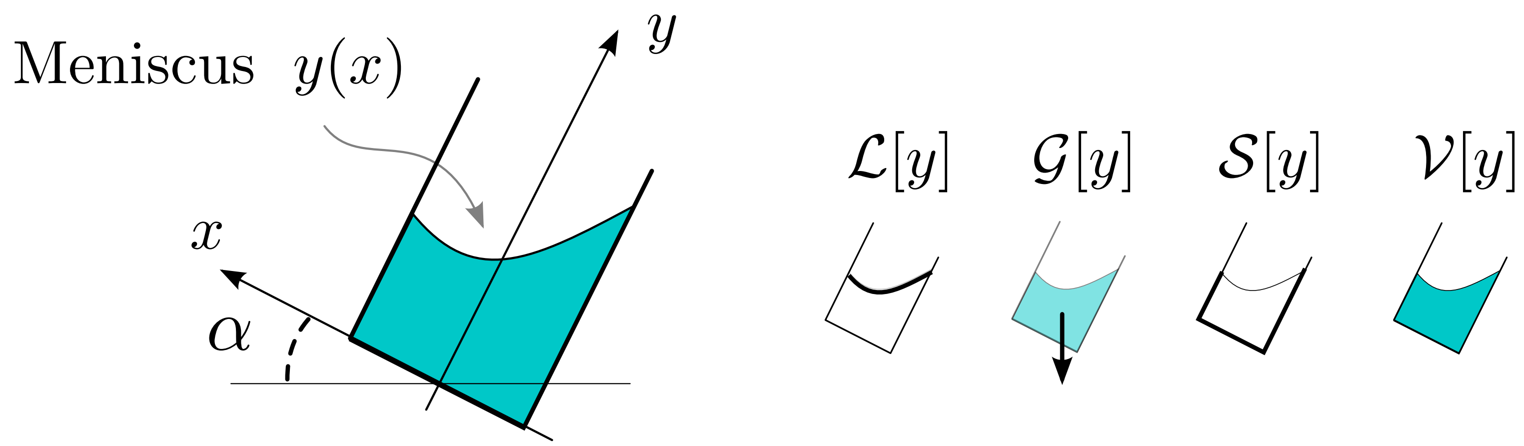

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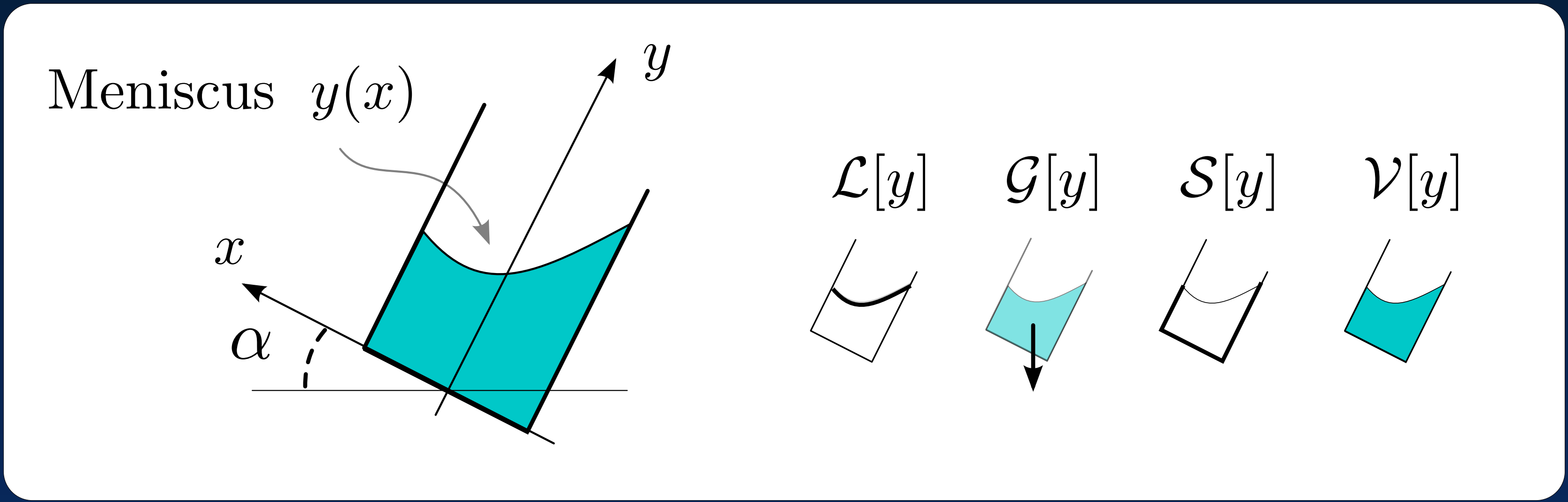

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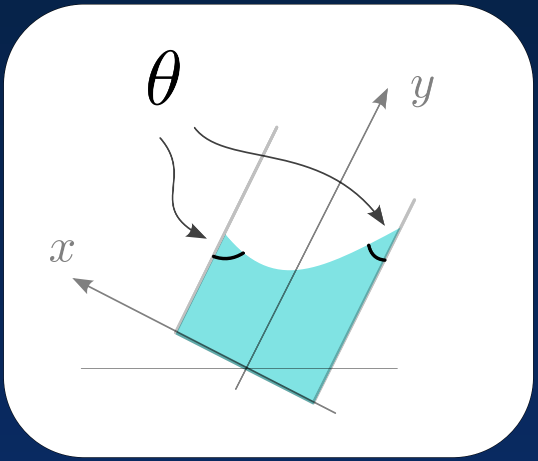

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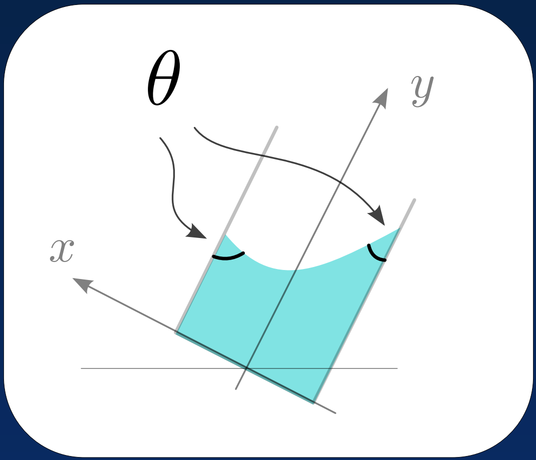

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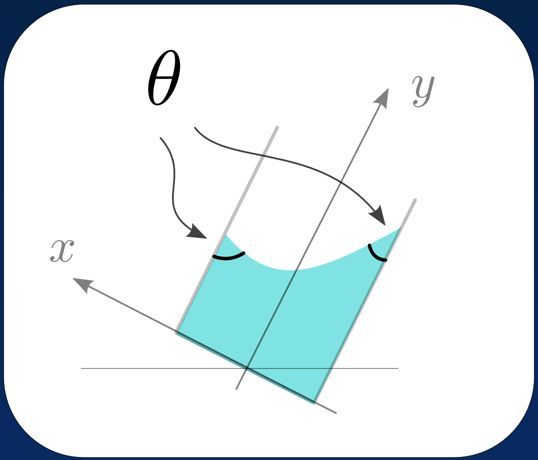

