Casimir Effect between Topological Insulators a proposal for quantum levitation.

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UC3M

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Outline

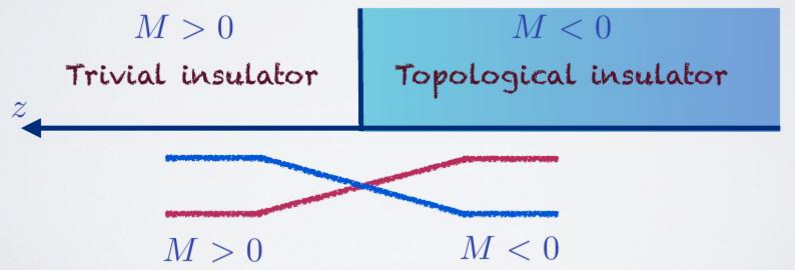
Outline

Topological Insulators

- The two key ideas behind TI (both 2D and 3D):

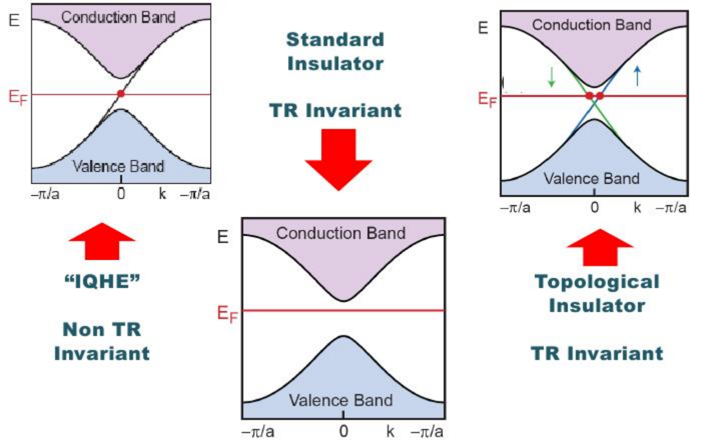
  [Kane & Hasan, Rev. Mod. Phys, **82** 3045 (2010)] [Qi & Zhang, Rev. Mod. Phys. **83**, 1057 (2011)]

  1. Strong spin orbit coupling: Band inversion (mass gap inversion)
  2. Time reversal symmetry: Massless Dirac fermions on the interface



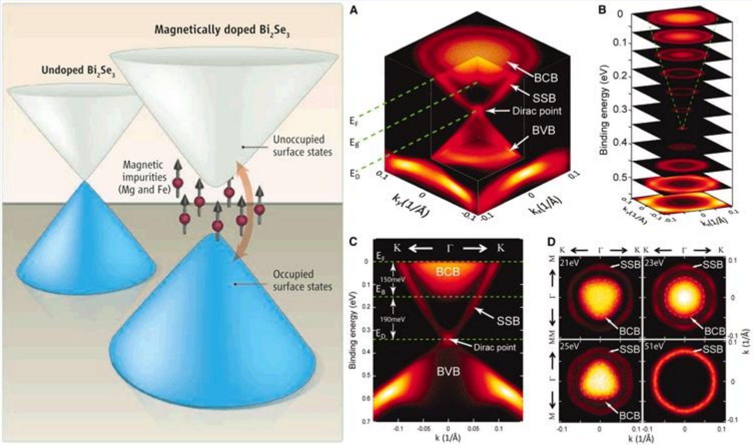
Topological Insulators

- A topological insulator is a quantum state with an energy gap in the bulk but supports robust conductor surface states at the boundaries.



Experiments

- ARPES shows this band structure in experiments. Topological Insulators exist! [Xia et al. Nature Phys. 5, 398–402 (2009)] [Chen et al. Science **329**, 659 (2010)] [Marcel Franz, Science **329**, 639 (2010)]



Topological Insulators from the QFT perspective

1. Start with a 3D T-invariant fermionic action describing an insulator:

$$S = \int dx^3 dt \bar{\Psi} \left(\not{D} - m \right) \Psi + \frac{1}{4\pi} F^{\mu\nu} F_{\mu\nu}$$

2. Perform a chiral transformation $\Psi = e^{i\theta\gamma_5/2}\Psi'$

$$S' = \int dx^3 dt \bar{\Psi}' \left(\not{D} - m e^{i\theta\gamma_5} \right) \Psi' + \frac{1}{4\pi} F^{\mu\nu} F_{\mu\nu}$$

3. $m e^{i\theta\gamma_5} = m \left(\cos(\theta) + i\gamma_5 \sin(\theta) \right)$
4. $m e^{i\theta\gamma_5}$ breaks T symmetry unless $\theta = \{0,\ \pi\} \mod(2\pi)$
5. We have to break T to modify θ. It is topologically protected.