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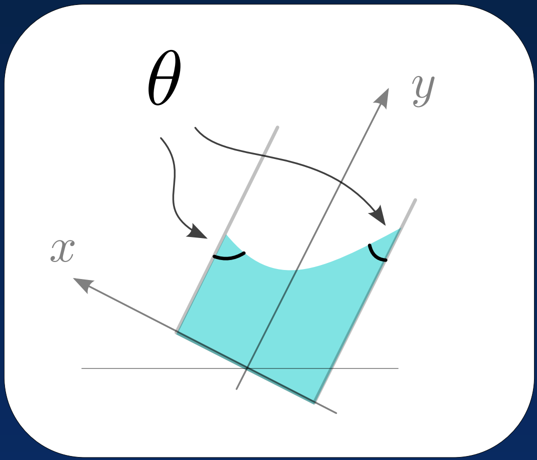

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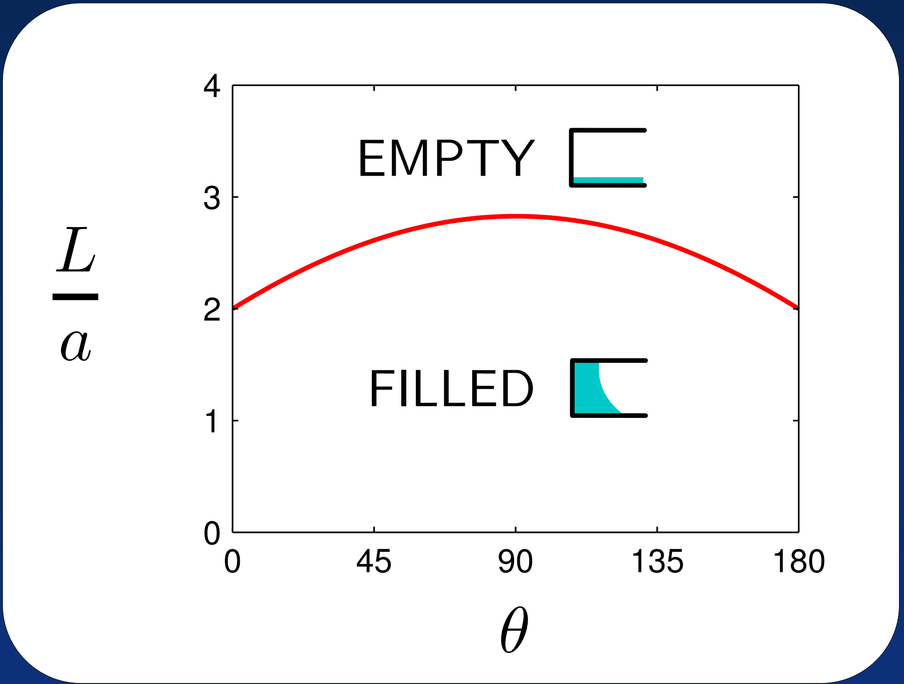

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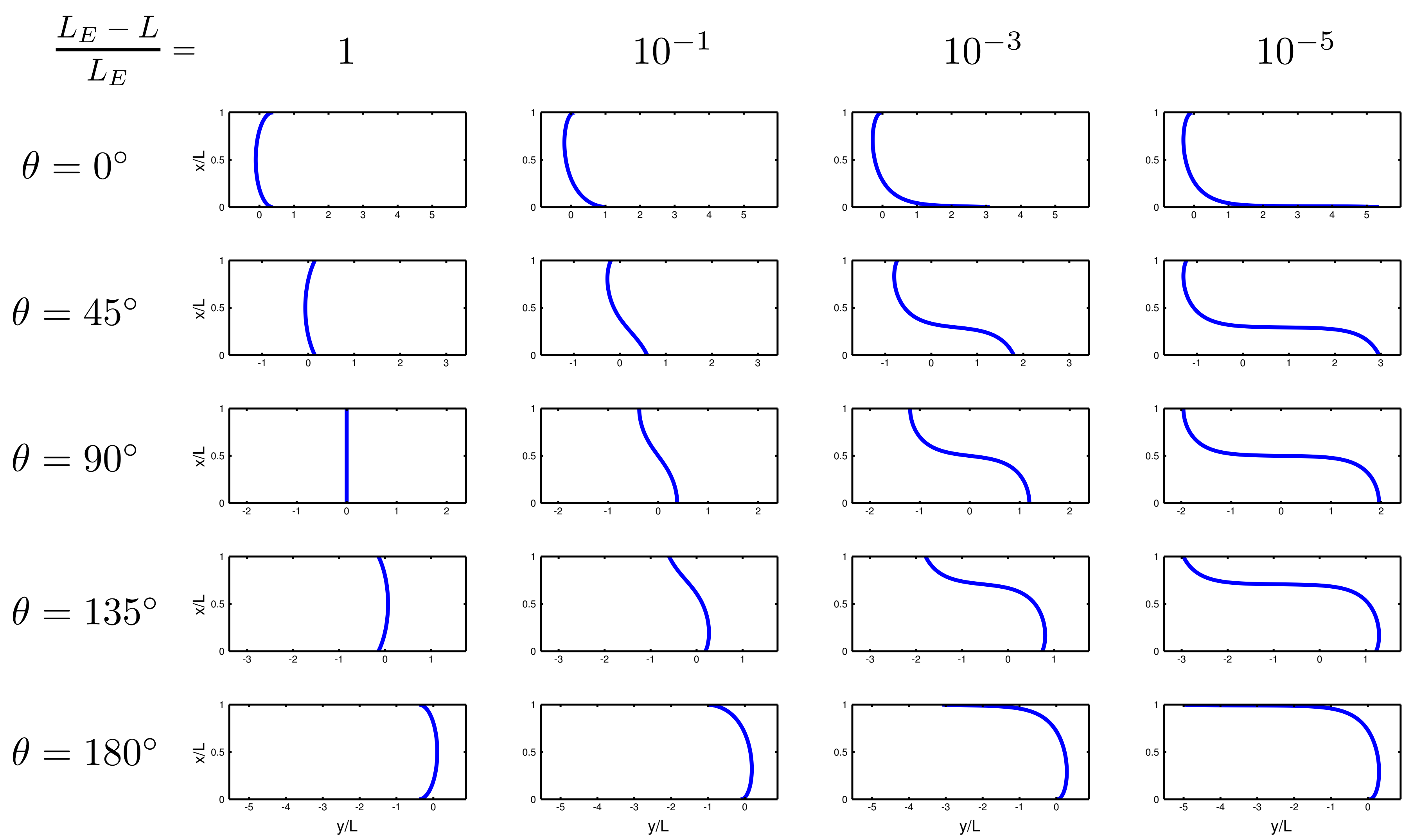
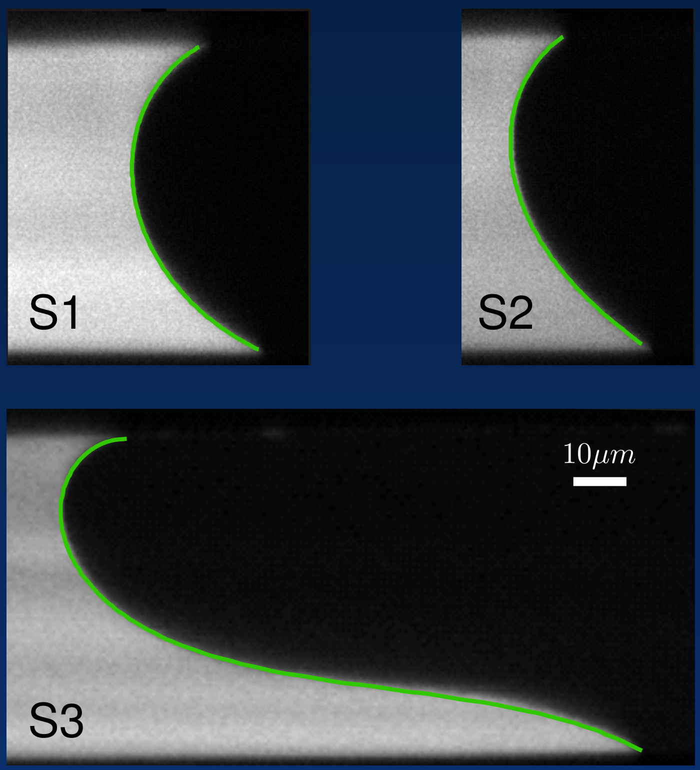