Topological Insulators from the QFT perspective

- θ clasifies T invariant insulators.

$$\theta = 0 \mod(2\pi) \quad \Rightarrow \quad m e^{i\theta\gamma_5} = \quad m > 0$$

Topologically trivial

$$\theta = \pi(2n+1) \quad \Rightarrow \quad m e^{i\theta\gamma_5} = -m < 0$$

Topological Insulator

Topological Insulators from the QFT perspective

- But there is more, the Dirac measure is not invariant under quiral transformation [Fujikawa, PRL **42**, 1195 (1979)] [Hosur et al. PRB **81**, 045120 (2010)] [R Bertlmann "Anomalies in QFT" Oxford science publications]

$$D\Psi^\dagger D\Psi = e^{i\int dx^3 dt \theta \frac{\alpha}{4\pi} F_{\mu\nu}\tilde{F}^{\mu\nu}} D\Psi'^\dagger D\Psi', \text{ where } \tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$$

- The action for a T−invariant Topological Insulator is

$$S_{TI} = \int dx^3 dt \bar{\Psi}' \left(\not{D} - m e^{i\theta\gamma_5} \right) \Psi' + \frac{1}{4\pi} F^{\mu\nu}F_{\mu\nu} + \theta \frac{\alpha}{4\pi} F_{\mu\nu}\tilde{F}^{\mu\nu},$$

- and the EM response of a TI is (integrating out fermions)

$$S_{TI} = \int dx^3 dt \frac{1}{4\pi} F^{\mu\nu}F_{\mu\nu} + \theta \frac{\alpha}{4\pi} F_{\mu\nu}\tilde{F}^{\mu\nu},$$

Axion electrodynamics

- em response of TI

$$\mathcal{L} = \frac{1}{4\pi} F^{\mu\nu} F_{\mu\nu} + \theta \frac{\alpha}{4\pi} F_{\mu\nu} \tilde{F}^{\mu\nu} + \mathcal{J}^{\mu} A_{\mu}$$

$$\mathcal{L} = \frac{1}{2} \left(\epsilon \mathbf{E}^2 + \mu^{-1} \mathbf{B}^2 \right) + \frac{\alpha\theta}{2\pi} \mathbf{E} \cdot \mathbf{B} + \mathcal{J}^{\mu} A_{\mu}$$

- Axion electrodynamics [F. Wilczek, PRL **58**, 1799 (1987)]

$$\boxed{\begin{array}{ll} \vec{\nabla} \cdot \mathbf{E} = \rho - \frac{\alpha}{\pi} \mathbf{B} \cdot \vec{\nabla}\theta & \vec{\nabla} \times \mathbf{E} = -\partial_t \mathbf{B} \\ \vec{\nabla} \cdot \mathbf{B} = 0 & \vec{\nabla} \times \mathbf{B} = \mathbf{J} + \partial_t \mathbf{E} + \frac{\alpha}{\pi} \dot{\theta} \mathbf{B} - \frac{\alpha}{\pi} \mathbf{E} \times \vec{\nabla}\theta \end{array}}$$

  - If θ is constant, electrodynamics do not change.
  - $\dot{\theta} = 0$ imply usual electrodynamics, but with magnetoelectric couplings

$$\boxed{\begin{array}{l} \mathbf{D} = \epsilon \mathbf{E} + \frac{\alpha\theta}{\pi} \mathbf{B} \\ \mathbf{H} = \mu^{-1} \mathbf{B} - \frac{\alpha\theta}{\pi} \mathbf{E} \end{array}}$$

Surface of a Topological Insulator

- Inside the bulk crystal, this action is valid

$$S_{TI} = \int dx^3 dt \frac{1}{2}\left(\epsilon \mathbf{E}^2 + \mu^{-1}\mathbf{B}^2\right) + \frac{\alpha\theta}{2\pi}\mathbf{E}\cdot\mathbf{B} = S_{\text{die}} + S_{\theta},$$

- What happens at the boundary?

$$S_{\theta} = \int dx^3 dt \theta \frac{\alpha}{4\pi} F_{\mu\nu}\tilde{F}^{\mu\nu} = \int dx^3 dt \epsilon^{\mu\nu\rho\sigma}\theta\frac{\alpha}{4\pi}\partial_{\mu}\left(A_{\nu}\partial_{\rho}A_{\sigma}\right),$$

- Stokes theorem gives the boundary contribution.

$$S_{\theta} = \theta\frac{\alpha}{4\pi}\int dx^2 dt \epsilon^{\nu\rho\sigma}\left(A_{\nu}\partial_{\rho}A_{\sigma}\right),$$

- In addition, the action of the Quantum Hall Effect is

$$S_{QHE} = \frac{\sigma_{xy}}{2}\int dx^2 dt \epsilon^{\nu\rho\sigma}\left(A_{\nu}\partial_{\rho}A_{\sigma}\right),$$

$$\theta = (2n+1)\pi \Rightarrow \boxed{\sigma_{xy} = \theta\frac{\alpha}{2\pi} = \frac{e^2}{h}\left(n+\frac{1}{2}\right),}$$

How to build a Topological Insulator?

$$\sigma_{xy} = \theta \frac{\alpha}{2\pi} = \frac{e^2}{h}\left(n + \frac{1}{2}\right),$$

- Hence the axion term is a description of both the bulk and the boundary only when T is broken at the boundary.
- Then we have a recipe to build a Topological Insulator.
- Break T at the surface with a magnetic coating, opening a gap though Zeeman coupling at the surface.
- Magnetic layer to break T and induce a QHE. [Qi **et al**. PRB. **78**, 195424 (2008)]

$$S = \frac{m}{4\pi} \int dx^2 dt \epsilon^{\nu\rho\sigma} A_\nu \partial_\rho A_\sigma = \int dx^3 dt \epsilon^{\mu\nu\rho\sigma} \partial_\mu \left(\frac{\alpha\theta}{2\pi} A_\nu \partial_\rho A_\sigma \right) \quad (1)$$

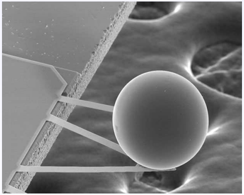
- Then the magnetic layer induces a QHE, equivalent to a TI with

$$\frac{m}{4\pi} = \frac{\sigma_{xy}}{2} = \frac{\alpha\theta}{2\pi} \Rightarrow \boxed{m = 2\alpha\theta} \quad (2)$$

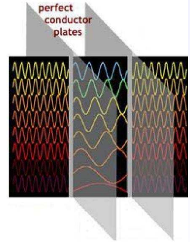
Outline

Historical Introduction

- Vacuum fluctuations of the electromagnetic field induce an attractive force between uncharged parallel plates. [Casimir, 1948. Proc. K. Ned. Akad. Wet. 51, 793]
- It has recently been measured with great precision: [Lamoreaux, 1997. PRL, 78, 5–8] [Mohideen and Roy, 1998. PRL, 81, 4549]



Casimir Calculation



$$E_{vac} = \frac{1}{2}\sum_{\mathbf{k}} \hbar\omega(\mathbf{k}) \qquad \omega(\mathbf{k}) = c|\mathbf{k}|$$

$$E_{plates} = \frac{1}{2}\sum_{n,\mathbf{k}_\perp} \hbar\omega_n(\mathbf{k}_\perp) \qquad \omega_n(\mathbf{k}_\perp) = c\sqrt{\mathbf{k}_\perp^2 + \left(\frac{\pi n}{d}\right)^2}$$

- Non compensation of vacuum density of energy out and between the plates.
- Both vacuum energies diverge, but their difference is finite:

$$\boxed{\langle E\rangle_{plates} - \langle E\rangle_{vac} = -\frac{\hbar c\pi^2}{720 d^3} A}$$

Experiments

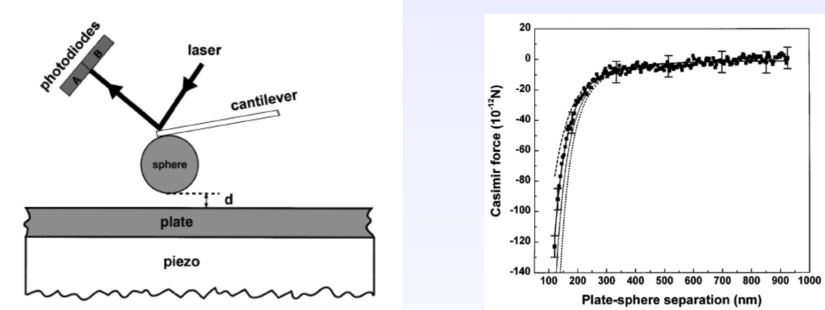


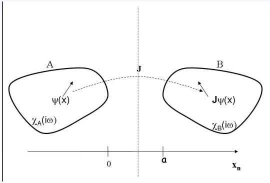
Figure: [Mohideen and Roy, 1998. PRL, 81, 4549]

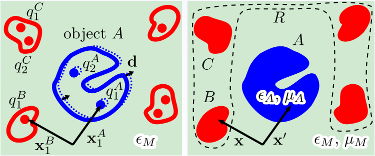
Outline

Looking for Casimir repulsion

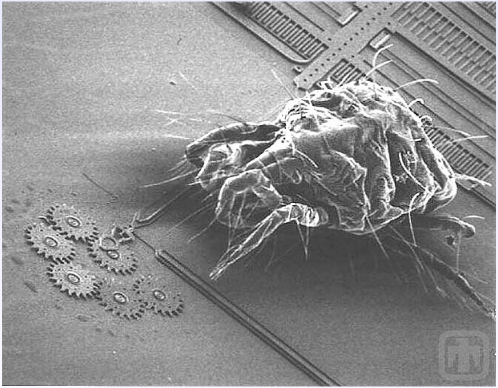
- Why?
  - Casimir interaction between usual dielectrics is **attractive**.
  - A proposal to avoid Stiction phenomena in vacuum
  - New designs for NEM and MEMs
  - Fundamental Physics: Is it possible to obtain repulsion with the Casimir effect?
- Stability theorems
  - Symmetric configurations attract [Kenneth & Klich. PRL **97**, 160401 (2006)]
  - Stable equilibria not accessible for dielectrics in vacuum.