- **Experiments** **(2D Square Capillary)**



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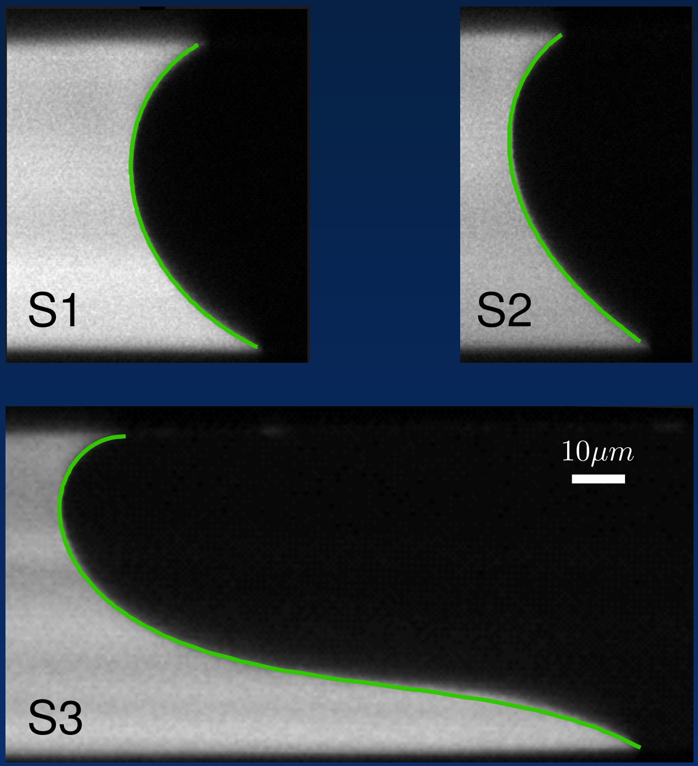
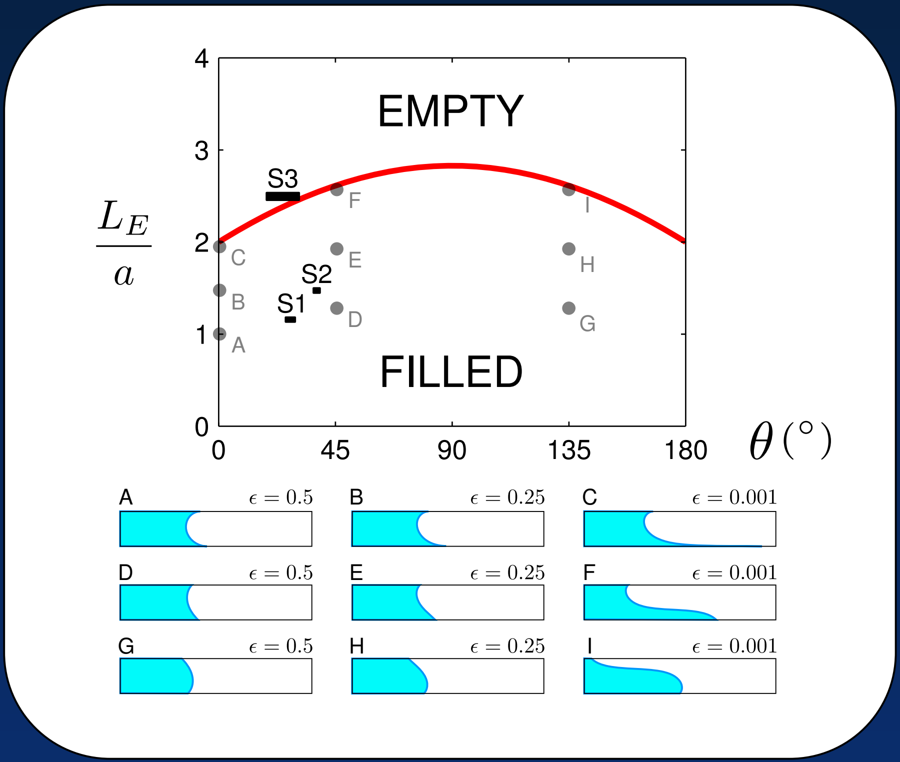
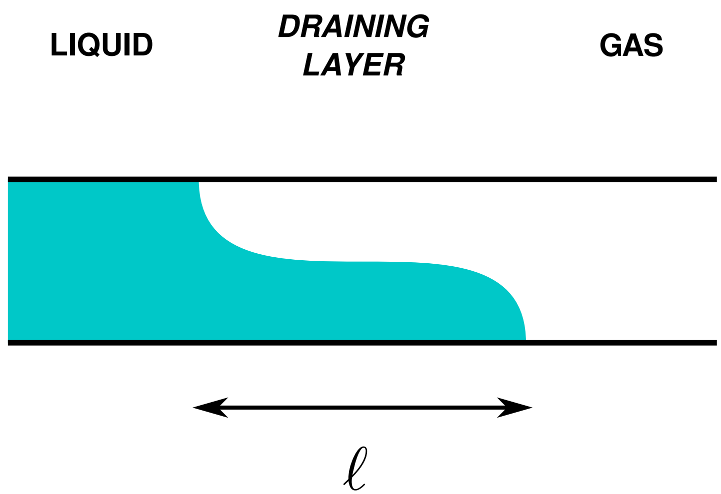




- **Draining as Macroscopic Wetting**



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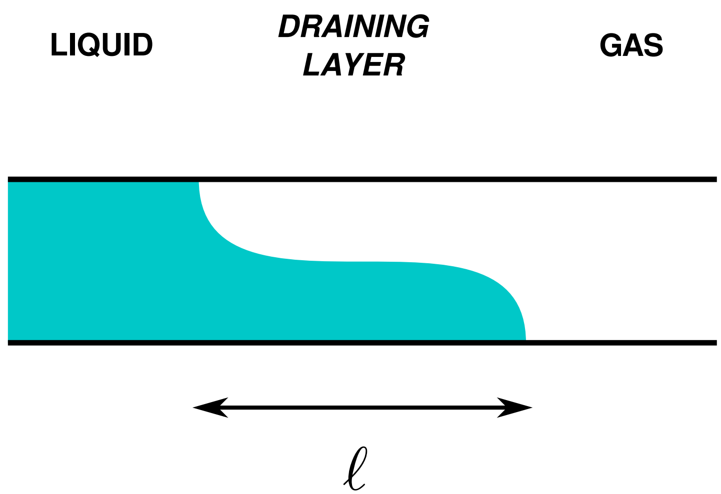

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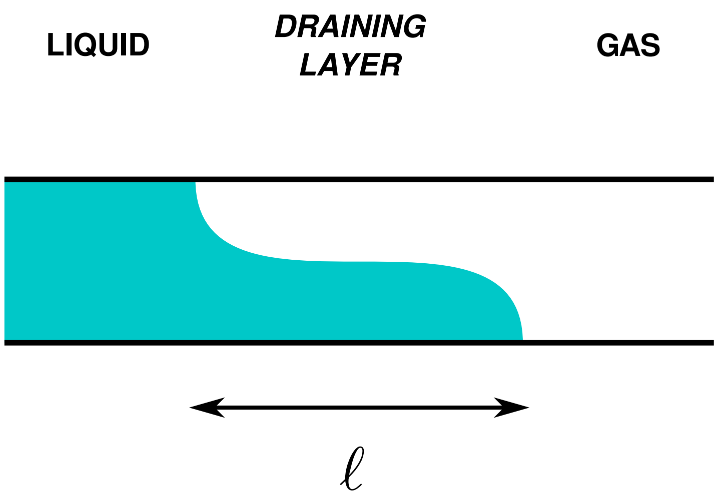

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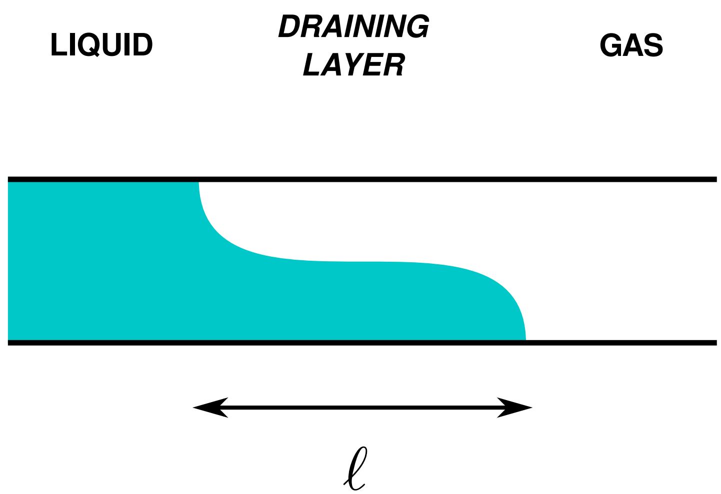

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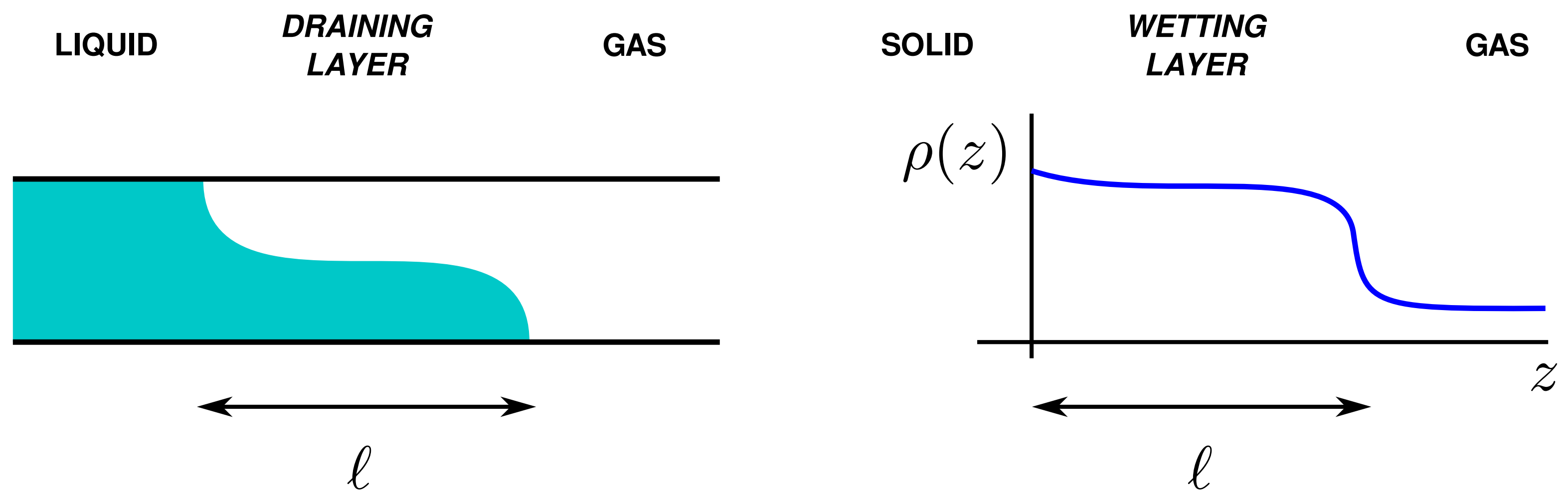

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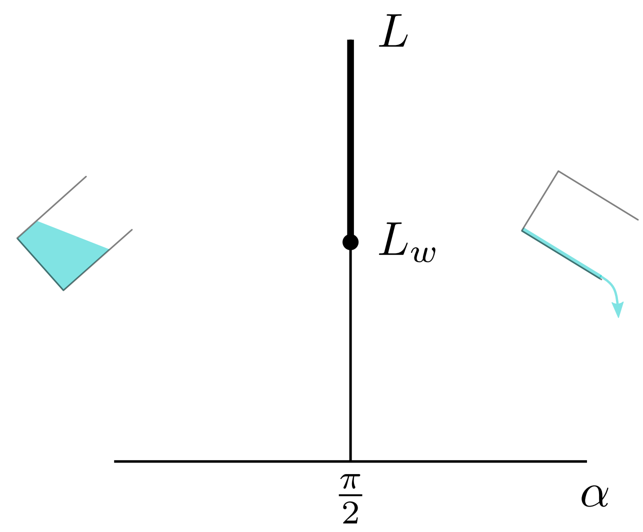