    [Rahi, Kardar & Emig. PRL **105**, 070404 (2010).]



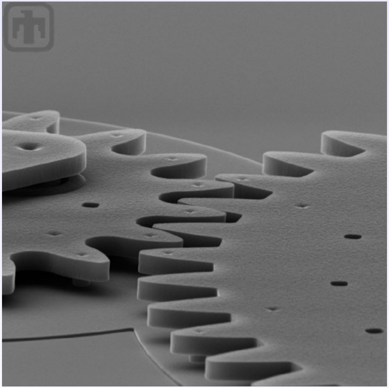


Human scale vs. Nanoscale



Same mechanism
Different scales } $\Rightarrow$ Different behavior!

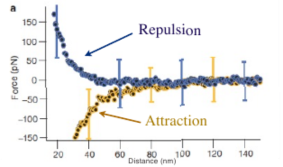
Stiction

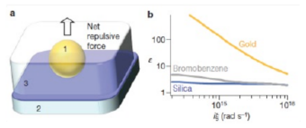


- At nanoscale, the gear sticks.
- The reason: The strong attractive Casimir force between gearwheels.
- Everyone is looking for ways to avoid it.

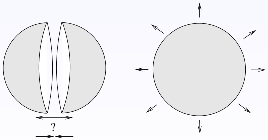
Several proposals to obtain repulsion

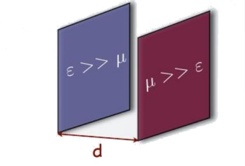
- Lifshitz formula, changes the sign of Casimir effect in dielectric media (proved) [Munday et al. Nature 457, 170-173 (2009)]





- Boyer: Sphere shell formula
- Boyer: dielectric ($\epsilon \to \infty$) vs. perfect magnetic ($\mu \to \infty$) [Boyer, PRA 9, 2078 (1974)]

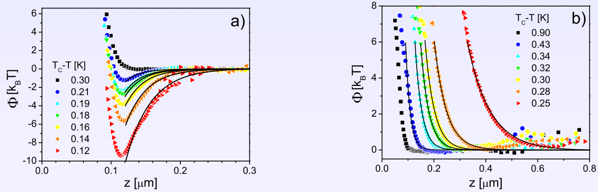




$$E_B(d) = -\frac{7}{8}E_c(d)$$

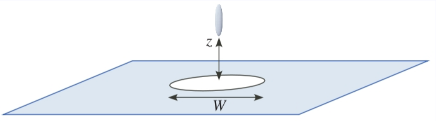
Several proposals to obtain repulsion

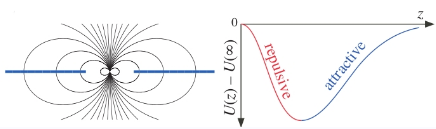
- Repulsion in Critical Casimir effect (proved) [Hertlein et al., Nature **451**, 172 (2008)]



- In some geometric configurations, unstable repulsion between perfect metals

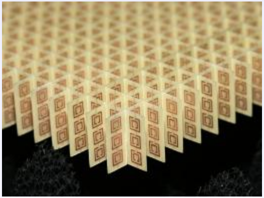
  [Levin et al. PRL **105**, 090403 (2010)]

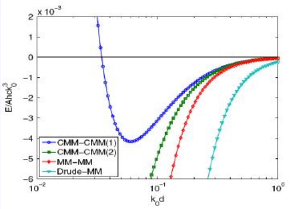




Several proposals to obtain repulsion

- Several proposals with metamaterials seem to give repulsion in vacuum, but several problems remains. [Zhao et al. PRL **103**, 103602 (2009)] [ULeonhardt and Philbin, New J. Phys. **9**, 254 (2007)] [Rosa et al. PRL. **100**, 183602 (2008), PRA **78**, 032117 (2008)] ...





- In this case, the materials are Veselago lensing material for all frequencies.

$$
\begin{aligned}
\mathbf{D} &= \epsilon \mathbf{E} + i\kappa \mathbf{H} \\
\mathbf{B} &= \mu^{-1}\mathbf{H} - i\kappa \mathbf{E}
\end{aligned}
$$

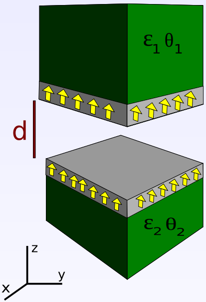
- Striking similarity to the constitutive equations in a 3DTI!

Simple model and em properties of TI

[A. G. Grushin, P. Rodriguez-Lopez and A. Cortijo. PRB **84**, 045119 (2011).] [P. Rodriguez-Lopez. PRB **84**, 165409 (2011).]

- Constitutive relations for Topological Insulators:

$$\begin{cases} \mathbf{D} = \epsilon \mathbf{E} + \alpha\theta/\pi \mathbf{B} \\ \mathbf{B} = \mu^{-1}\mathbf{H} - \alpha\theta/\pi \mathbf{E} \end{cases}$$



- $\mu = \mu_0$
- $\alpha = \frac{e^2}{\hbar c}$
- $\theta = (2n+1)\pi$ for $n \in \mathbb{Z}$
- Oscillator model (Pseudo–Drude) for $\epsilon(k)$:

$$\epsilon(i\kappa) = \epsilon_0 + \sum_i \frac{\omega_{e,i}^2}{\omega_{R,i}^2 + \gamma_{R,i} c\kappa + c^2\kappa^2}.$$

- $w = \frac{\omega_e}{\omega_R}$

$$\epsilon(i\kappa) = \epsilon_0 + \frac{w^2}{1 + \frac{c^2}{\omega_R^2}\kappa^2}.$$

Casimir effect between TIs plates: exact calculations

- Casimir energy for two parallel plates [Dzyaloshinskii, Lifshitz & Pitaevskii, Adv. in Phys. **10**, 38, (1961)] [Rahi et al. PRD **80**, 085021 (2009)]

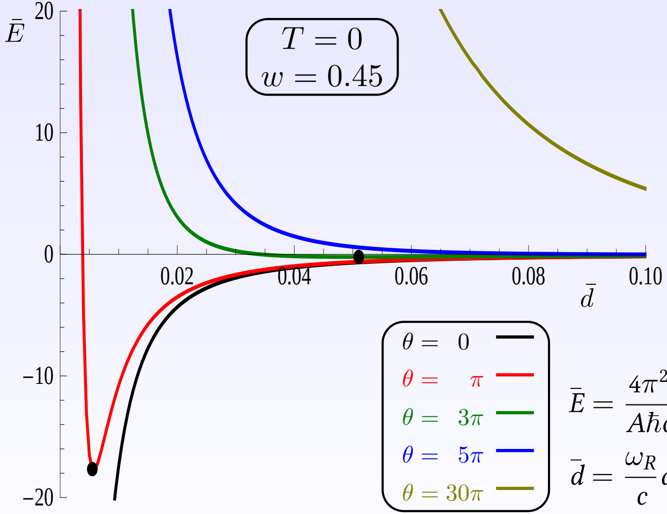
$$E = k_B T {\sum_{n=0}^{\infty}}' \int \frac{d\mathbf{k}_\perp^2}{(2\pi)^2} \log \left| \mathbb{I} - \mathbf{R}_1 \mathbf{R}_2 e^{-2d\sqrt{\kappa_n^2 + \mathbf{k}_\perp^2}} \right|$$

- Fresnel coefficients for an isotropic TI plate at imaginary frequencies [Chang and Yang, PRB **80**, 113304 (2009)] [Grushin, Rodriguez-Lopez and Cortijo. PRB **84**, 045119 (2011).]

$$\mathbf{R}(ic\kappa) = \frac{1}{\Delta} \begin{pmatrix} (\mu k_z - q)(\epsilon k_z + q) - q k_z \mu \bar{\alpha}^2 & 2\mu\bar{\alpha} q k_z \\ 2\mu\bar{\alpha} q k_z & (\mu k_z + q)(\epsilon k_z - q) + q k_z \mu \bar{\alpha}^2 \end{pmatrix}$$

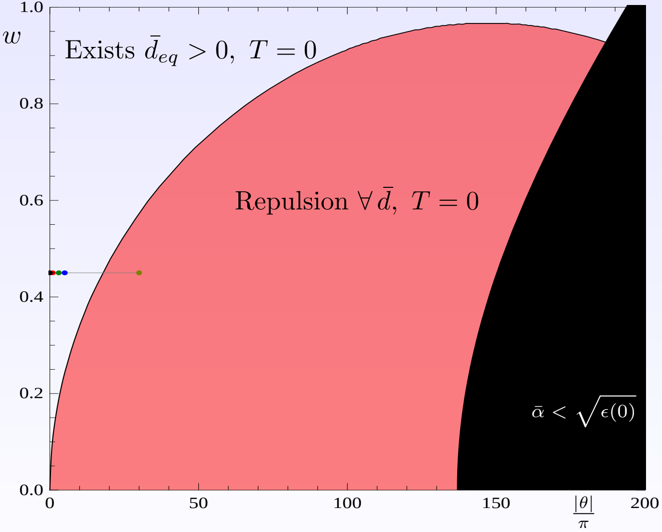
  - $\Delta = (\mu k_z + q)(\epsilon k_z + q) + q k_z \mu \bar{\alpha}^2$
  - $k_z^2 = \kappa^2 + \mathbf{k}_\perp^2$
  - $q^2 = \kappa^2 \mu\epsilon + \mathbf{k}_\perp^2$
  - $\bar{\alpha} = \frac{\alpha\theta}{\pi}$