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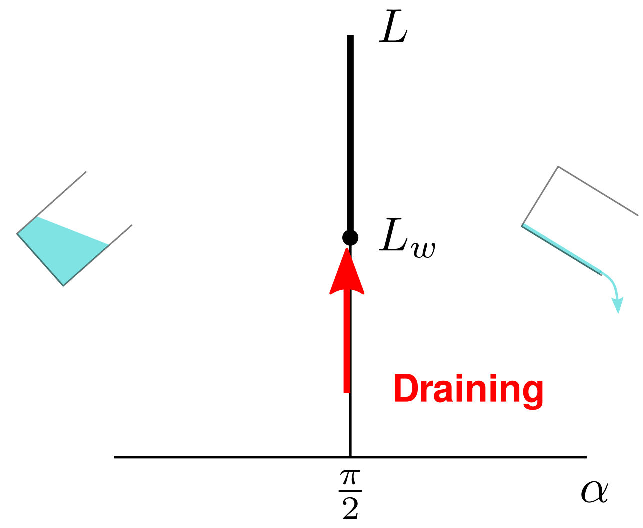

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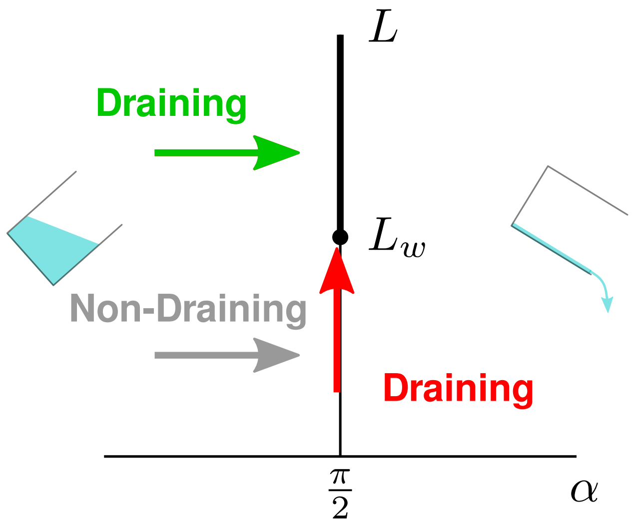

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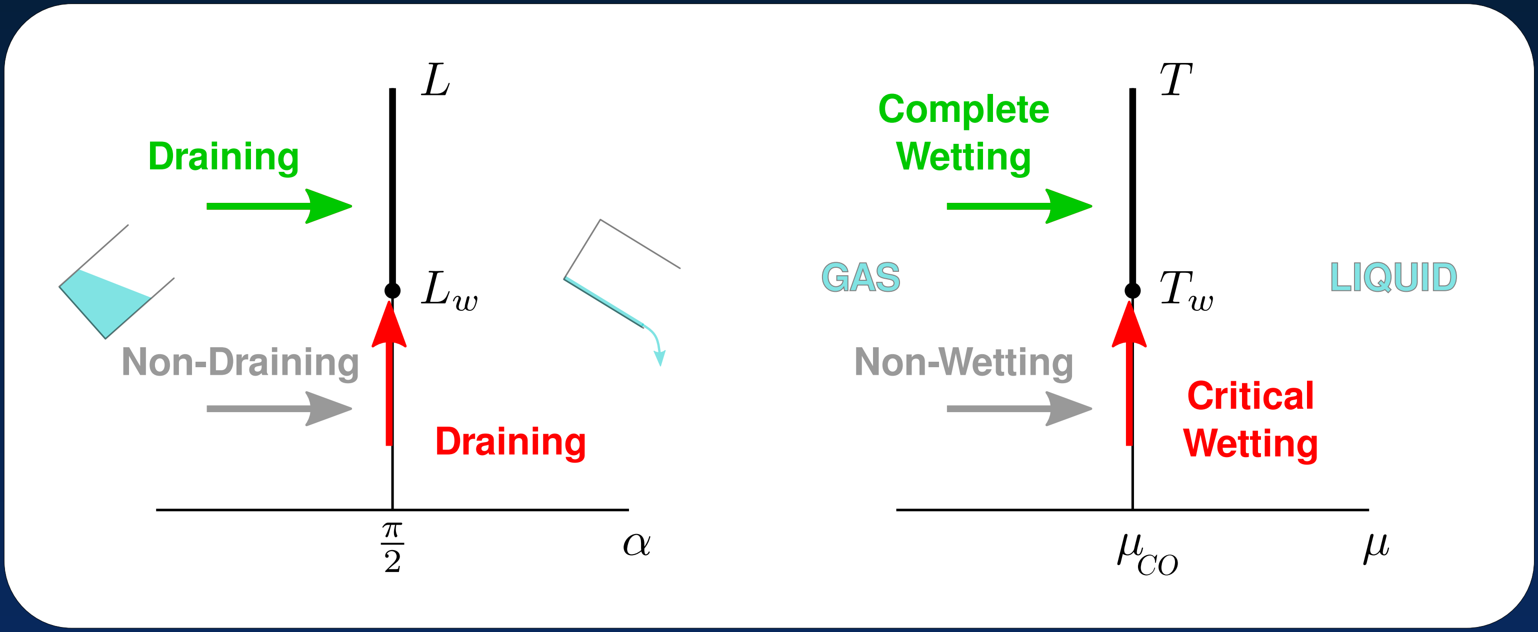