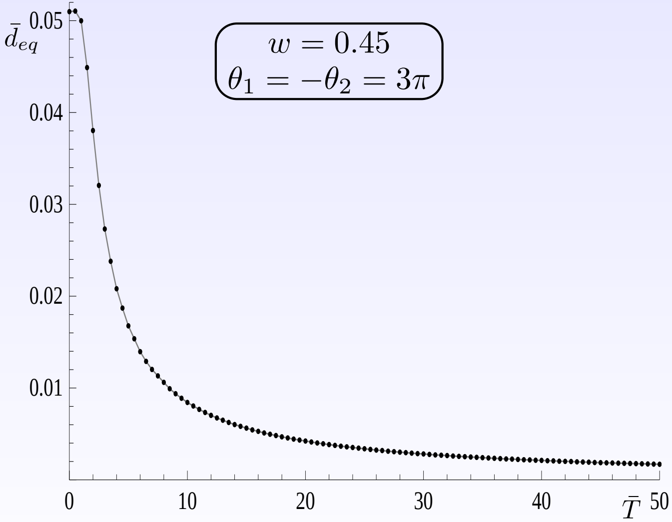
- If $\mathrm{sign}(\theta_1) = \mathrm{sign}(\theta_2)$, Kenneth & Klich theorem imply attraction.
- If $\mathrm{sign}(\theta_1) \neq \mathrm{sign}(\theta_2)$, any theorem constrain the dynamics, thus we can expect any behavior.

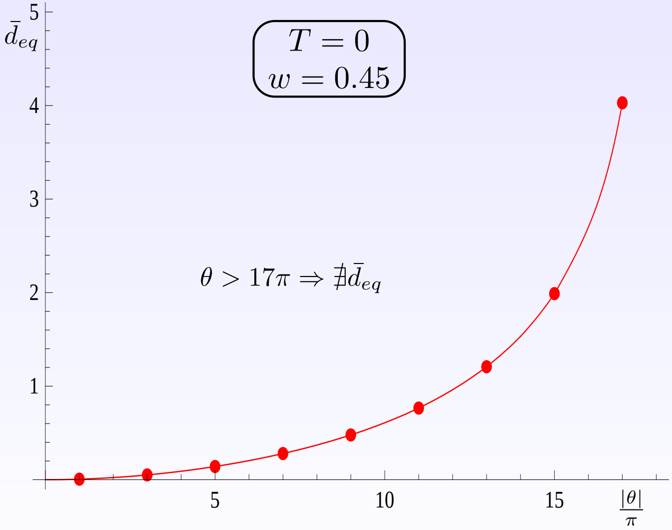


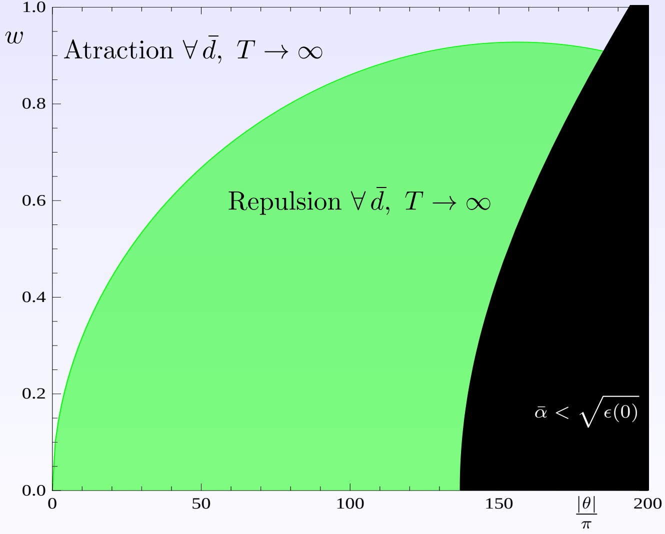
$$\bar{E} = \frac{4\pi^2}{A\hbar c}\left(\frac{\omega_R}{c}\right)^3 E$$

$$\bar{d} = \frac{\omega_R}{c} d$$


1.0
w
Exists $\bar{d}_{eq} > 0, \ T = 0$
0.8
0.6
Repulsion $\forall \, \bar{d}, \ T = 0$
0.4
0.2
$\bar{\alpha} < \sqrt{\epsilon(0)}$
0.0
0
50
100
150
$\frac{|\theta|}{\pi}$
200


$\bar{d}_{eq}$
0.05
0.04
0.03
0.02
0.01
0
10
20
30
40
50
$\bar{T}$
$w = 0.45$
$\theta_1 = -\theta_2 = 3\pi$


$\bar{d}_{eq}$
5
4
3
2
1
$T = 0$
$w = 0.45$
$\theta > 17\pi \Rightarrow \nexists \bar{d}_{eq}$
5
10
15
$\frac{|\theta|}{\pi}$


1.0
0.8
0.6
0.4
0.2
0.0
w
0
50
100
150
200
$\frac{|\theta|}{\pi}$
Atraction $\forall\,\bar{d},\ T\to\infty$
Repulsion $\forall\,\bar{d},\ T\to\infty$
$\bar{\alpha}<\sqrt{\epsilon(0)}$

Outline

Casimir effect between arbitrary shaped TIs: PSA

1. Generalization of PSA to magnetoelectric couplings
2. Why? PSA covers the interesting regime
3. $T = 0$: Demonstration of Repulsion and atraction behaviors
4. $T \to \infty$: Demonstration of Repulsion

Pairwise Summation Approximation

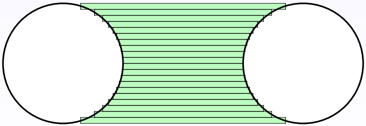
[P. Rodriguez-Lopez. PRE **80**, 061128 (2009).] [A. G. Grushin, P. Rodriguez-Lopez and A. Cortijo. PRB **84**, 045119 (2011).] [P. Rodriguez-Lopez. PRB **84**, 165409 (2011).]

- Approximations to Casimir Interaction: PFA and PSA.
- Could we add some correction terms?
- When PSA is a valid approximation?

Proximity Force Approximation (**PFA**)

$$\frac{d^2E}{dA} = -\frac{\hbar c\pi^2}{720R^3}$$

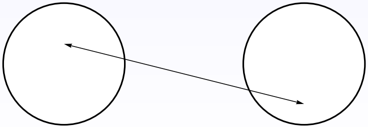
$$E_{PFA} = \int_s dA \frac{d^2E}{dA}$$



Pairwise Summation Approximation (**PSA**)

$$\frac{d^6E}{dV_1 dV_2} = -\frac{23\hbar c}{4\pi R^7}\alpha_1^E \alpha_2^E$$

$$E_{PSA} = \int_1 dV_1 \int_2 dV_2 \frac{d^6E}{dV_1 dV_2}$$



Our approximation to the problem

- $\mathbb{T}_\alpha \approx \mathbb{V}_\alpha$, Valid approximation in the diluted limit $(\epsilon_\alpha \to \epsilon_0, \mu_\alpha \to \mu_0, \alpha_\alpha \to 0, \beta_\alpha \to 0)$

$$\begin{cases} \mathbf{D}_\alpha = \epsilon_\alpha \mathbf{E} + \alpha_\alpha \mathbf{H}, \\ \mathbf{B}_\alpha = \beta_\alpha \mathbf{E} + \mu_\alpha \mathbf{H} \end{cases}$$

$$\mathbb{V}_\alpha = \begin{pmatrix} V_{EE} & V_{EM} \\ V_{ME} & V_{MM} \end{pmatrix} = \begin{pmatrix} \tilde{\epsilon}_\alpha & \alpha_\alpha \\ \beta_\alpha & \tilde{\mu}_\alpha \end{pmatrix} \chi_\alpha(\mathbf{r}) \Rightarrow \chi_\alpha(\mathbf{r}) = \begin{cases} 1 \ \forall \mathbf{r} \in \Omega_\alpha \\ 0 \ \forall \mathbf{r} \notin \Omega_\alpha \end{cases}$$

- $\mathbb{U}_{\alpha\beta} = \mathbb{G}_{0,\alpha\beta}$
- Multiscattering formula for small $\mathbb{N}$ $(\log|\mathbb{1} - \mathbb{N}| \approx -\mathrm{Tr}(\mathbb{N}))$:

$$E = \frac{\hbar c}{2\pi} \int_0^\infty d\kappa \log|\mathbb{1} - \mathbb{N}| \approx -\frac{\hbar c}{2\pi} \int_0^\infty d\kappa \mathrm{Tr}\left(\mathbb{T}_1 \mathbb{U}_{12} \mathbb{T}_2 \mathbb{U}_{21}\right)$$

PSA energy for dielectrics

- Lowest order Born expansion of Casimir energy

$$E_{PSA} = -\frac{\hbar c}{2\pi}\int_0^\infty d\kappa \mathrm{Tr}\left(\mathbb{N}\right) = -\frac{\hbar c}{2\pi}\int_0^\infty d\kappa \mathrm{Tr}\left(\mathbb{V}_1\mathbb{G}_{0,12}\mathbb{V}_2\mathbb{G}_{0,21}\right)$$

- Performing the trace and κ integration, we obtain the PSA energy for 2 objects at $T = 0$ as

$$E = \frac{-\hbar c}{(4\pi)^3}\left[23\tilde{\epsilon}_1\tilde{\epsilon}_2 - 7\tilde{\epsilon}_1\tilde{\mu}_2 - 7\tilde{\mu}_1\tilde{\epsilon}_2 + 23\tilde{\mu}_1\tilde{\mu}_2\right]\int_1\int_2 \frac{d\mathbf{r}_1 d\mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|^7}$$

PSA for two Topological Insulators at $T \to \infty$

$$E_{cl}^{PSA} = -\frac{3k_B T}{(4\pi)^2}\gamma_{cl}\int_1\int_2 \frac{d\mathbf{r}_1 d\mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|^6}$$

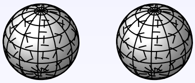
$$\gamma_{cl} = w_1^2 w_2^2 + \bar{\alpha}_1^2 w_2^2 + \bar{\alpha}_2^2 w_1^2 + \bar{\alpha}_1^2\bar{\alpha}_2^2 + 2\bar{\alpha}_1\bar{\alpha}_2$$