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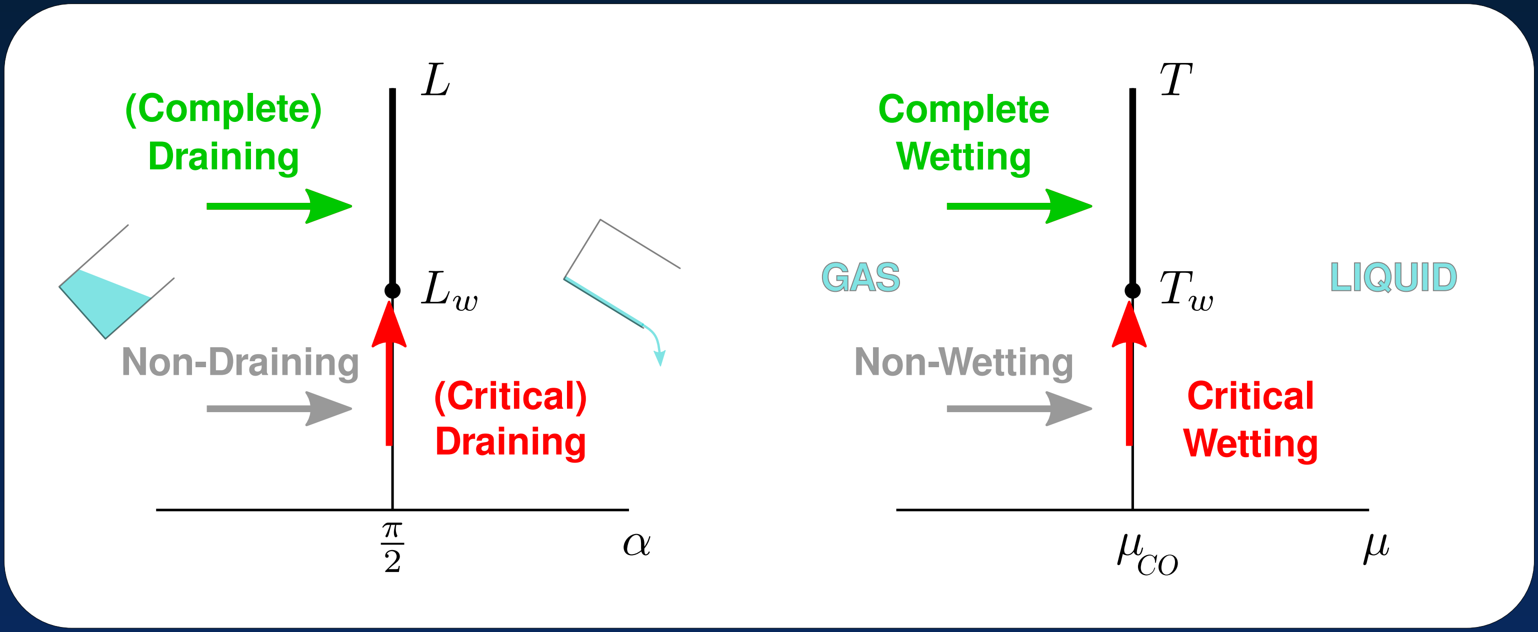

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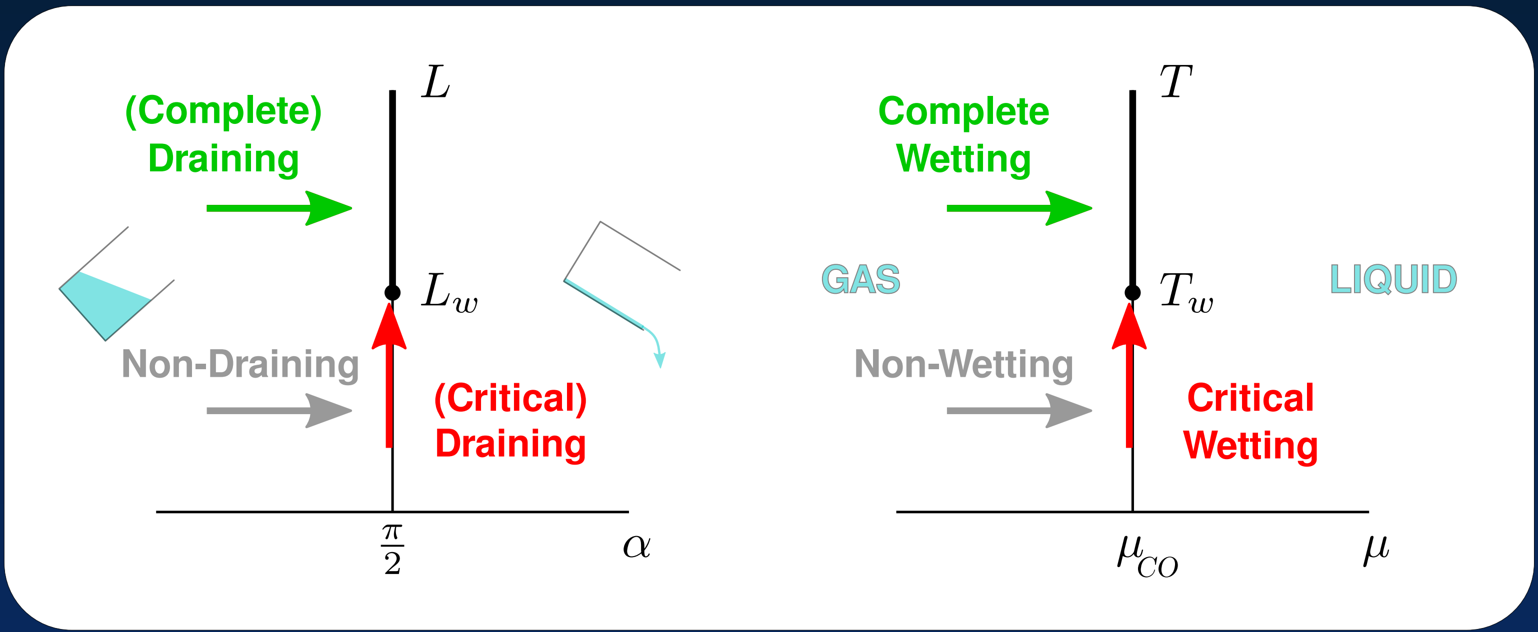

- **Complete (LR):**

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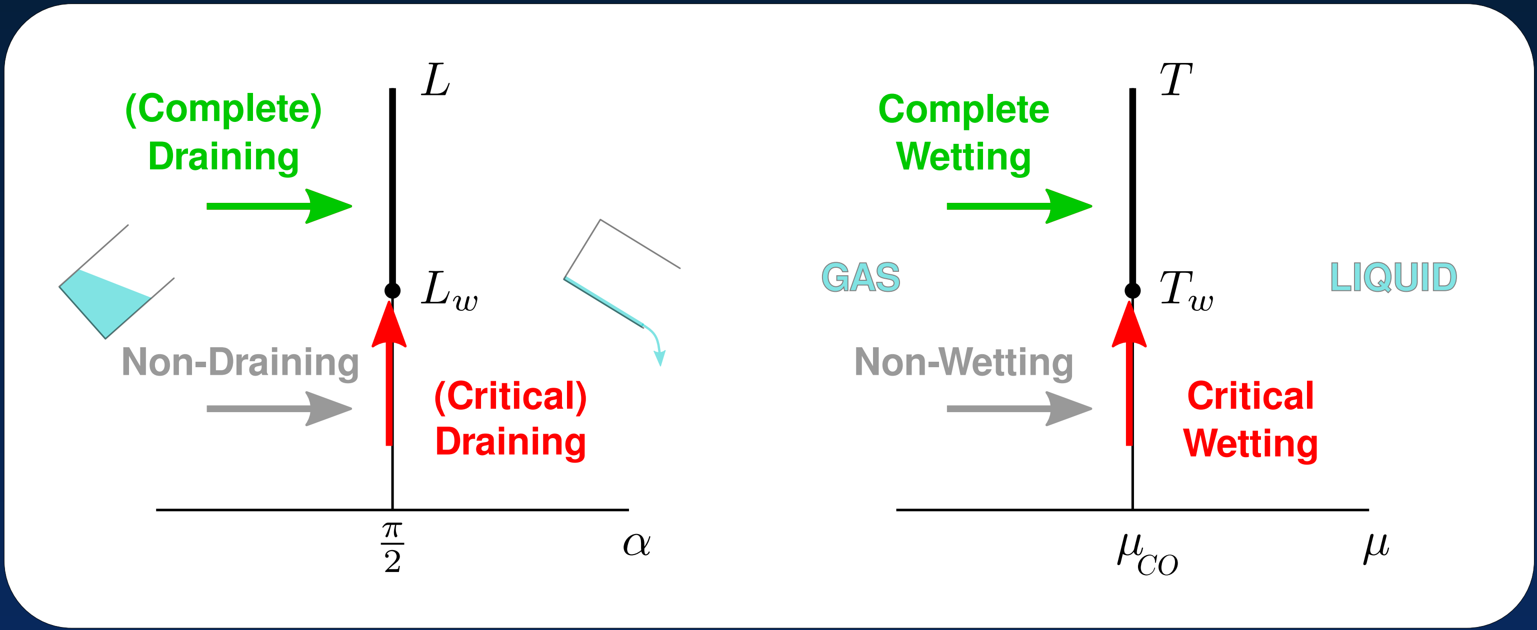

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$$\ell \sim \frac{1}{|\alpha - \frac{\pi}{2}|}$$

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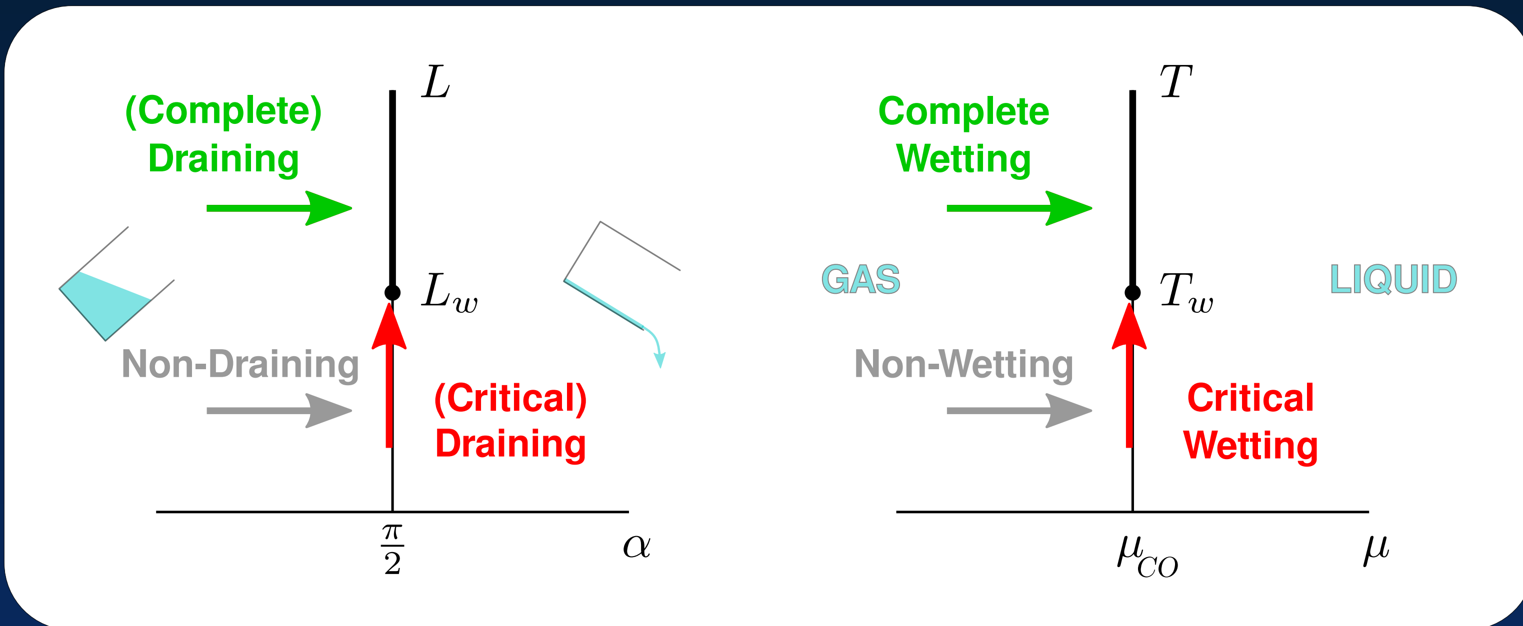

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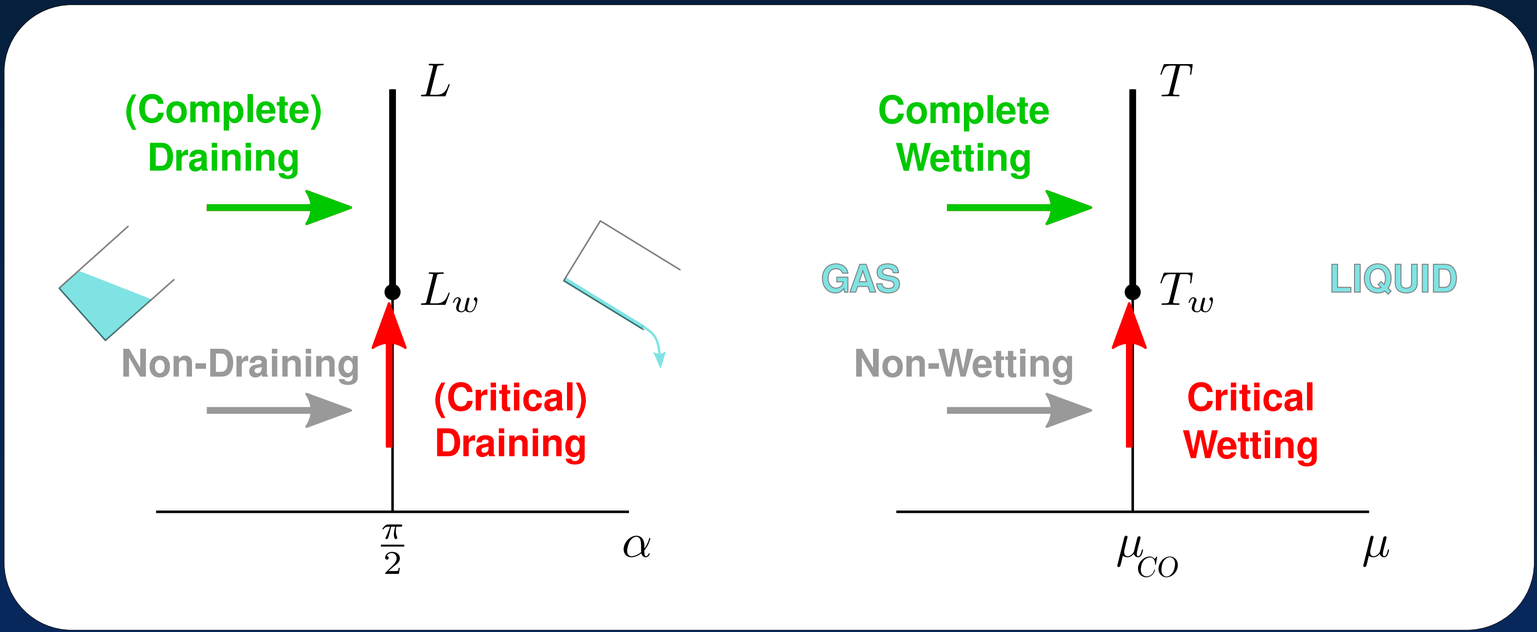