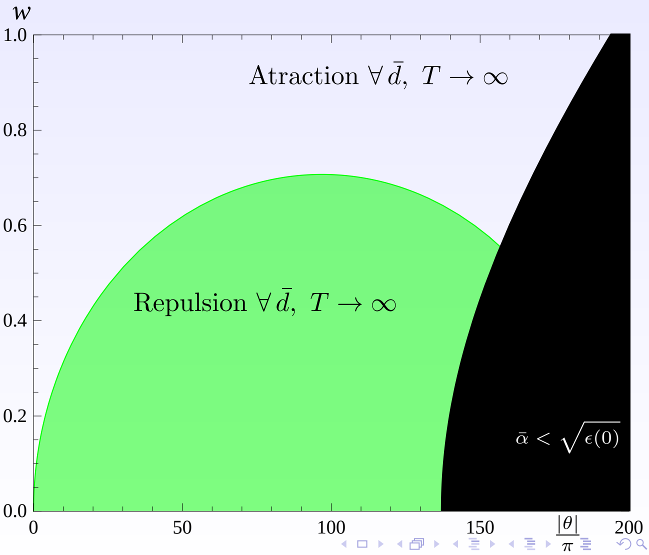
- $\bar{\alpha}_i = \alpha\frac{\theta_i}{\pi}$
- $w_1 = w_2 = w$
- Repulsion ($E > 0$) for all distances if

$$w^2 < \frac{1}{2}\left(-\bar{\alpha}_1^2 - \bar{\alpha}_2^2 + \sqrt{\bar{\alpha}_1^4 + \bar{\alpha}_2^4 - 8\bar{\alpha}_1\bar{\alpha}_2 - 2\bar{\alpha}_1^2\bar{\alpha}_2^2}\right)$$

PSA for two Topological Insulators at $T \to \infty$

$\theta_1 = -\theta_2$





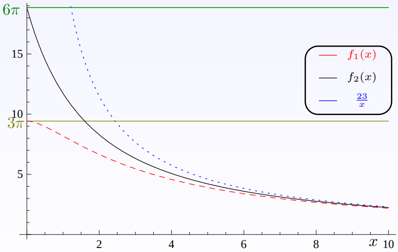
PSA for two Topological Insulators at $T = 0$

- $\bar{\alpha}_i = \alpha\frac{\theta_i}{\pi}$
- $x = \frac{\omega_R}{c}|\mathbf{r}_1 - \mathbf{r}_2|$

$$E_0^{PSA} = -\frac{\hbar c}{(4\pi)^3}\int_1\int_2 \frac{d\mathbf{r}_1 d\mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|^7}\gamma_0(x)$$

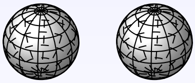
$$\gamma_0(x) = w_1^2 w_2^2 x f_1(x) + 60\bar{\alpha}_1\bar{\alpha}_2 + 23\bar{\alpha}_1^2\bar{\alpha}_2^2 + \left(\bar{\alpha}_1^2 w_2^2 + \bar{\alpha}_2^2 w_1^2\right) x f_2(x)$$

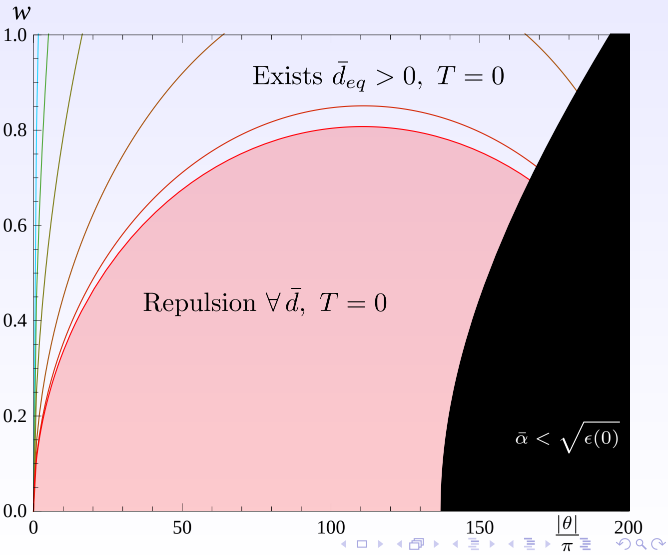
- The condition of repulsion depends on the distance.



PSA for two Topological Insulators at $T = 0$

$\theta_1 = -\theta_2$





Outline

Conclussions

1. Definition and properties of Topological Insulators
2. Stability theorems in Casimir effect
3. Topological Insulators and Casimir effect
    - Exact results between parallel plates.
    - PSA for arbitrary shaped objects.
    - Controlling θ let us control the sign and intensity of the Casimir force.
    - New phenomena: Stable Casimir configurations
    - A proposal: Avoid Stiction with TI?

Casimir Effect between Topological Insulators a proposal for quantum levitation.

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