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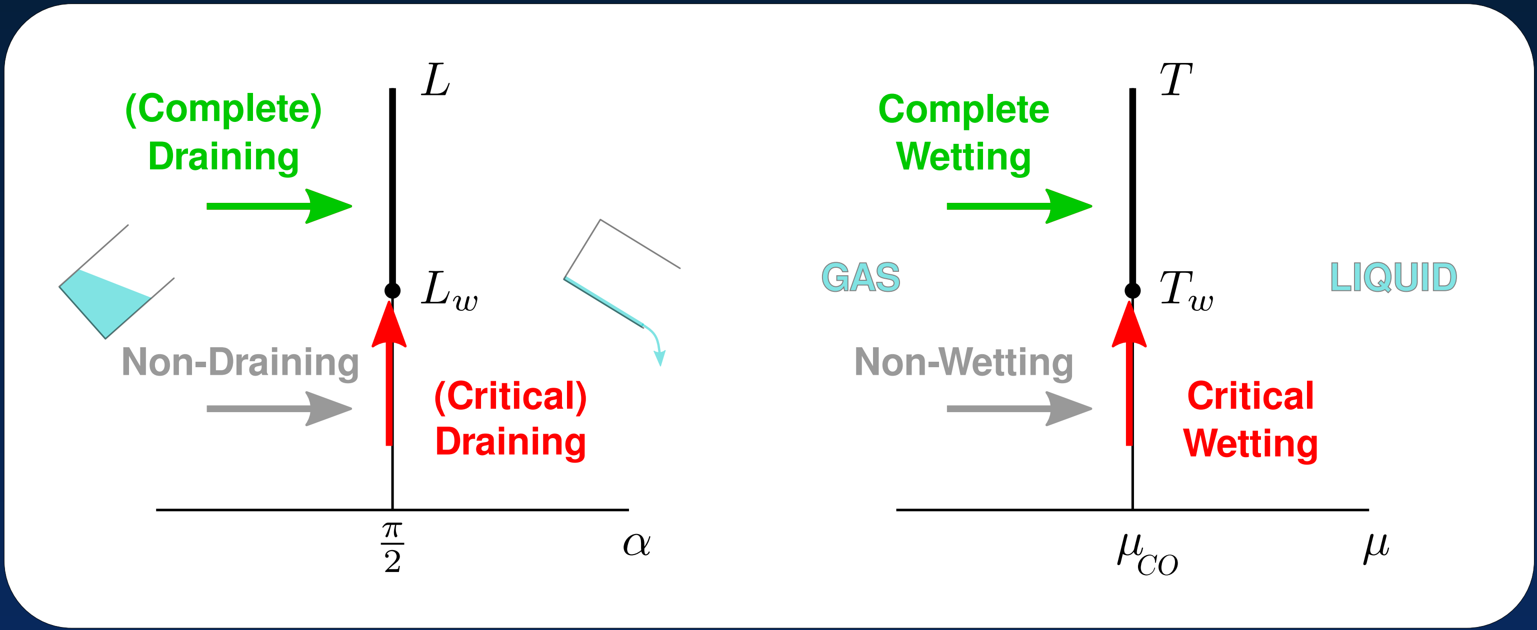

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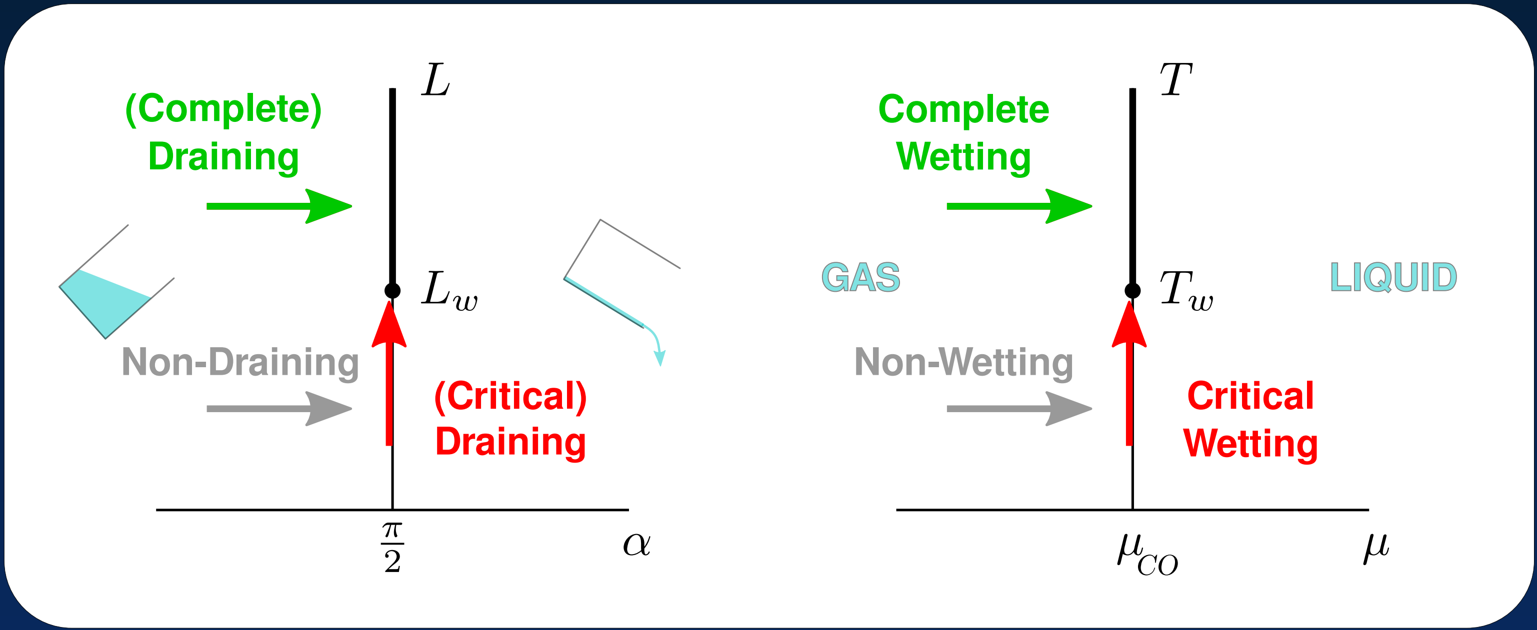

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  - **Long-ranged / Short-ranged**
  - **Many capillary shapes to explore**

THANK YOU FOR YOUR ATTENTION